2025 NHT

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

Write your **student number** in the boxes above. **Letter**

Specialist Mathematics Examination 2

Question and Answer Book

VCE (NHT) Examination – Friday 23 May 2025

- Reading time is **15 minutes**: 2.00 pm to 2.15 pm
- Writing time is **2 hours**: 2.15 pm to 4.15 pm

Approved materials

- Protractors, set squares and aids for curve sketching
- One bound reference
- One approved CAS calculator or CAS software, and one scientific calculator

Materials supplied

- Question and Answer Book of 28 pages
- Formula Sheet
- Multiple-Choice Answer Sheet

Instructions

- Follow the instructions on your Multiple-Choice Answer Sheet.
- At the end of the examination, place your Multiple-Choice Answer Sheet inside the front cover of this book.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

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Section A – Multiple-choice questions

Instructions

- Answer **all** questions in pencil on your Multiple-Choice Answer Sheet.
- Choose the response that is **correct** for the question.
- A correct answer scores 1; an incorrect answer scores 0.
- Marks will **not** be deducted for incorrect answers.
- No marks will be given if more than one answer is completed for any question.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- Take the **acceleration due to gravity** to have magnitude g m s^{-2}, where $g = 9.8$

Question 1

Consider the statement

‘if $n^2 + 5$ is odd, then n is even’, where $n \in Z$.

The contrapositive of this statement is

A. if n is even then $n^2 + 5$ is odd.

B. if $n^2 + 5$ is even then n is even.

C. if $n^2 + 5$ is even then n is odd.

D. if n is odd then $n^2 + 5$ is even.

Question 2

The number of straight-line asymptotes of the graph of the function with

rule $f(x) = \dfrac{2x^4 - 6x^3 - x + 3}{x^2 + 2x - 15}$ is

A. 0

B. 1

C. 2

D. 3

Question 3

How many integer values of a can $\dfrac{1}{x^2 + ax - 21}$ be expressed in the form $\dfrac{B}{x+b} + \dfrac{C}{x+c}$,

where b and c are integers and $B, C \in R$?

A. 2

B. 3

C. 4

D. 5

Question 4

Consider the complex numbers z and w, where $w = -1 + \sqrt{3}i$, $|zw| = 2\sqrt{2}$ and $\text{Arg}(zw) = -\dfrac{7\pi}{12}$.
The value of $\text{Arg}(z^3)$ is

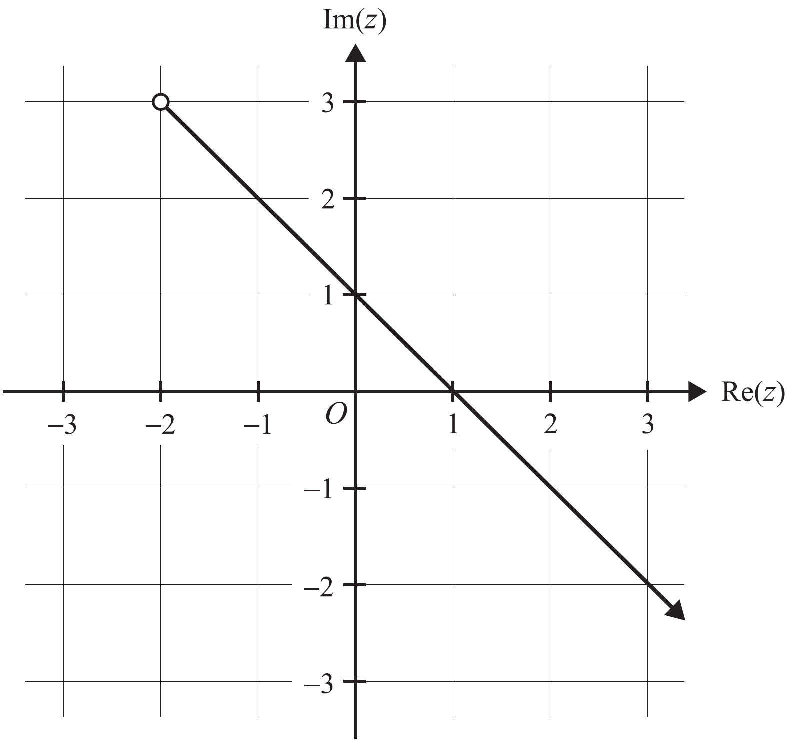
A. $-\dfrac{3\pi}{4}$

B. $-\dfrac{\pi}{4}$

C. $\dfrac{\pi}{4}$

D. $\dfrac{3\pi}{4}$

Question 5



The equation of the ray on the Argand plane above is

A. $\text{Arg}(z - 2 + 3i) = \dfrac{3\pi}{4}$

B. $\text{Arg}(z + 2 - 3i) = -\dfrac{\pi}{4}$

C. $\text{Arg}(z + 2 - 3i) = \dfrac{3\pi}{4}$

D. $\text{Arg}(z - 2 + 3i) = -\dfrac{\pi}{4}$

Question 6

Euler's method is applied to $\frac{dy}{dx} = e^x$ with initial values $(0, y_0)$.

