2025 VCE Specialist Mathematics 1 (NHT) external assessment report

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

Question 1a

$$\begin{aligned} x^2+y^2 &= \frac{16t^2}{\left(t^2+1\right)^2}+\frac{4\left(1-t^2\right)^2}{\left(t^2+1\right)^2} \\ &= \frac{4t^4+8t^2+4}{\left(t^2+1\right)^2} \\ &= \frac{4\left(t^2+1\right)^2}{\left(t^2+1\right)^2} \\ &= 4 \end{aligned}$$

Therefore, $c=4$.

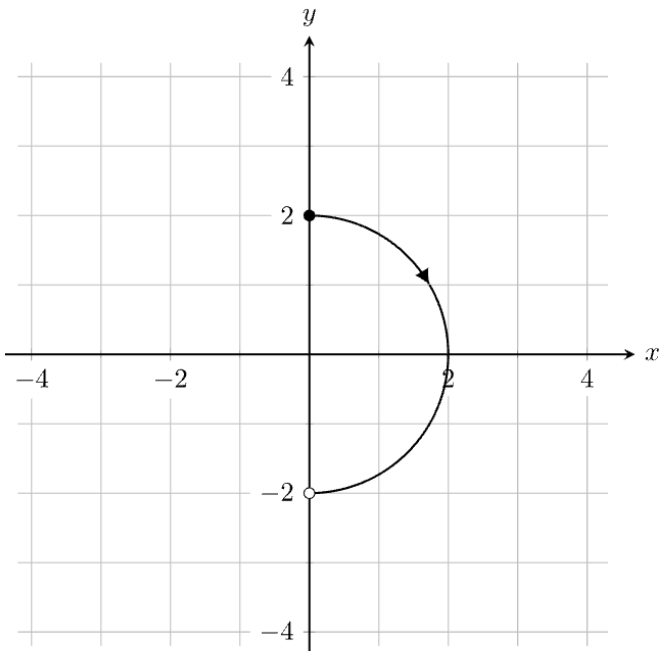
Question 1b

$h=\lim\limits_{t\to\infty}\frac{4t}{t^2+1}=0$ and $k=\lim\limits_{t\to\infty}\frac{2\left(1-t^2\right)}{t^2+1}=-2$

Therefore, $h=0$ and $k=-2$.

Question 1c

The path is circular, starting at $(0, 2)$ and moving clockwise. The path approaches the point $(0, -2)$, as shown below.



Question 2a

The sample mean is the middle of the confidence interval:

$$\bar{x} = \frac{67.35 + 52.65}{2} = \frac{120}{2} = 60$$

Question 2b

Let s be the population standard deviation. Then (using $z = 2$):

$$2 \times \frac{s}{\sqrt{16}} = 7.35$$
$$\Rightarrow s = 2 \times 7.35 = 14.7$$

Question 3

Let $P(n)$ be the proposition that $2^3+4^3+6^3+\ldots+(2n)^3=2n^2(n+1)^2,\ n\in N$.

The left-hand side of $P(1)$ is $2^3=8$. The right-hand side of $P(1)$ is $2\times1^2\times2^2=8$ and so $P(1)$ is true.

Assuming $P(k)$ is true: $2^3+4^3+6^3+\ldots+(2k)^3=2k^2(k+1)^2$ where k is some element of N.

Then

$$\begin{aligned}\text{LHS of } P(k+1) &= 2^3+4^3+6^3+\ldots+(2k)^3+(2(k+1))^3\\ &= 2k^2(k+1)^2+8(k+1)^3\\ &= 2(k+1)^2\left(k^2+4(k+1)\right)\\ &= 2(k+1)^2(k+2)^2\\ &= \text{RHS of } P(k+1)\end{aligned}$$

Therefore, by mathematical induction, $2^3+4^3+6^3+\ldots+(2n)^3=2n^2(n+1)^2,\ n\in N$

Question 4a

Using the difference formula for tan:

$$\begin{aligned}\tan\left(\frac{\pi}{12}\right) &= \tan\left(\frac{\pi}{3}-\frac{\pi}{4}\right)\\ &= \frac{\tan\left(\frac{\pi}{3}\right)-\tan\left(\frac{\pi}{4}\right)}{1+\tan\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{4}\right)}\\ &= \frac{\sqrt{3}-1}{1+\sqrt{3}} \quad \text{as required.}\end{aligned}$$

Question 4b

The modulus of z is

$$|z| = \sqrt{\left(\sqrt{3}+1\right)^2 + \left(\sqrt{3}-1\right)^2}$$
$$= 2\sqrt{2}$$

The argument of z is $\frac{\pi}{12}$.

