2024.M28

2024L003G2EL

Coimisiún na Scrúduithe Stáit

State Examinations Commission

Leaving Certificate Examination 2024

Mathematics

Paper 2

Ordinary Level

Monday 10 June Morning 9:30 - 12:00

300 marks

Examination Number	
Date of Birth	/ /
Centre Stamp	

For example, 3rd February 2005 is entered as 03 02 05

The 2024 examination papers were adjusted to compensate for disruptions to learning due to COVID-19. This examination paper does not necessarily reflect the same structure and format as the examination papers of past or subsequent years.

Do not write on this page

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	4 questions

Answer questions as follows:

- any **five** questions from Section A – Concepts and Skills
- any **three** questions from Section B – Contexts and Applications.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet. If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

In general, diagrams are not to scale.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if you do not include appropriate units of measurement, where relevant.

You may lose marks if you do not give your answers in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A	Concepts and Skills	150 marks

Answer **any five questions** from this section.

Question 1 **(30 marks)**

(a) The scores for 9 students in a history test are given in the table below.
X is the lowest score and Y is the highest score, where $X, Y \in \mathbb{N}$.

X	91	76
82	37	42
74	54	Y

(i) Write the scores in order, from the lowest to the highest in the spaces below.

X								Y

(ii) Find the median of these scores **and** explain what this score means in the context of the question.

Median =

Explanation:

(iii) All students received **different scores** in the test.
The range of the scores is 61.

Work out the maximum possible value of X and the corresponding value of Y .

$X =$ _______ $Y =$ _______

(b) A different group of five students did a maths question.
Their scores were as follows, where $t \in \mathbb{N}$:

$$15, \ 16, \ 19, \ t, \ 26$$

The mean of the five scores is $19{\cdot}6$.

Work out the value of t.

Question 2 **(30 marks)**

Seán is playing a game with 14 cards.
7 of the cards are green (**G**), numbered 1 to 7.
The other 7 cards are red (**R**), also numbered 1 to 7.

(a) Complete the table below to show all possible outcomes when a card is chosen.
Some have already been completed for you.
For example, **G2** means the green card with the number 2.

		Number						
		1	2	3	4	5	6	7
Colour	Green (**G**)		**G2**					
	Red (**R**)					**R5**		

(b) A card is chosen at random.
Work out the probability of getting a card with an even number.

(c) Seán picks a card at random and **doesn't replace it**. It is Red 5 (**R5**).
He then picks a second card at random.

What is the probability that the second card is a green card?

Seán puts back the two cards.

(d) Seán picks a card at random and **doesn't replace it**.
He then picks a second card at random.

Find the probability that the second card is a different colour **and** a different number from the first card.

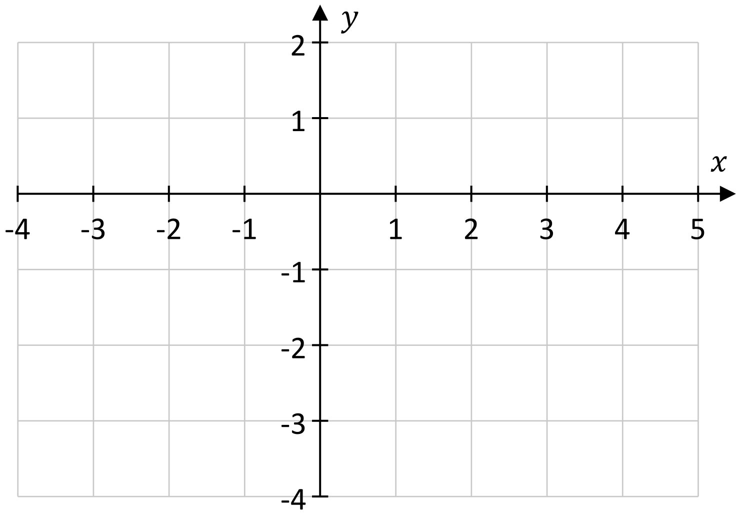
Question 3 **(30 marks)**

(a) A line, l, has the equation:

