2025
NHT

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Write your **student number** in the boxes above. **Letter**

Specialist Mathematics Examination 1

Question and Answer Book

VCE (NHT) Examination – Thursday 22 May 2025

- Reading time is **15 minutes**: 10.30 am to 10.45 am
- Writing time is **1 hour**: 10.45 am to 11.45 am

Materials supplied

- Question and Answer Book of 16 pages
- Formula Sheet

Students are **not** permitted to bring any technology (calculators or software) or notes of any kind into the examination room.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- Unless otherwise specified, an **exact** answer is required for each question.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- Take the **acceleration due to gravity** to have a magnitude g m s^{-2}, where $g = 9.8$

Question 1 (5 marks)

The position of a particle with respect to the origin, O, at time t seconds, $t \geq 0$, is given by the parametric equations $x = \dfrac{4t}{t^2+1}$ and $y = \dfrac{2(1-t^2)}{t^2+1}$, where x and y are measured in centimetres.

a. The Cartesian equation of the path of the particle is given by $x^2 + y^2 = c$.

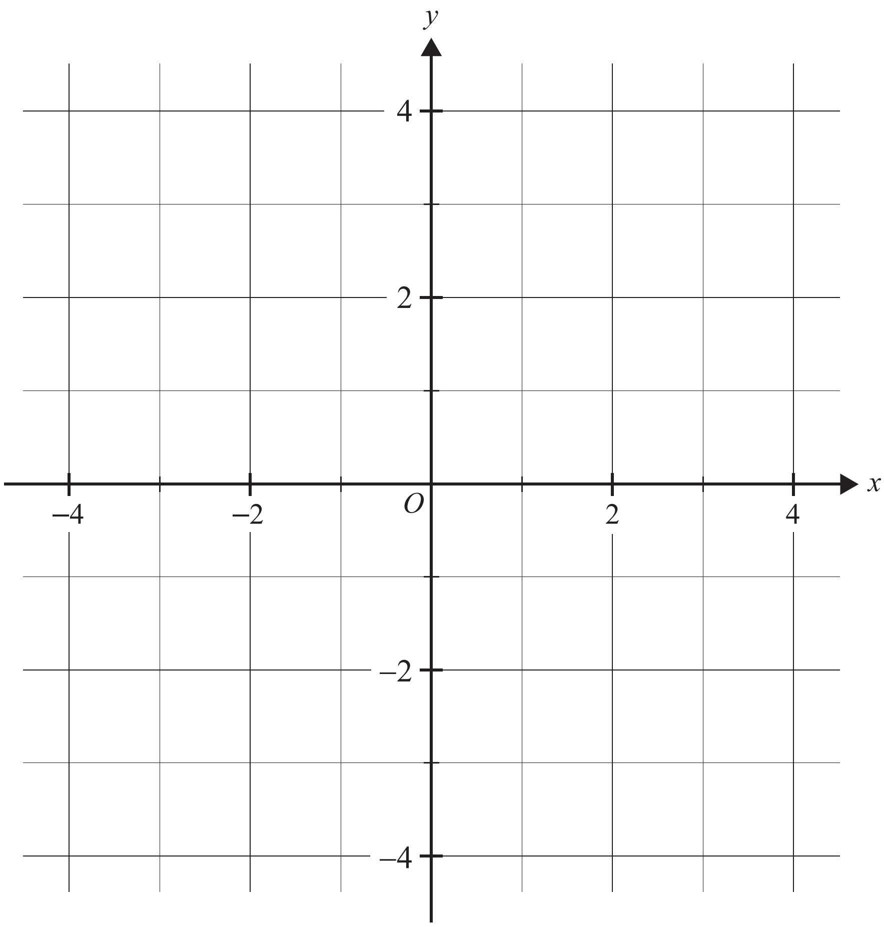
Find the value of c. 2 marks

b. As the value of t approaches infinity, the position of the particle approaches the point (h, k).

Find the values of h and k. 1 mark

c. Sketch the graph of the path of the particle for $t \geq 0$ on the axes below.

Use an arrow to indicate the direction of the particle's motion. 2 marks



Question 2 (3 marks)

It may be assumed that the mass of precious metal found in a randomly selected cubic metre of soil is normally distributed throughout a mine.

Sixteen samples are taken from random locations within the mine. Each sample consists of one cubic metre of soil, independently extracted and analysed. The number of grams of precious metal in each sample is then recorded.