Therefore $z = 2\sqrt{2}\,\text{cis}\left(\frac{\pi}{12}\right)$

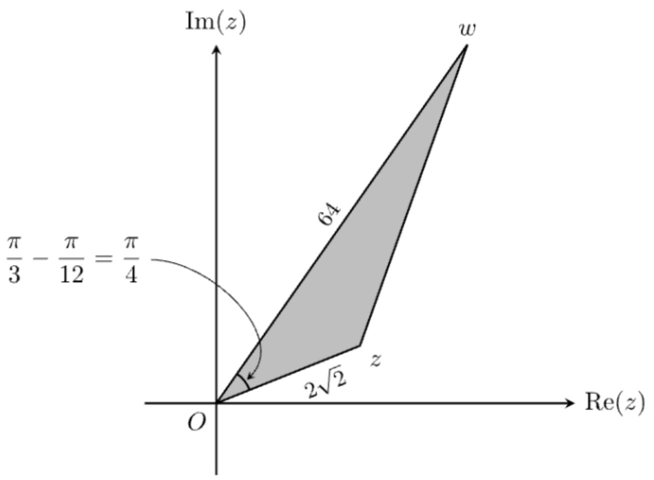
Question 4c

By de Moivre's theorem:

$$w = z^4 = \left(2\sqrt{2}\right)^4 \text{cis}\left(\frac{4\pi}{12}\right)$$
$$= 64\,\text{cis}\left(\frac{\pi}{3}\right)$$
$$= 64\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$
$$= 32 + 32\sqrt{3}i \quad \text{as required.}$$

Question 4d

The angle between z and w is $\frac{\pi}{3}-\frac{\pi}{12}=\frac{\pi}{4}$



The area of the triangle is $\frac{1}{2}\cdot 64\cdot 2\sqrt{2}\sin\left(\frac{\pi}{4}\right)=\frac{1}{2}\cdot 64\cdot 2\sqrt{2}\cdot\frac{1}{\sqrt{2}}=64$.

Question 5a

The differential equation is $\frac{dB}{dt} = \frac{3}{100} \cdot \frac{1}{500} B(500 - B)$.

Then $\int dt = \frac{50\ 000}{3} \int \frac{1}{B(500 - B)} dB$.

Now apply partial fractions and integrate:

$$
\begin{aligned}
t &= \frac{100}{3} \int \left(\frac{1}{B} + \frac{1}{500 - B} \right) dB \\
&= \frac{100}{3} \left(\log_e |B| - \log_e |500 - B| \right) + c \\
&= \frac{100}{3} \log_e \left(\frac{B}{500 - B} \right) + c, \text{ as } 200 < B < 500
\end{aligned}
$$

When $t = 0$, $B = 200$, $c = -\frac{100}{3} \log_e \left(\frac{2}{3} \right)$.

Therefore

$$
\begin{aligned}
t &= \frac{100}{3} \log_e \left(\frac{B}{500 - B} \div \frac{2}{3} \right) \\
&= \frac{100}{3} \log_e \left(\frac{3B}{1000 - 2B} \right) \quad \text{as required.}
\end{aligned}
$$

Question 5b

The population is increasing most rapidly when $B = 250$:

$$t = \frac{100}{3} \log_e \left(\frac{750}{1000 - 500} \right) = \frac{100}{3} \log_e \left(\frac{3}{2} \right).$$

Question 6a

Suppose the lines meet. Then for some t, s, we have the system of equations:

$$\begin{aligned} 1+t &= 2s \\ -2+3t &= 3+s \\ 4-t &= -3+4s \end{aligned}$$

Solving the first two equations gives $t=\frac{11}{5}$ and $s=\frac{8}{5}$. These values do not satisfy the third equation and so the lines do not meet.

The direction vectors $\underset{\sim}{i}+3\underset{\sim}{j}-\underset{\sim}{k}$ and $2\underset{\sim}{i}+\underset{\sim}{j}+4\underset{\sim}{k}$ are not parallel.

Therefore, the lines are shown to be skew as required.

Question 6b

$$\underset{\sim}{n}=\begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 1 & 3 & -1 \\ 2 & 1 & 4 \end{vmatrix}=13\underset{\sim}{i}-6\underset{\sim}{j}-5\underset{\sim}{k} \text{ and } |\underset{\sim}{n}|=\sqrt{169+36+25}=\sqrt{230}$$

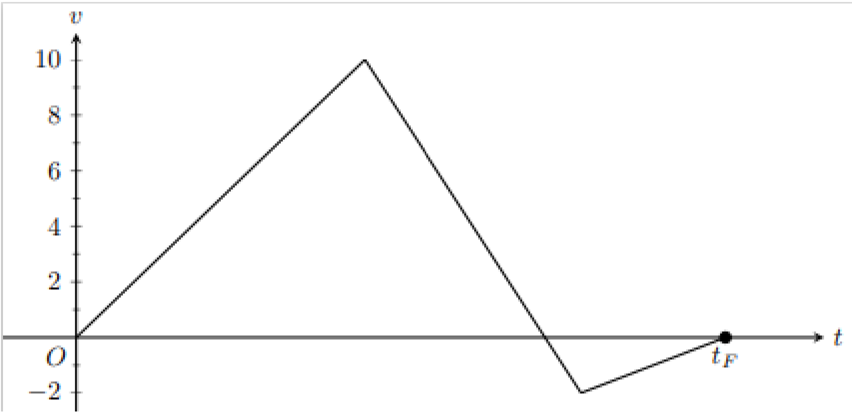
$A(1,-2,4)$ and $B(0,3,-3)$ are points on L_1 and L_2, respectively. $\overrightarrow{AB}=-\underset{\sim}{i}+5\underset{\sim}{j}-7\underset{\sim}{k}$

$$\text{distance}=\left|\overrightarrow{AB}\cdot\hat{\underset{\sim}{n}}\right|=\left|\frac{1}{\sqrt{230}}(-13-30+35)\right|=\left|\frac{-8}{\sqrt{230}}\right|$$

Therefore, the distance between the lines is $\frac{8}{\sqrt{230}}$.