For step size h, the value of y_3 will be $\left(e^h + \frac{1}{2}\right)^2 h$ when

A. $y_0 = -\frac{3}{4}h$

B. $y_0 = -\frac{1}{2}h$

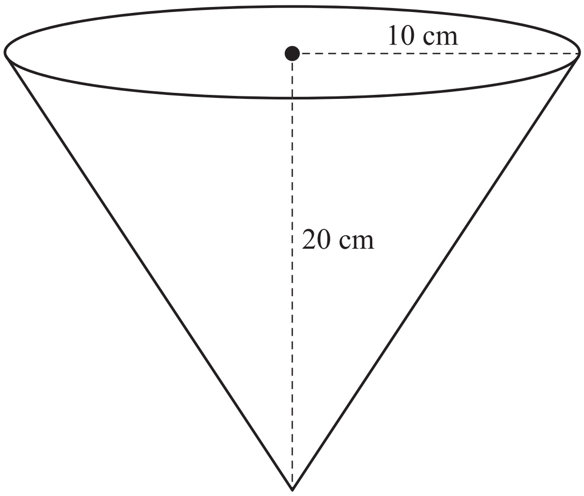
C. $y_0 = \frac{1}{4}h$

D. $y_0 = \frac{1}{2}h$

Question 7

Water is poured at a rate of 5 $\text{cm}^3\,\text{s}^{-1}$ into an inverted right circular cone of height 20 cm and radius 10 cm, as shown below.

Water flows through a hole at the vertex of the cone at a rate of $k\sqrt{h}$ $\text{cm}^3\,\text{s}^{-1}$, where h cm is the depth of water in the cone at time t seconds, and k is an arbitrary positive constant.



The rate at which the depth of water is increasing when the depth of water is h cm is

A. $\dfrac{5 - k\sqrt{h}}{3\pi h^2}$

B. $\dfrac{5 - k\sqrt{h}}{4\pi h^2}$

C. $\dfrac{4\left(5 - k\sqrt{h}\right)}{\pi h^2}$

D. $\dfrac{5 - k\sqrt{h}}{\pi h^3}$

Question 8

Using the substitution $x = \tan(\theta)$, then $\int_{-1}^{1} \frac{1}{(1+x^2)^2}\, dx$ is equivalent to

A. $\int_{0}^{\frac{\pi}{4}} \frac{1}{\sec^2(\theta)}\, d\theta$

B. $\int_{0}^{\frac{\pi}{4}} 2\cos^2(\theta)\, d\theta$

C. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 2\cos^4(\theta)\, d\theta$

D. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{\sec^4(\theta)}\, d\theta$

Question 9

The length of the curve defined by $x = e^t \sin(t)$ and $y = e^t \cos(t)$, where $t \in \left[0, \frac{\pi}{2}\right]$, can be found by evaluating

A. $\sqrt{2}\int_{0}^{\frac{\pi}{2}} e^{\sqrt{2}t}\, dt$

B. $2\int_{0}^{\frac{\pi}{2}} e^{2t}\, dt$

C. $\sqrt{2}\int_{0}^{\frac{\pi}{2}} e^{t}\, dt$

D. $\int_{0}^{\frac{\pi}{2}} e^{t}\, dt$

Question 10

The curve given by $y = \frac{1}{3}x^3$, where $x \in [0, 1]$, is rotated about the x-axis to form a surface of revolution.

The area of this surface is

A. $\frac{2\sqrt{2}\pi}{9}$

B. $\frac{\pi}{9}\left(2\sqrt{2}-1\right)$

C. $\frac{\pi}{18}\left(2\sqrt{2}-1\right)$

D. $\frac{4\pi}{9}\left(2\sqrt{2}-1\right)$

Question 11

A drone is rising vertically. When it reaches a height of h metres above the ground it is travelling at a speed of u metres per second. At height h metres the drone stops working and it subsequently falls to the ground.

Assuming air resistance is negligible, the time, in seconds, taken for the drone to fall to the ground after it stops working is given by

A. $\frac{\sqrt{2h}}{g}$

B. $\frac{\sqrt{2h}}{g} + \frac{2u}{g}$

C. $-\frac{u}{g} + \frac{\sqrt{u^2 + 2gh}}{g}$

D. $\frac{u}{g} + \frac{\sqrt{u^2 + 2gh}}{g}$

Question 12

An object starts with an initial velocity of 4 m s^{-1} at time $t = 0$ seconds and moves in a straight line from a fixed point O.
At time t seconds its displacement is x metres from O and its velocity is v m s^{-1} where $\frac{dv}{dx} = \frac{4 + v^2}{2v^2}$.