$$y = \frac{1}{2}x - 1$$

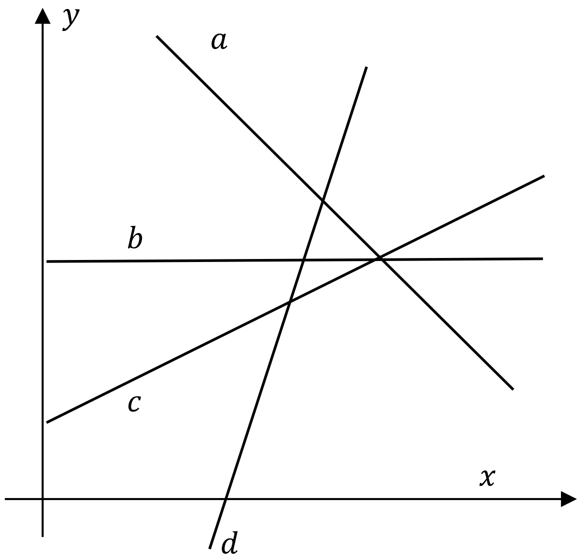
(i) Show that the point $(4, 1)$ lies on l.

(ii) Draw the line l on the co-ordinate diagram below, in the domain $-4 \leq x \leq 5,\ x \in \mathbb{R}$.



(b) Four lines are drawn on the co-ordinate diagram below, labelled a, b, c and d.
Their slopes are given in the table below.

Write a, b, c and d in the correct place in the table to match each line to its slope.



Line (a, b, c or d)	Slope
	3
	-1
	$\frac{1}{2}$
	0

(c) Find the equation of the line through the points $(0, 6)$ and $(2, 9)$.

Question 4 **(30 marks)**

(a) The circle k has equation:

$$(x-5)^2 + (y+3)^2 = 25$$

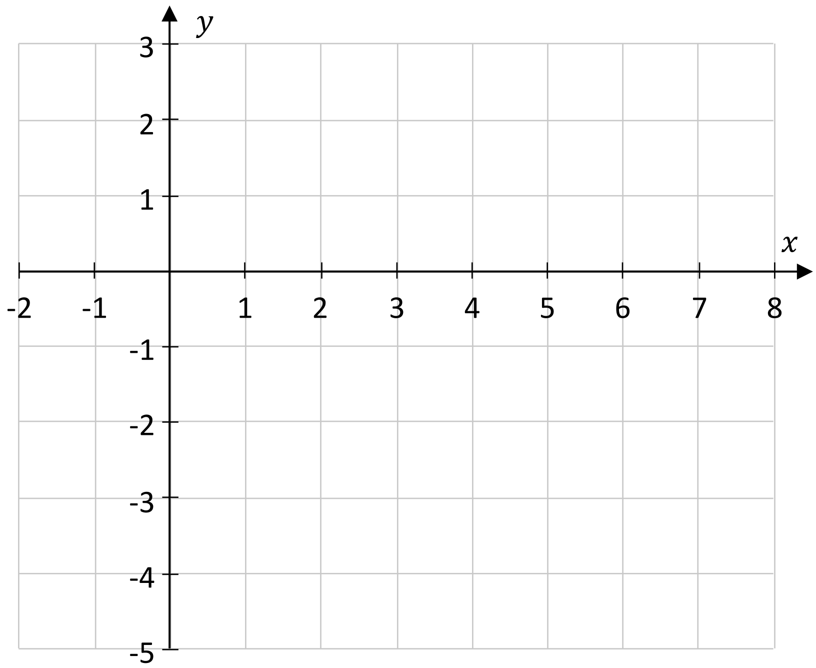
(i) Write down the centre and radius of the circle k.

Centre = (,) Radius = ____________

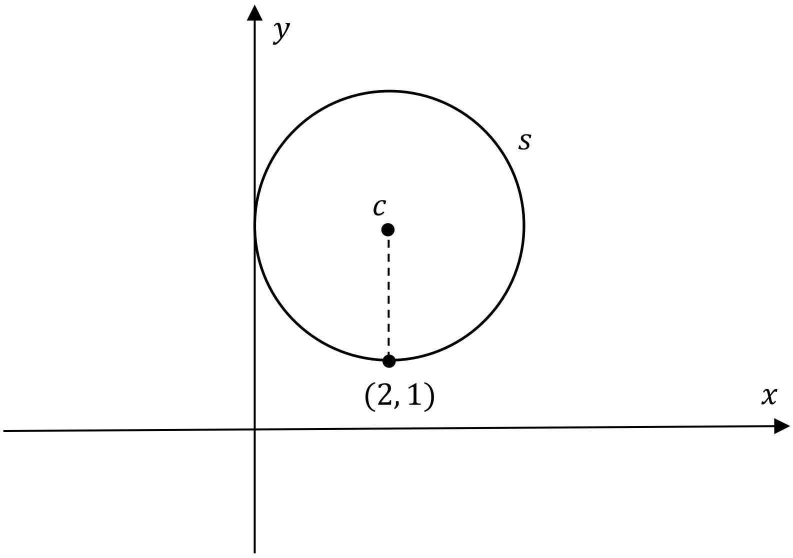
(ii) Use **algebra** to investigate if the point $(9, 2)$ is on, inside, or outside the circle k.