Question 7a

The velocity–time graph is shown below.



Question 7b.i

The time spent decelerating from 10 m s^{-1} to -2 m s^{-1} at -4 m s^{-2} is

$$t = \frac{v-u}{a} = \frac{-2-10}{-4} = 3 \text{ seconds}.$$

If the acceleration is positive for twice the amount of time that the acceleration is negative, then the acceleration is positive for 6 seconds.

Given 2 seconds for the last stage, the first stage takes 4 seconds.

Therefore, the total time taken to reach line F from line O is $4+3+2=9$ seconds, as required.

Question 7b.ii

The equation for the velocity of the car is $v = u - 4t$ when the acceleration is negative.

The car starts to decelerate when $t = 4$. At this time, it has a velocity of 10 m s^{-1}.

Therefore $10 = u - 16 \Rightarrow u = 26$, so the velocity is shown to be $v = 26 - 4t$, as required.

Question 7b.iii

$26-4t=0 \Rightarrow t=\frac{26}{4}=\frac{13}{2}$

Therefore, the displacement is

$\frac{1}{2}\times\frac{13}{2}\times10-\frac{1}{2}\times\left(9-\frac{13}{2}\right)\times2=\frac{65}{2}-\frac{5}{2}=\frac{60}{2}=30$ metres as required.

Question 7c

The toy train travels at a constant speed of $\frac{30}{9}=\frac{10}{3}$ m s^{-1}.

Suppose that the train and car have the same displacement in the first 4 seconds.

- The distance in metres travelled by the car is $\frac{1}{2}\cdot\frac{5}{2}t^2$ (the acceleration of the car is $\frac{10}{4}=\frac{5}{2}$ m s^{-2} during the first 4 seconds).
- The distance in metres travelled by the train is $\frac{10}{3}t$.

Solving $\frac{10}{3}t=\frac{5}{4}t^2$ gives $t=\frac{8}{3}$ seconds.

Question 8

$\frac{dx}{dt}=-12\sin\theta\cos^2\theta$ and $\frac{dy}{dt}=12\cos\theta\sin^2\theta$.

The surface area is given by

$$\int_0^{\frac{\pi}{2}}2\pi\cdot4\cos^3\theta\sqrt{144\sin^2\theta\cos^4\theta+144\cos^2\theta\sin^4\theta}\,d\theta$$

$$=96\pi\int_0^{\frac{\pi}{2}}\cos^3\theta\sqrt{\sin^2\theta\cos^2\theta\left(\cos^2\theta+\sin^2\theta\right)}\,d\theta$$

$$=96\pi\int_0^{\frac{\pi}{2}}\cos^4\theta\sin\theta\,d\theta$$

Let $u=\cos\theta$, $\frac{du}{d\theta}=-\sin\theta$. The integral becomes

$$-96\pi\int_1^0u^4\,du=96\pi\int_0^1u^4\,du=\frac{96\pi}{5}$$

Question 9a

Using integration by parts:

$$u = \arctan\left(\sqrt{x}\right) \qquad \frac{dv}{dx} = 1$$

$$\frac{du}{dx} = \frac{1}{2\sqrt{x}(1+x)} \qquad v = x$$

$\int \arctan\left(\sqrt{x}\right) dx = x \arctan\left(\sqrt{x}\right) - \frac{1}{2}\int \frac{\sqrt{x}}{1+x} dx$ as required.

Question 9b

From Question 9a:

$$\int_1^3 \arctan\left(\sqrt{x}\right) dx = \left[x \arctan\left(\sqrt{x}\right)\right]_1^3 - \frac{1}{2}\int_1^3 \frac{\sqrt{x}}{1+x} dx$$

Let $u = \sqrt{x} \Rightarrow \frac{du}{dx} = \frac{1}{2\sqrt{x}}$, $dx = 2u\,du$:

$$\begin{aligned}
\int_1^3 \arctan\left(\sqrt{x}\right) dx &= \frac{3\pi}{4} - \int_1^{\sqrt{3}} \frac{u^2}{1+u^2} du \\
&= \frac{3\pi}{4} - \int_1^{\sqrt{3}} \frac{1+u^2-1}{1+u^2} du \\
&= \frac{3\pi}{4} - \int_1^{\sqrt{3}} \left(1 - \frac{1}{1+u^2}\right) du \\
&= \frac{3\pi}{4} - \left[u - \arctan(u)\right]_1^{\sqrt{3}} \\
&= \frac{3\pi}{4} - \left(\sqrt{3} - \frac{\pi}{3} - \left(1 - \frac{\pi}{4}\right)\right) \\
&= \frac{5\pi}{6} + 1 - \sqrt{3}
\end{aligned}$$