The object's velocity, in metres per second, at time $t = \log_e(10)$ is

A. $\sqrt{6}$

B. 6

C. 14

D. 196

Question 13

Consider two vectors, $\underset{\sim}{a}$ and $\underset{\sim}{b}$, where $\underset{\sim}{a} = -2\underset{\sim}{i} + 3\underset{\sim}{j} + m\underset{\sim}{k}$, $m \in R^+$ and $\underset{\sim}{b}$ is an arbitrary non-zero vector.

The angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is $\frac{\pi}{3}$ and the scalar resolute of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$ is 4 units.

The value of m is

A. $\frac{5\sqrt{3}}{3}$

B. $\sqrt{51}$

C. $\frac{8\sqrt{3} - 3}{3}$

D. $2\sqrt{2}$

Question 14

Consider the vectors

$\underset{\sim}{a} = \underset{\sim}{i} - p\underset{\sim}{j}$ and $\underset{\sim}{b} = p\underset{\sim}{i} - 4\underset{\sim}{j}$, where $p \in R$, and $\underset{\sim}{a}$ is parallel to $\underset{\sim}{b}$.

The magnitude of p is

A. 1

B. 2

C. 3

D. 4

Question 15

A cricket ball is hit from an origin O at ground level, on a horizontal oval. Its position at time t seconds is given by $\underset{\sim}{r}(t) = 12t\,\underset{\sim}{i} + \left(19.6t - 4.9t^2\right)\underset{\sim}{j}$, where $t \geq 0$, $\underset{\sim}{i}$ is a unit vector in the forward direction and $\underset{\sim}{j}$ is a unit vector vertically up. Displacement components are measured in metres.

A fielder is located at the position $60\underset{\sim}{i}$ when the ball is hit.

The distance, in metres, the fielder must run to catch the descending ball at a height of 1.5 m above ground level is closest to

A. 11

B. 12

C. 13

D. 14

Question 16

Consider two vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ such that $|\underset{\sim}{a}| > 0$, $|\underset{\sim}{b}| > 0$ and $\underset{\sim}{a} \cdot \underset{\sim}{b} = 0$.

The expression $(\underset{\sim}{a} \times \underset{\sim}{b}) \cdot (\underset{\sim}{b} \times \underset{\sim}{a})$ is equivalent to

A. $-|\underset{\sim}{a}||\underset{\sim}{b}|$

B. $-|\underset{\sim}{a}|^2 |\underset{\sim}{b}|^2$

C. $|\underset{\sim}{a}||\underset{\sim}{b}|$

D. $|\underset{\sim}{a}|^2 |\underset{\sim}{b}|^2$

Question 17

Two sides of a triangle are spanned by the vectors $2\underset{\sim}{i} + \underset{\sim}{j} - 3\underset{\sim}{k}$ and $\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$.

The area of this triangle is

A. $\dfrac{5\sqrt{3}}{2}$

B. $5\sqrt{3}$

C. 5

D. $\dfrac{3\sqrt{10}}{2}$

Question 18

For two non-parallel planes Π_1 and Π_2, the cross product of their respective normal vectors $\underset{\sim}{n}_1$ and $\underset{\sim}{n}_2$

A. will be parallel to both Π_1 and Π_2.

B. is the vector resolute of $\underset{\sim}{n}_2$ perpendicular to $\underset{\sim}{n}_1$.

C. forms a linearly dependent set with $\underset{\sim}{n}_1$ and $\underset{\sim}{n}_2$.

D. will be perpendicular to both Π_1 and Π_2.

Question 19

W is a random variable such that $W = aX + bY$, where X and Y are independent random variables and $a, b \in R$.

Given that $\text{E}(X) = 1$, $\text{Var}(X) = 1$, $\text{E}(Y) = 2$, $\text{Var}(Y) = 2$, $E(W) = 1$ and $\text{Var}(W) = 6$, then b satisfies

A. $2b^2 - b - \frac{35}{4} = 0$

B. $18b^2 - 8b - 5 = 0$

C. $8b^2 - 4b - 5 = 0$

D. $6b^2 - 4b - 5 = 0$

Question 20

Certain electronic components have resistances that can be assumed to be normally distributed with a mean of 1000 ohms and a standard deviation of 20 ohms. Two independent random samples of 25 of these components are taken and the sample mean for each is calculated.

What is the probability, correct to four decimal places, that the two sample means differ by less than 5 ohms?

A. 0.1403

B. 0.3116

C. 0.6232

D. 0.7887

End of Section A

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Examination continues on the next page.