Answer: (Tick (✓) **one** box only)	$(9, 2)$ is **on** k	$(9, 2)$ is **inside** k	$(9, 2)$ is **outside** k
	☐	☐	☐

(b) A different circle has centre $(4, -1)$ and radius 3.
Construct this circle on the co-ordinate diagram below.



(c) The point $(2, 1)$ is the lowest point on the circle s with centre c, as shown in the diagram below.
The y-axis is a **tangent** to the circle s.



Work out the radius and the centre of the circle s.

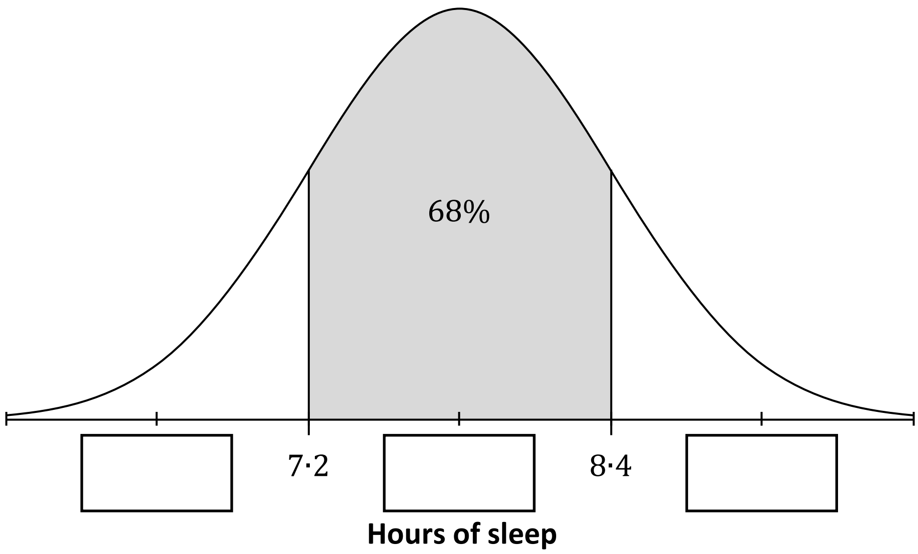
Radius = ____________ Centre = $(\quad , \quad)$

Question 5 **(30 marks)**

(a) A large group of 17-year-olds were asked how many hours of sleep they got on the previous night. The results were normally distributed.

The **middle 68%** slept between $7 \cdot 2$ and $8 \cdot 4$ hours, as shown on the diagram below.
Use the empirical rule to answer **parts (a)(i)** and **(a)(ii)**.

(i) Fill in the three missing numbers on the horizontal axis.



(ii) Research says that a 17-year-old should get at least 9 hours of sleep each night.
What percentage of this sample got **at least 9 hours** of sleep on the previous night?

(b) Owen records the number of hours of sleep that he gets each night for several weeks. Based on this, he calculates that he gets the recommended amount of sleep 10% of the time.

(i) What is the probability that he does **not** get the recommended amount of sleep on a particular night?

(ii) Beginning on a Sunday night, Owen records his sleep each night for a week. Find the probability that Owen gets the recommended amount of sleep for the **first time** on Tuesday night (the third night).

Assume that the number of hours of sleep Owen gets on different nights are independent.