From the sample results, a 95% confidence interval of (52.65, 67.35) is calculated for the population mean using $z = 2$.

a. Write down the sample mean. 1 mark

b. Find the population standard deviation used to calculate the confidence interval. 2 marks

Question 3 (4 marks)

Prove by mathematical induction that $2^3 + 4^3 + 6^3 + \ldots + (2n)^3 = 2n^2\,(n + 1)^2$, where $n \in N$.

Question 4 (6 marks)

a. Show that $\tan\left(\frac{\pi}{12}\right) = \frac{\sqrt{3}-1}{\sqrt{3}+1}$. 2 marks

Consider the complex number $z = \left(\sqrt{3}+1\right) + \left(\sqrt{3}-1\right)i$.

b. Express z in polar form. 1 mark

Consider the complex number $w = z^4$.

c. Show that $w = 32 + 32\sqrt{3}\,i$. 1 mark

d. On the complex plane, the origin, O, and the points representing the complex numbers z and w form a triangle.

Calculate the area of the triangle. 2 marks

Question 5 (5 marks)

The growth of a population of bees is modelled by the logistic differential equation

$\frac{dB}{dt} = 0.03B\left(1 - \frac{B}{500}\right)$, where B is the number of bees after t weeks.

The initial population is $B_0 = 200$.

a. Use integration to show that $t = \frac{100}{3}\log_e\left(\frac{3B}{1000 - 2B}\right)$. 4 marks

b. Find the value of t for which the population is increasing most rapidly. 1 mark

Question 6 (4 marks)

Consider the lines L_1 and L_2 with respective equations

$\underset{\sim}{r}_1(t) = \underset{\sim}{i} - 2\underset{\sim}{j} + 4\underset{\sim}{k} + t(\underset{\sim}{i} + 3\underset{\sim}{j} - \underset{\sim}{k})$ and $\underset{\sim}{r}_2(s) = 3\underset{\sim}{j} - 3\underset{\sim}{k} + s(2\underset{\sim}{i} + \underset{\sim}{j} + 4\underset{\sim}{k})$, where $s, t \in R$.

a. Show that L_1 and L_2 are skew, that is they do not intersect and they are not parallel. 2 marks

b. Find the shortest distance between L_1 and L_2. 2 marks

Question 7 (6 marks)

A toy car travels along a straight, horizontal track.

At time $t = 0$ seconds, the toy car starts from rest at line O and accelerates with positive constant acceleration until it reaches a velocity of 10 m s^{-1}.

It then undergoes a constant acceleration of –4 m s^{-2} until it attains a velocity, v, of –2 m s^{-1}.

Finally, the toy car accelerates with constant acceleration for 2 seconds, until it arrives at a line F with zero velocity, at time t_F seconds.

Lines O and F are perpendicular to the track.

a. Sketch the velocity-time graph of the toy car on the axes below. 1 mark

Question 7 continues on the next page.

b. The acceleration of the toy car is positive for twice the amount of time that the acceleration of the toy car is negative.

i. Show that it takes 9 seconds for the toy car to reach line F from line O. 1 mark

ii. Show that the equation for the velocity of the toy car when it has negative acceleration is given by $v = 26 - 4t$. 1 mark

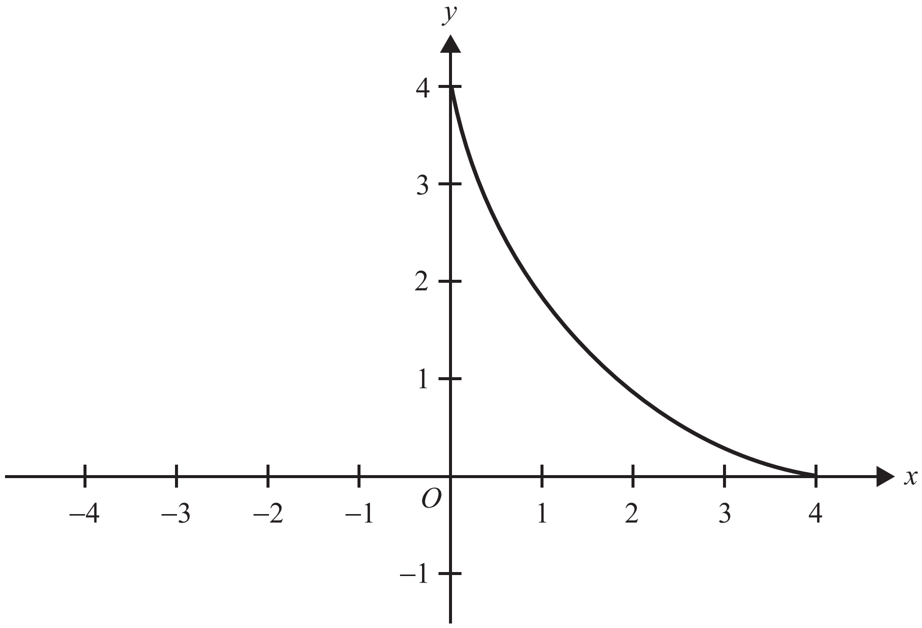
iii. Show that the displacement of the toy car, from line O, when it reaches line F, is 30 m. 1 mark

c. A toy train moves with a constant velocity on an adjacent, parallel track such that the toy car and the toy train are at line O at the same time, and both arrive at line F at time t_F.