Section B

Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- Unless otherwise specified, an **exact** answer is required to a question.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- Take the **acceleration due to gravity** to have magnitude g m s^{-2}, where $g = 9.8$

Question 1 (10 marks)

Consider the family of functions f with rule $f(x) = \dfrac{x^3 + ax^2 + ax + 3}{x^2 - 1}$, where a is a real constant.

a. Write down the implied domain of f given that $a \neq -2$. 1 mark

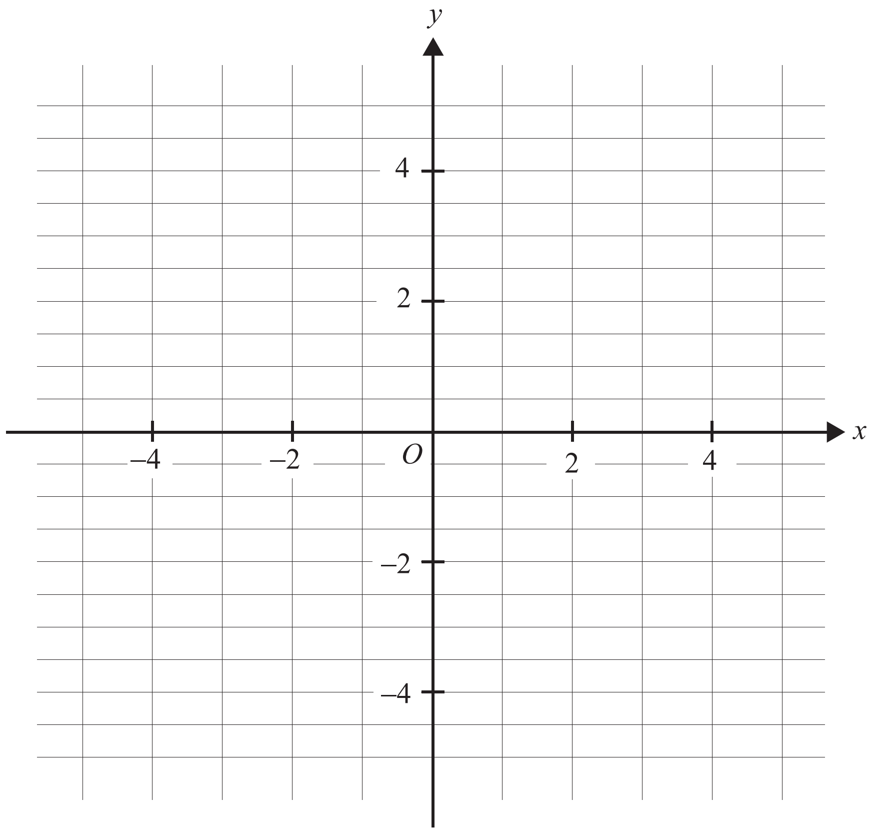
b. The rule for f can be written as $f(x) = x + a + g(x)$.

i. Using partial fractions, show that $g(x) = \dfrac{a+2}{x-1} - \dfrac{1}{x+1}$. 2 marks

ii. Show that when $a = -2$ the graph of $y = f(x)$ has no stationary points. 1 mark

c. Sketch the graph of $y = f(x)$ when $a = -2$ on the axes below.

Label all asymptotes with their equations, and label any intercepts with the coordinate axes with their values. 3 marks



Question 1 continues on the next page.

d. **i.** Using $f(x) = x + a + \frac{a+2}{x-1} - \frac{1}{x+1}$, find $f''(x)$ where a is an arbitrary real constant.

Give your answer in the form $f''(x) = \frac{A(a+2)}{(x-1)^p} + \frac{B}{(x+1)^p}$ where A, B and $p \in R$. 1 mark

ii. Hence, verify that the graph of $y = f(x)$ has no points of inflection when $a = -2$ and $a = -1$. 2 marks

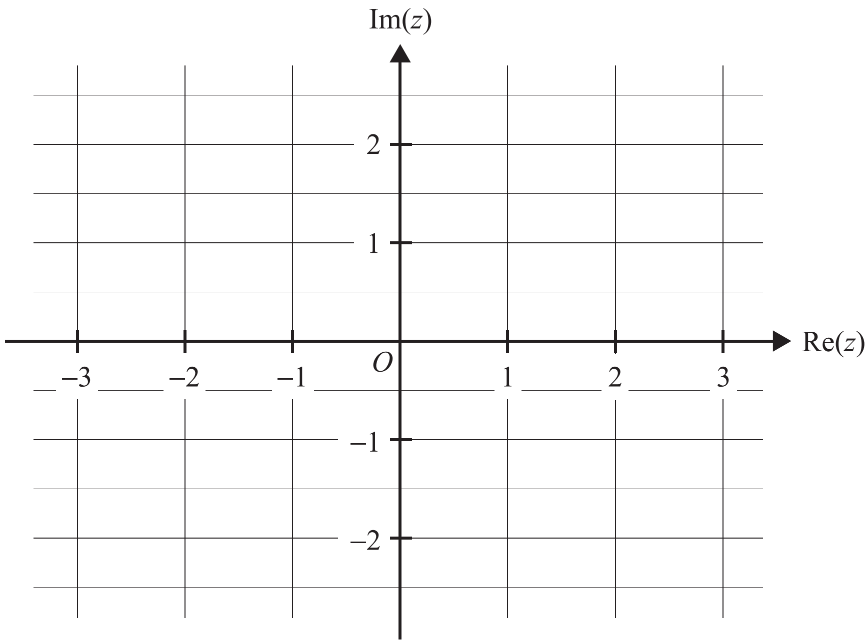
Question 2 (10 marks)