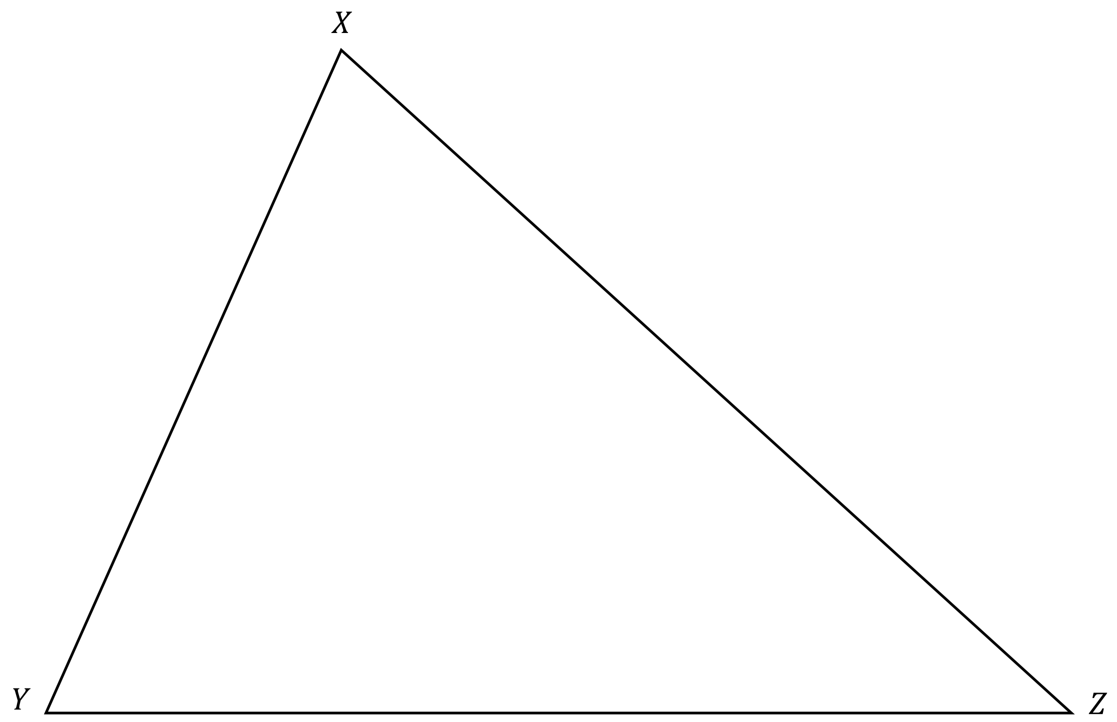
Question 6 **(30 marks)**

(a) The diagram below shows the triangle XYZ.

(i) **Construct** the bisector of the angle XYZ, using only a compass and straight edge.

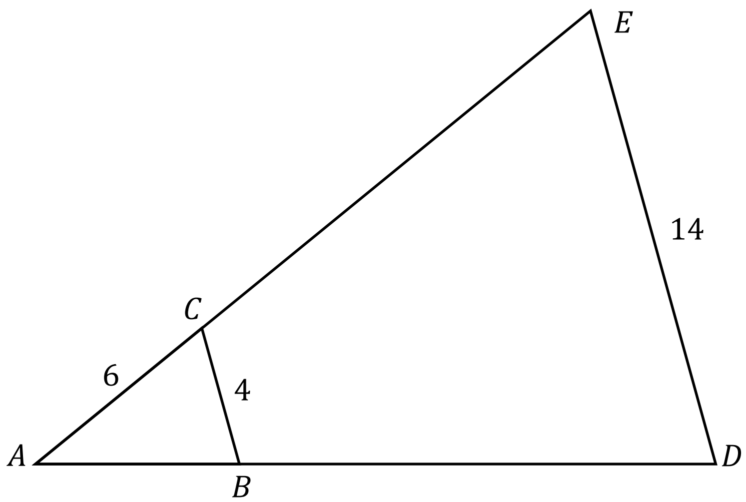
Show all your construction lines and arcs clearly.

(ii) Hence, construct the incentre of the triangle XYZ below.



(b) The diagram below shows the triangles ABC and ADE (not to scale).
The triangle ADE is the image of the triangle ABC under enlargement.

Some of the lengths are shown in the diagram.



(i) Use $|BC|$ and $|DE|$ to show the scale factor is 3·5.

(ii) $|AC| = 6$ units. Use the scale factor to find the distance from C to E, that is, find $|CE|$.

(iii) The area of the triangle ABC is 11 units^2.
Find the area of the triangle ADE.

Section B — Contexts and Applications — 150 marks

Answer **any three** questions from this section.

Question 7 **(50 marks)**

(a) A company was interested in investigating the fitness of its employees.

The table below shows the heart rates of ten employees measured before and after exercise. Their heart rates are measured in beats per minute (bpm).

Employee	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
Heart Rate (bpm) (**before** exercise)	60	76	68	89	67	65	77	83	88	70
Heart Rate (bpm) (**after** exercise)	71	81	79	108	91	83	102	113	118	87

(i) Complete the back-to-back stem-and-leaf plot below to show this information. Two values are already filled in.