Find the value of t, where $t > 0$, when the toy car and the toy train first have the same displacement from O. 2 marks

Question 8 (4 marks)

Consider the curve shown below.



The parametric equations for this curve are

$$x = 4\cos^3(\theta) \text{ and } y = 4\sin^3(\theta), \text{ where } \theta \in \left[0, \frac{\pi}{2}\right].$$

The curve is rotated about the y-axis to form a surface of revolution.

Find the area of this surface.

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Do not write in this area.

Question 9 (3 marks)

a. Show that $\int \arctan\left(\sqrt{x}\right) dx = x\arctan\left(\sqrt{x}\right) - \frac{1}{2}\int \frac{\sqrt{x}}{1+x}\, dx$. 1 mark

b. Hence, evaluate $\int_1^3 \arctan\left(\sqrt{x}\right) dx$. 2 marks

End of examination questions

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2025 Formula Sheet

You may keep this Formula Sheet.

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Mensuration

area of a circle segment	$\frac{r^2}{2}(\theta - \sin(\theta))$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$	sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
volume of a pyramid	$\frac{1}{3}Ah$	cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Algebra, number and structure (complex numbers)

$z = x + iy = r\left(\cos(\theta) + i\sin(\theta)\right) = r\operatorname{cis}(\theta)$	$\lvert z \rvert = \sqrt{x^2 + y^2} = r$	
$-\pi < \operatorname{Arg}(z) \le \pi$	$z_1 z_2 = r_1 r_2 \operatorname{cis}\left(\theta_1 + \theta_2\right)$	
$\frac{z_1}{z_2} = \frac{r_1}{r_2}\operatorname{cis}\left(\theta_1 - \theta_2\right)$	de Moivre's theorem	$z^n = r^n \operatorname{cis}(n\theta)$

Data analysis, probability and statistics

for independent random variables $X_1, X_2, \ldots, X_n$	$\mathrm{E}(aX_1 + b) = a\,\mathrm{E}(X_1) + b$ $\mathrm{E}(a_1X_1 + a_2X_2 + \ldots + a_nX_n)$ $= a_1\mathrm{E}(X_1) + a_2\mathrm{E}(X_2) + \ldots + a_n\mathrm{E}(X_n)$	
	$\mathrm{Var}(aX_1 + b) = a^2\mathrm{Var}(X_1)$ $\mathrm{Var}(a_1X_1 + a_2X_2 + \ldots + a_nX_n)$ $= a_1^2\mathrm{Var}(X_1) + a_2^2\mathrm{Var}(X_2) + \ldots + a_n^2\mathrm{Var}(X_n)$	
for independent identically distributed variables $X_1, X_2, \ldots, X_n$	$\mathrm{E}(X_1 + X_2 + \ldots + X_n) = n\mu$	
	$\mathrm{Var}(X_1 + X_2 + \ldots + X_n) = n\sigma^2$	
approximate confidence interval for μ	$\left(\bar{x} - z\dfrac{s}{\sqrt{n}},\ \bar{x} + z\dfrac{s}{\sqrt{n}}\right)$	
distribution of sample mean $\bar{X}$	mean	$E(\bar{X}) = \mu$
	variance	$\mathrm{Var}(\bar{X}) = \dfrac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$	$\int \frac{1}{x}dx=\log_e\lvert x\rvert+c$
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$	$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$	$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}\left(\tan(ax)\right)=a\sec^2(ax)$	$\int \sec^2(ax)\,dx=\frac{1}{a}\tan(ax)+c$
$\frac{d}{dx}\left(\cot(ax)\right)=-a\,\mathrm{cosec}^2(ax)$	$\int \mathrm{cosec}^2(ax)dx=-\frac{1}{a}\cot(ax)+c$
$\frac{d}{dx}\left(\sec(ax)\right)=a\sec(ax)\tan(ax)$	$\int \sec(ax)\tan(ax)\,dx=\frac{1}{a}\sec(ax)+c$
$\frac{d}{dx}\left(\mathrm{cosec}(ax)\right)=-a\,\mathrm{cosec}(ax)\cot(ax)$	$\int \mathrm{cosec}(ax)\cot(ax)\,dx=-\frac{1}{a}\mathrm{cosec}(ax)+c$
$\frac{d}{dx}\left(\sin^{-1}(ax)\right)=\frac{a}{\sqrt{1-(ax)^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\cos^{-1}(ax)\right)=\frac{-a}{\sqrt{1-(ax)^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\tan^{-1}(ax)\right)=\frac{a}{1+(ax)^2}$	$\int \frac{a}{a^2+x^2}dx=\tan^{-1}\left(\frac{x}{a}\right)+c$
	$\int (ax+b)^n\,dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\quad n\neq -1$
	$\int \frac{1}{ax+b}dx=\frac{1}{a}\log_e\lvert ax+b\rvert+c$