a. Show that the relation $|z - 2i| = \text{Im}(z + i)$ can be expressed in Cartesian form as $6y = x^2 + 3$, where $z = x + iy,\ x, y \in R$. 1 mark

Question 2 continues on the next page.

b. Sketch the graph of the relation $|z - 2i| = \text{Im}(z + i)$ on the Argand diagram below for $-3 \le \text{Re}(z) \le 3$.

Label the endpoints with their coordinates. 1 mark



c. A ray from the origin O is tangential to the part of the curve where $\text{Re}(z) > 0$.

i. Sketch the ray on the Argand diagram above. 1 mark

ii. Find the equation of this ray in Cartesian form. 2 marks

iii. Find the point of intersection of the ray with the curve.

Express your answer in the form $a + ib$, where $a, b \in R$. 1 mark

d. Sketch the graph of the relation $|\overline{z} - 2i| = \text{Im}(\overline{z} + i)$, where $z = x + iy$, $x, y \in R$, on the Argand diagram provided in **part b**, labelling it with its Cartesian equation. 1 mark

e. **i.** The line that passes through the points $z_1 = -1$ and $z_2 = 3 + 2i$ has the equation $|z| = |z - (c + id)|$, where $c, d \in R \backslash \{0\}$.

Find the values of c and d. 1 mark

ii. Find the area of the region enclosed by the curves $|z| = |z - (c + id)|$ and $|z - 2i| = \text{Im}(z + i)$. 2 marks

Question 3 (10 marks)

Let $I_3(x) = \int \left(\sec^3(x)\right) dx$

$= \int \left(\sec^2(x)\ \sec(x)\right) dx$.

a. Use integration by parts to find an expression for $I_3(x)$ involving $\sec(x)$, $\tan(x)$ and $I_1(x)$, where $I_1(x) = \int \left(\sec(x)\right) dx$. 3 marks

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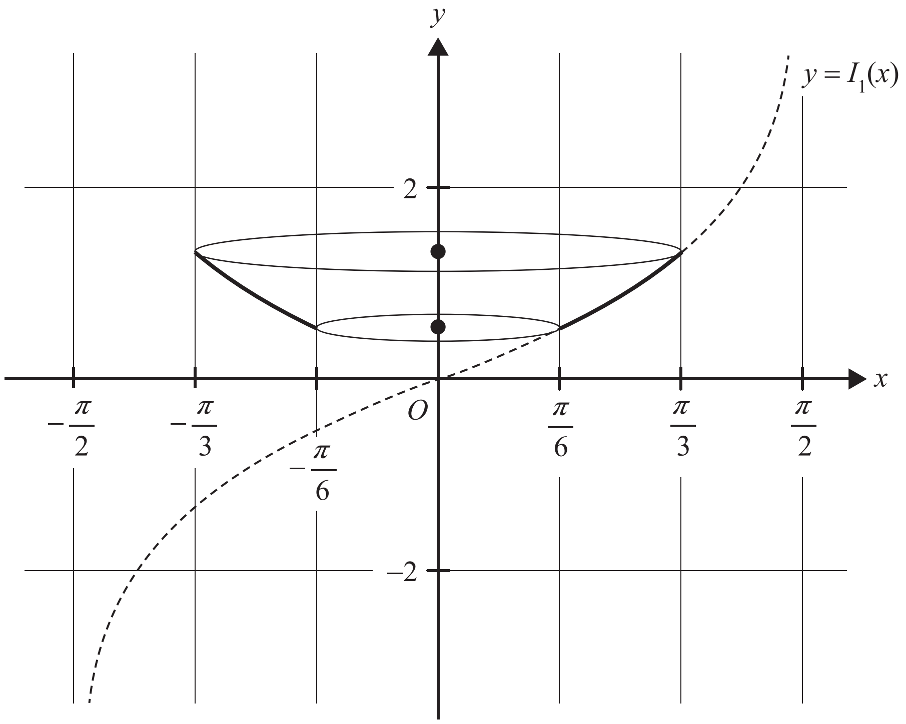
The graph of $y = I_1(x)$ for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is shown below. The part of the graph where $x \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ is rotated about the y-axis to model the curved surface of a small laboratory dish. The dish has a flat circular base and an open top. One unit on each axis represents one centimetre.



b. Using $V(x) = \pi \int x^2 \frac{dy}{dx} dx$, where $y = I_1(x)$, find the volume of fluid needed to completely fill the dish.