Heart Rate (before exercise)					**Heart Rate** (after exercise)			
			0	6				
				7	1			
				8				
				9				
				10				
				11				
Key: 0 | 6 = 60 bpm					Key: 7 | 1 = 71 bpm			

The values for the heart rates after exercise are shown again below.

Employee	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
Heart Rate (bpm) (after exercise)	71	81	79	108	91	83	102	113	118	87

(ii) Work out the **mean** and **standard deviation** of the heart rate after exercise. Give each answer correct to 1 decimal place.

Mean = ______________

Standard deviation = _____________

(iii) Based on your stem-and-leaf plot, how has the standard deviation changed **following exercise**? Give a reason for your answer.

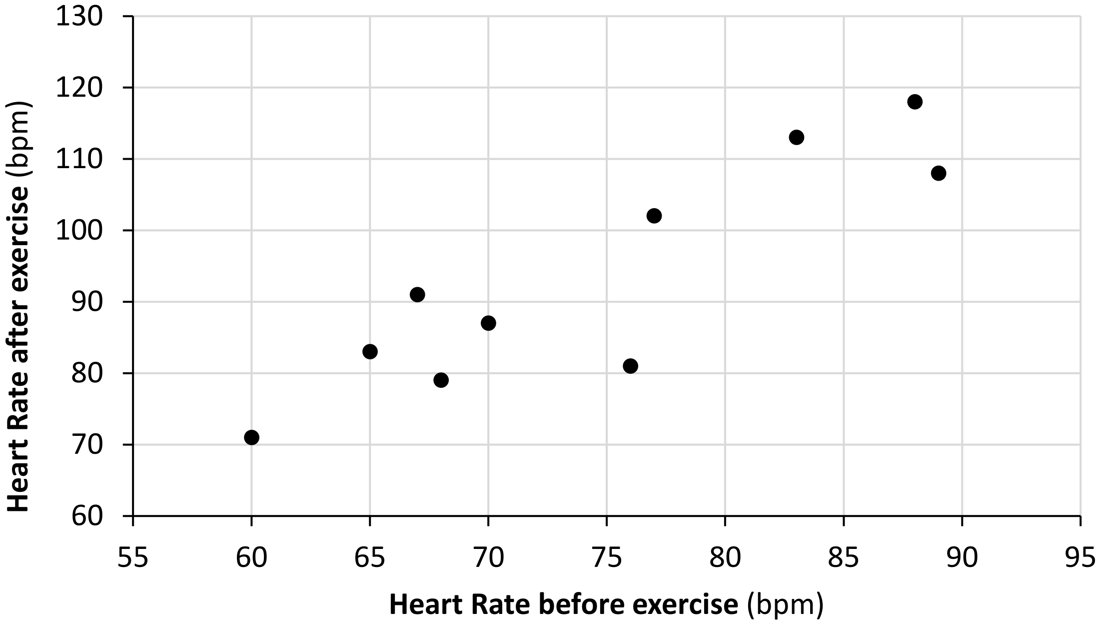
The standard deviation: (Tick (✓) **one** box only)

Increased ☐ Decreased ☐ Stayed the same ☐

Reason:

This question continues on the next page.

A scatter plot of the data is shown in the diagram below.



(iv) r is the correlation coefficient between the heart rate before exercise and the heart rate after exercise.

Based on the diagram above, pick the value from the list below that is closest to r. Give a reason for your answer.

Closest value of r:	$-0{\cdot}8$	$-0{\cdot}2$	$0{\cdot}3$	$0{\cdot}9$
(Tick (✓) **one** box only)	☐	☐	☐	☐

Reason:

(b) A random sample of 355 people was picked from everyone who took part in a park run in 2023 in Galway.

(i) Show that, for this sample, the margin of error for a population proportion is 5·3%, correct to 1 decimal place.

(ii) 96 of the 355 runners finished the run in under 25 minutes.
Work out the percentage of runners in this sample who finished the park run in under 25 minutes. Give your answer correct to the nearest percent.

(iii) A running club commentator said that Galway park run times have changed since 2019. In 2019 it was reported that 24% of all runners finished the park run in under 25 minutes.

Use your answers to **parts (b)(i)** and **(b)(ii)** to test the claim that the percentage of runners who finish the park run in under 25 minutes has changed since 2019, at the 5% level of significance.