Calculus – continued

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
integration by parts	$\int u\frac{dv}{dx}\,dx = u\,v - \int v\frac{du}{dx}\,dx$
Euler's method	If $\frac{dy}{dx} = f(x,\ y)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + h \times f(x_n, y_n)$.
arc length parametric	$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area Cartesian about x-axis	$\int_{x_1}^{x_2} 2\pi y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$
surface area Cartesian about y-axis	$\int_{y_1}^{y_2} 2\pi x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$
surface area parametric about x-axis	$\int_{t_1}^{t_2} 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area parametric about y-axis	$\int_{t_1}^{t_2} 2\pi x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

Kinematics

acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$	
constant acceleration formulas	$v = u + at$	$s = ut + \frac{1}{2}at^2$
	$v^2 = u^2 + 2as$	$s = \frac{1}{2}(u+v)t$

Vectors in two and three dimensions

$\underset{\sim}{r}(t) = x(t)\underset{\sim}{i} + y(t)\underset{\sim}{j} + z(t)\underset{\sim}{k}$	$\lvert\underset{\sim}{r}(t)\rvert = \sqrt{x(t)^2 + y(t)^2 + z(t)^2}$
	$\dot{\underset{\sim}{r}}(t) = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
for $\underset{\sim}{r}_1 = x_1\underset{\sim}{i} + y_1\underset{\sim}{j} + z_1\underset{\sim}{k}$ and $\underset{\sim}{r}_2 = x_2\underset{\sim}{i} + y_2\underset{\sim}{j} + z_2\underset{\sim}{k}$	vector scalar product $\underset{\sim}{r}_1 . \underset{\sim}{r}_2 = \lvert\underset{\sim}{r}_1\rvert\lvert\underset{\sim}{r}_2\rvert\cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$
	vector cross product $\underset{\sim}{r}_1 \times \underset{\sim}{r}_2 = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = (y_1z_2 - y_2z_1)\underset{\sim}{i} + (x_2z_1 - x_1z_2)\underset{\sim}{j} + (x_1y_2 - x_2y_1)\underset{\sim}{k}$
vector equation of a line	$\underset{\sim}{r}(t) = \underset{\sim}{r}_1 + t\underset{\sim}{r}_2 = (x_1 + x_2t)\underset{\sim}{i} + (y_1 + y_2t)\underset{\sim}{j} + (z_1 + z_2t)\underset{\sim}{k}$
parametric equation of a line	$x(t) = x_1 + x_2t \qquad y(t) = y_1 + y_2t \qquad z(t) = z_1 + z_2t$
vector equation of a plane	$\underset{\sim}{r}(s, t) = \underset{\sim}{r}_0 + s\underset{\sim}{r}_1 + t\underset{\sim}{r}_2$ $= (x_0 + x_1s + x_2t)\underset{\sim}{i} + (y_0 + y_1s + y_2t)\underset{\sim}{j} + (z_0 + z_1s + z_2t)\underset{\sim}{k}$
parametric equation of a plane	$x(s, t) = x_0 + x_1s + x_2t,\ y(s, t) = y_0 + y_1s + y_2t,\ z(s, t) = z_0 + z_1s + z_2t$
Cartesian equation of a plane	$ax + by + cz = d$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \dfrac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$	$\tan(x-y) = \dfrac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$
$\sin(2x) = 2\sin(x)\cos(x)$	
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	$\tan(2x) = \dfrac{2\tan(x)}{1 - \tan^2(x)}$
$\sin^2(ax) = \dfrac{1}{2}\left(1 - \cos(2ax)\right)$	$\cos^2(ax) = \dfrac{1}{2}\left(1 + \cos(2ax)\right)$

End of Formula Sheet