Give your answer in cubic centimetres correct to one decimal place. 2 marks

Question 3 continues on the next page.

The dish is used to study the growth of a certain type of bacteria. Initially 500 bacteria are in the dish and the number of bacteria, N, present after t minutes grows according to the differential equation

$$\frac{dN}{dt} = 0.05N\left(1 - \frac{N}{15\,000}\right).$$

c. What maximum value will N approach? 1 mark

d. **i.** Express as a definite integral the time it would take for N to reach 7500. 1 mark

ii. Find the time it would take for the number of bacteria to grow to 7500.

Give your answer in minutes correct to one decimal place. 1 mark

e. Given that $\frac{d^2N}{dt^2} = \frac{N}{400}\left(1 - \frac{N}{7500}\right)\left(1 - \frac{N}{15\,000}\right)$, show calculations to verify that the graph of N against t has a non-stationary point of inflection where $N = 7500$. 2 marks

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Examination continues on the next page.

Question 4 (10 marks)

A duck swims on a lake. The duck's position vector, relative to an origin, O, at time t seconds, is given by

$$\underset{\sim}{r}_D(t) = \left(2 - 5\cos(t)\right)\underset{\sim}{i} + \left(1 - 6\sin(t)\right)\underset{\sim}{j}, \text{ where } 0 \le t \le 10.$$

Displacement components are measured in metres.

a. Find the Cartesian equation of the path of the duck. 2 marks

b. How far does the duck travel along the path in the first 5 seconds?

Give your answer in metres correct to one decimal place. 2 marks

N
H
T

The duck is to be monitored by a drone. The drone is dispatched at $t = 0$ and its velocity vector is given by

$\dot{\underset{\sim}{r}}_R(t) = (at + 20)\underset{\sim}{i} + (bt + 30)\underset{\sim}{j} + (ct^2 + 2t)\underset{\sim}{k}$, where $t \geq 0$.

Components are measured in metres per second and a, b and c are real constants.

The initial position of the drone is $20\underset{\sim}{i} + 30\underset{\sim}{j}$ relative to the origin O used for the duck's position vector.

c. Use integration to show that the position vector of the drone is

$$\underset{\sim}{r}_R(t) = \left(\frac{a}{2}t^2 + 20t + 20\right)\underset{\sim}{i} + \left(\frac{b}{2}t^2 + 30t + 30\right)\underset{\sim}{j} + \left(\frac{c}{3}t^3 + t^2\right)\underset{\sim}{k}.$$

1 mark

d. Find the values of a, b and c if the drone reaches the duck when $t = 10$.

Give your answers correct to one decimal place. 2 marks

e. At what time is the duck closest to the initial position of the drone?

Give your answer in seconds correct to one decimal place. 3 marks

Do not write in this area.

Question 5 (10 marks)

The points $P(a, -4, -4)$, $Q(-3, -4, -2)$, $S(5, 4, -2)$ and $T(2, 3, 2)$, where $a \in R$, lie on the plane Π_1 relative to an origin, O.

a. Given $\overrightarrow{OP} = m\overrightarrow{OQ} + n\overrightarrow{OT}$, where $m, n \in R$, find the value of a. 2 marks

b. **i.** Find the vector cross product $\overrightarrow{QT} \times \overrightarrow{QS}$. 1 mark

ii. Find the Cartesian equation of the plane Π_1.

Express your answer in the form $Ax + By + z = C$, where $A, B, C \in R$. 2 marks

c. A second plane Π_2 has the equation $u^2x + 9uy + 3z = 13$, where $u \in R$.

Given that the vector $\overrightarrow{ST}$ is parallel to this plane, find the possible values of u. 2 marks

d. What is the shortest distance from point Q to the line that passes through the points S and T?

Give your answer in the form $\frac{b\sqrt{2c}}{c}$ where $b, c \in Z$. 3 marks

Question 6 (10 marks)

A manufacturer produces a bicycle in three stages. The times taken to complete each stage, T_1, T_2 and T_3 hours, may be assumed to be independent and normally distributed. The mean and standard deviation of each stage are shown in the following table, where a is a real constant.