Show relevant calculations, state your conclusion, and give a reason for your conclusion.

Calculations:

Conclusion:

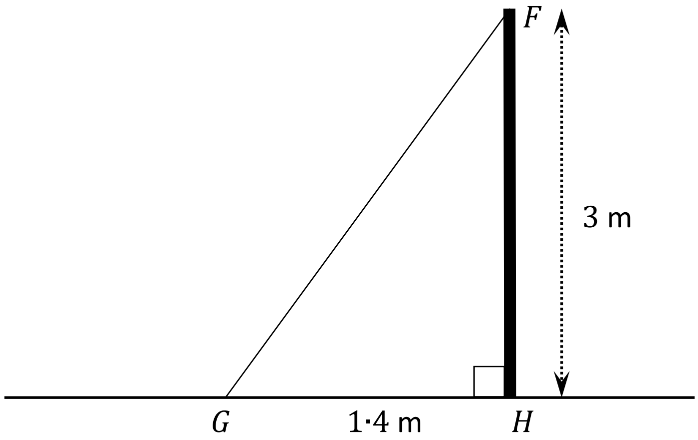
Reason for the conclusion:

Question 8 **(50 marks)**

(a) A fish shop has a vertical sign outside its front door.

In the diagram below, the sign is represented by $[FH]$, with $|FH| = 3$ m.

A cable is tied at the top of the sign, F, and is secured at the point G on the horizontal ground. G is $1{\cdot}4$ m from the base of the sign H, as shown in the diagram below (not to scale).

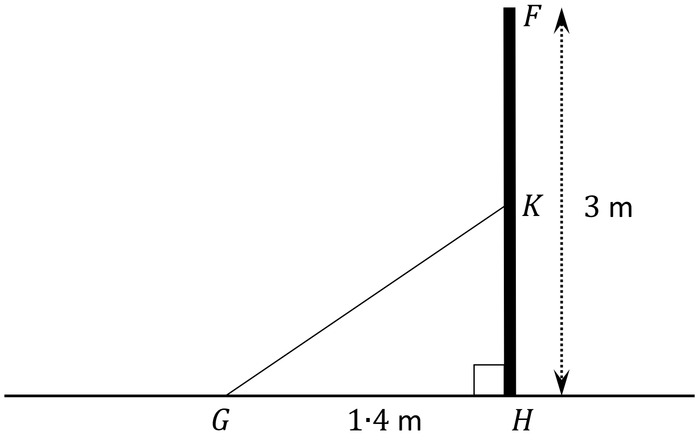


(i) Use the Theorem of Pythagoras to find the length of the cable, $|GF|$.
Give your answer in metres, correct to 1 decimal place.

(ii) Show that the size of the angle $\angle FGH = 65°$, correct to the nearest degree.

(iii) A second cable from G is tied to the sign, at the point K, where K is half way up the sign $[FH]$.

Mairéad says that the size of the new angle $\angle KGH$ is half the size of the angle $\angle FGH$.



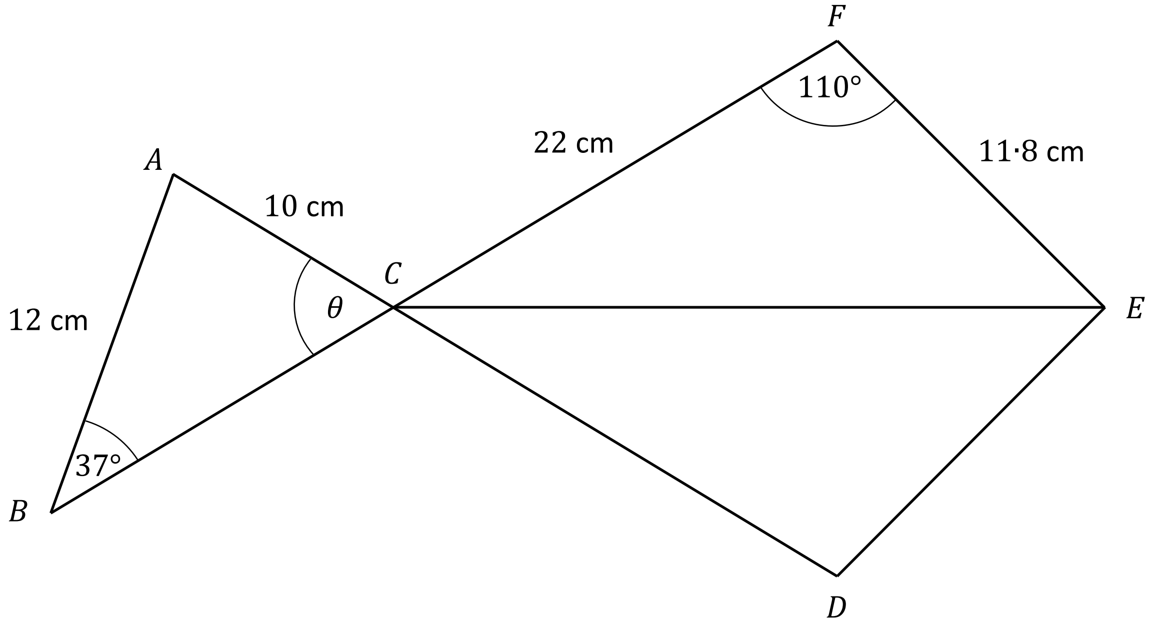
Is Mairéad correct?
Use calculations to support your answer.