Stage	Mean (hours)	Standard deviation (hours)
1	5	1
2	8	a
3	3	0.2

The standard deviation of the total processing time to produce a bicycle is 1.5 hours.

a. Find the value of a. 1 mark

b. What is the probability that the total time taken to produce a bicycle exceeds 20 hours?

Give your answer correct to four decimal places. 1 mark

The heights of people in Australia may be assumed to be normally distributed, with a standard deviation of 10 cm. The bicycles are designed for people of varying heights. The manufacturer takes a random sample of 150 people, the mean height for which is 169 cm.

c. Find an approximate 99% confidence interval for the mean height of people in Australia.

Give your answer in centimetres rounded to one decimal place. 1 mark

d. If the manufacturer took 200 such samples, how many of the 99% confidence intervals would be expected to contain the value of the population mean? 1 mark

e. The manufacturer wants to get a better estimate of the population mean by decreasing the width of the confidence interval found in **part c** by at least 60%.

What minimum sample size should be taken to achieve this? 1 mark

Question 6 continues on the next page.

Bicycle tyre life is defined as the number of kilometres for which a bicycle tyre is safe to use. The life of bicycle tyres may be assumed to be normally distributed.

The manufacturer claims that the mean life of bicycle tyres is 3200 km and the standard deviation is 450 km. Over the years a large number of customers reported that their bicycle tyres had a tyre life of more than 3200 km. To assess these reports, the manufacturer performs a one-sided statistical test at the 5% level of significance.

A random sample of 100 tyres is taken and the mean bicycle tyre life is found to be 3300 km. Assume that the standard deviation is still 450 km.

f. Write down the null and alternative hypotheses for this test. 1 mark

g. Find the p value for the test. Give your answer correct to four decimal places. 1 mark

h. At the 5% level of significance, should the manufacturer change the claim that the mean bicycle tyre life is 3200 km? Give a reason for your conclusion. 1 mark

i. Suppose that the true mean bicycle tyre life is 3350 km.

Find the probability, correct to three decimal places, that the manufacturer will still conclude that the mean bicycle tyre life is 3200 km.

Assume a 5% level of significance, a sample size of 100 and that $\sigma = 450$ km. 2 marks

End of examination questions

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2025

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Specialist Mathematics
Examination 2

2025 Formula Sheet

You may keep this Formula Sheet.

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Mensuration

area of a circle segment	$\frac{r^2}{2}(\theta - \sin(\theta))$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$	sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
volume of a pyramid	$\frac{1}{3}Ah$	cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Algebra, number and structure (complex numbers)

$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r\text{cis}(\theta)$	$\lvert z \rvert = \sqrt{x^2 + y^2} = r$	
$-\pi < \text{Arg}(z) \le \pi$	$z_1 z_2 = r_1 r_2 \text{cis}(\theta_1 + \theta_2)$	
$\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$	de Moivre's theorem	$z^n = r^n \text{cis}(n\theta)$

Data analysis, probability and statistics

for independent random variables $X_1, X_2, \ldots, X_n$	$\mathrm{E}(aX_1 + b) = a\,\mathrm{E}(X_1) + b$ $\mathrm{E}(a_1X_1 + a_2X_2 + \ldots + a_nX_n)$ $= a_1\mathrm{E}(X_1) + a_2\mathrm{E}(X_2) + \ldots + a_n\mathrm{E}(X_n)$	
	$\mathrm{Var}(aX_1 + b) = a^2\mathrm{Var}(X_1)$ $\mathrm{Var}(a_1X_1 + a_2X_2 + \ldots + a_nX_n)$ $= a_1^2\mathrm{Var}(X_1) + a_2^2\mathrm{Var}(X_2) + \ldots + a_n^2\mathrm{Var}(X_n)$	
for independent identically distributed variables $X_1, X_2, \ldots, X_n$	$\mathrm{E}(X_1 + X_2 + \ldots + X_n) = n\mu$	
	$\mathrm{Var}(X_1 + X_2 + \ldots + X_n) = n\sigma^2$	
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$	
distribution of sample mean $\bar{X}$	mean	$E(\bar{X}) = \mu$
	variance	$\mathrm{Var}(\bar{X}) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}\left(x^n\right) = n x^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, \ n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = a e^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e \lvert x \rvert + c$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$	$\int \sec^2(ax)\, dx = \frac{1}{a}\tan(ax) + c$
$\frac{d}{dx}\left(\cot(ax)\right) = -a\,\text{cosec}^2(ax)$	$\int \text{cosec}^2(ax) dx = -\frac{1}{a}\cot(ax) + c$
$\frac{d}{dx}\left(\sec(ax)\right) = a\sec(ax)\tan(ax)$	$\int \sec(ax)\tan(ax)\, dx = \frac{1}{a}\sec(ax) + c$
$\frac{d}{dx}\left(\text{cosec}(ax)\right) = -a\,\text{cosec}(ax)\cot(ax)$	$\int \text{cosec}(ax)\cot(ax)\, dx = -\frac{1}{a}\text{cosec}(ax) + c$
$\frac{d}{dx}\left(\sin^{-1}(ax)\right) = \frac{a}{\sqrt{1-(ax)^2}}$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, \ a > 0$
$\frac{d}{dx}\left(\cos^{-1}(ax)\right) = \frac{-a}{\sqrt{1-(ax)^2}}$	$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, \ a > 0$
$\frac{d}{dx}\left(\tan^{-1}(ax)\right) = \frac{a}{1+(ax)^2}$	$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n\, dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, \quad n \neq -1$
	$\int \frac{1}{ax+b} dx = \frac{1}{a}\log_e \lvert ax+b \rvert + c$

Calculus – continued

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
integration by parts	$\int u\frac{dv}{dx}\,dx = u\,v - \int v\frac{du}{dx}\,dx$
Euler's method	If $\frac{dy}{dx} = f(x,\ y)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + h \times f(x_n, y_n)$.
arc length parametric	$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area Cartesian about *x*-axis	$\int_{x_1}^{x_2} 2\pi y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$
surface area Cartesian about *y*-axis	$\int_{y_1}^{y_2} 2\pi x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$
surface area parametric about *x*-axis	$\int_{t_1}^{t_2} 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area parametric about *y*-axis	$\int_{t_1}^{t_2} 2\pi x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

Kinematics

acceleration	$a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$	
constant acceleration formulas	$v = u + at$	$s = ut + \dfrac{1}{2}at^2$
	$v^2 = u^2 + 2as$	$s = \dfrac{1}{2}(u+v)t$

Vectors in two and three dimensions

$\underset{\sim}{r}(t) = x(t)\underset{\sim}{i} + y(t)\underset{\sim}{j} + z(t)\underset{\sim}{k}$	$\lvert\underset{\sim}{r}(t)\rvert = \sqrt{x(t)^2 + y(t)^2 + z(t)^2}$
	$\dot{\underset{\sim}{r}}(t) = \dfrac{d\underset{\sim}{r}}{dt} = \dfrac{dx}{dt}\underset{\sim}{i} + \dfrac{dy}{dt}\underset{\sim}{j} + \dfrac{dz}{dt}\underset{\sim}{k}$
for $\underset{\sim}{r}_1 = x_1\underset{\sim}{i} + y_1\underset{\sim}{j} + z_1\underset{\sim}{k}$ and $\underset{\sim}{r}_2 = x_2\underset{\sim}{i} + y_2\underset{\sim}{j} + z_2\underset{\sim}{k}$	vector scalar product $\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = \lvert\underset{\sim}{r}_1\rvert\lvert\underset{\sim}{r}_2\rvert\cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$
	vector cross product $\underset{\sim}{r}_1 \times \underset{\sim}{r}_2 = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = (y_1z_2 - y_2z_1)\underset{\sim}{i} + (x_2z_1 - x_1z_2)\underset{\sim}{j} + (x_1y_2 - x_2y_1)\underset{\sim}{k}$
vector equation of a line	$\underset{\sim}{r}(t) = \underset{\sim}{r}_1 + t\underset{\sim}{r}_2 = (x_1 + x_2t)\underset{\sim}{i} + (y_1 + y_2t)\underset{\sim}{j} + (z_1 + z_2t)\underset{\sim}{k}$
parametric equation of a line	$x(t) = x_1 + x_2t \qquad y(t) = y_1 + y_2t \qquad z(t) = z_1 + z_2t$
vector equation of a plane	$\underset{\sim}{r}(s, t) = \underset{\sim}{r}_0 + s\underset{\sim}{r}_1 + t\underset{\sim}{r}_2$ $= (x_0 + x_1s + x_2t)\underset{\sim}{i} + (y_0 + y_1s + y_2t)\underset{\sim}{j} + (z_0 + z_1s + z_2t)\underset{\sim}{k}$
parametric equation of a plane	$x(s, t) = x_0 + x_1s + x_2t,\ y(s, t) = y_0 + y_1s + y_2t,\ z(s, t) = z_0 + z_1s + z_2t$
Cartesian equation of a plane	$ax + by + cz = d$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x + y) = \dfrac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$	$\tan(x - y) = \dfrac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$
$\sin(2x) = 2\sin(x)\cos(x)$	
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	$\tan(2x) = \dfrac{2\tan(x)}{1 - \tan^2(x)}$
$\sin^2(ax) = \dfrac{1}{2}\left(1 - \cos(2ax)\right)$	$\cos^2(ax) = \dfrac{1}{2}\left(1 + \cos(2ax)\right)$

End of Formula Sheet