	Yes	No
Answer: (Tick (✓) **one** box only)	☐	☐

Calculations:

This question continues on the next page.

(b) The fish shop has a logo with some measurements as shown in the diagram below (not to scale).



(i) $|AB| = 12$ cm, $|AC| = 10$ cm, and $|\angle ABC| = 37°$.

Work out the size of the angle θ, where θ is the angle $\angle BCA$.
Give your answer correct to the nearest degree.

(ii) $|CF| = 22$ cm, $|EF| = 11{\cdot}8$ cm, and $|\angle EFC| = 110°$.

Use the cosine rule to work out the distance $|CE|$.
Give your answer correct to 1 decimal place.

(iii) $CDEF$ is symmetrical about $[CE]$.
Work out the **total** area of $CDEF$.
Give your answer correct to the nearest cm^2.

Question 9 **(50 marks)**

Diarmuid owns an ice cream van.

(a) Diarmuid sells:

- 7 different flavours of ice cream
- 4 different toppings
- 2 containers: a choice of a cone or a tub.

(i) How many different choices are available if you must choose **only** one flavour, one topping, and either a cone or a tub?

(ii) Diarmuid wants to maximise the number of choices available to his customers. He can only add either an extra flavour or an extra topping.

Should he add an extra flavour or an extra topping?
Use calculations to support your answer.

Answer:	Extra flavour	Extra topping
(Tick (✓) **one** box only)	☐	☐

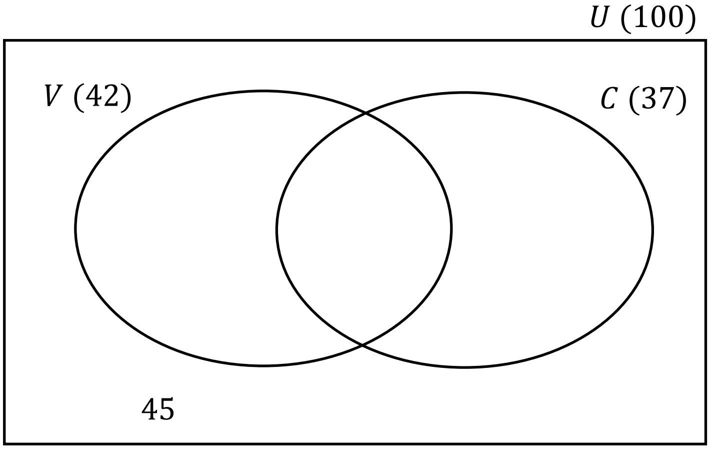
Calculations:

(b) One day, Diarmuid recorded the ice cream choices of the first 100 customers.

- Vanilla (V) flavour was bought by 42 customers.
- A cone (C) was bought by 37 customers.
- 45 customers bought neither vanilla nor a cone.

This information is shown in the Venn diagram below.

(i) Complete the Venn diagram below, to show the number of customers in each region.



(ii) A customer is picked at random from the first 100 customers.
Find the probability that this customer bought a cone with a flavour other than vanilla.

This question continues on the next page.

(c) In general, sales of ice creams increase as weather conditions improve.

Diarmuid's typical ice cream sales for different weather conditions are shown in the table below. The table also shows the probability of each weather condition next Sunday.

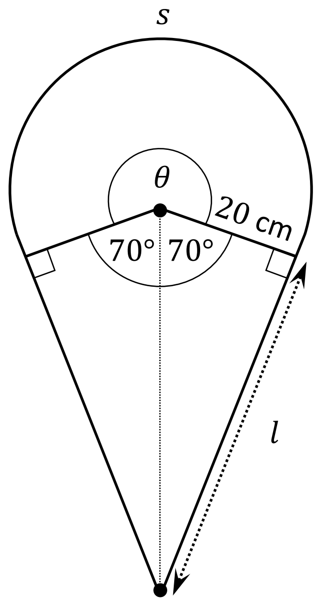
	Wet	**Dry and cold**	**Dry and warm**
Typical ice cream sales	€150	€200	€450
Probability of each weather condition	30%	45%	25%

Use the information in the table to work out the **expected value** of Diarmuid's ice cream sales next Sunday.

(d) Diarmuid has a logo of an ice cream cone on the side of his van.

The logo is made up of a sector of a circle with radius 20 cm, and two identical right-angled triangles, as shown in the diagram (not to scale).

The arc of the sector is labelled s.
The sizes of some of the angles are shown.



(i) Find the size of the angle θ **and** hence, find the length of the arc s.

Give the length in cm, correct to 1 decimal place.

$\theta =$ ______ Length of the arc $s =$ _______

(ii) Use trigonometry to work out the length of the side labelled l.
Give your answer correct to the nearest cm.

Question 10 **(50 marks)**

(a) In Wimbledon Tennis Tournament 2023, the fastest women's serve was by Aryna Sabalenka at 121 miles per hour. (*Source www.wimbledon.org*)

(i) Use the conversion rate of $1 \text{ mile} = 1{\cdot}61$ km to convert 121 miles to km.

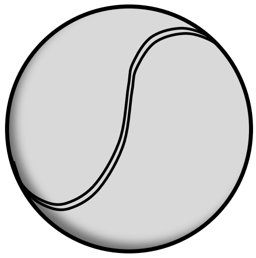
(ii) Convert one hour to seconds.

(iii) Use your answers to **parts (a)(i)** and **(a)(ii)** to convert 121 miles per hour to metres per second.
Give your answer correct to 2 decimal places.

(b) Tennis balls are in the shape of a sphere. For professional tennis matches, tennis balls can be of slightly different sizes.

The smallest size allowed has a radius of $3{\cdot}27$ cm.

Find the **surface area** of this tennis ball.
Give your answer in cm^2, correct to 2 decimal places.

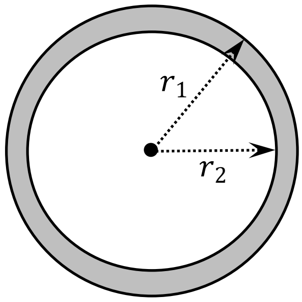


(c) The cross-section of a tennis ball is shown in the diagram on the right.

The ball is made up of an air-filled centre and a rubber layer of uniform thickness.

The **external** radius (r_1) of the tennis ball is $3{\cdot}4$ cm.
The **internal** radius (r_2) of the rubber layer is $3{\cdot}1$ cm.

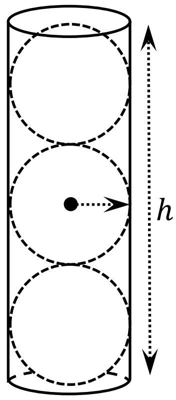
Find the volume of the rubber layer.
Give your answer in cm^3, correct to 2 decimal places.



This question continues on the next page.

(d) The diagram on the right shows the smallest closed cylindrical container that contains three identical tennis balls, arranged one directly on top of the other.

The radius of each tennis ball is $3{\cdot}4$ cm.
The height of the cylinder is h.

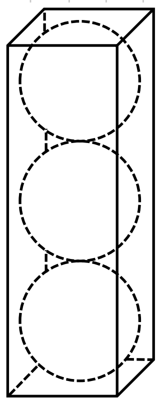


(i) Show that $h = 20{\cdot}4$ cm.

(ii) Hence, calculate the **volume** of the cylinder.
Give your answer correct to nearest whole number.

(iii) The three tennis balls could also be arranged, one directly on top of the other, inside a **closed rectangular box**, as shown below.

Find the **surface area** of the smallest such rectangular box, in cm^2.



Page for extra work.
Label any extra work clearly with the question number and part.

Do not write on this page

Leaving Certificate – Ordinary Level

Mathematics Paper 2

Monday 10 June

Morning 9:30 - 12:00