Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2015

Mathematics

Paper 1

Ordinary Level

Friday 5 June   Afternoon 2:00 – 4:30

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

| Section A | Concepts and Skills | 150 marks | 6 questions |
| Section B | Contexts and Applications | 150 marks | 3 questions |

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. You may ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A	Concepts and Skills	150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

Padraic works in America and travels between Ireland and America.

(a) In Ireland, he exchanged €2000 for US dollars when the exchange rate was €1 = $1·29. Find how many US dollars he received.

(b) Padraic returned to Ireland and exchanged $21 000 for euro. He received €15 000. Write the exchange rate for this transaction in the form €1 = $□·□□.

€1 = $ _____

(c) Padraic wants to exchange some dollars for sterling. On a day when the euro to dollar exchange rate is €1 = $1·24 and the euro to sterling exchange rate is €1 = £0·83, find the dollar to sterling exchange rate. Write your answer in the form $1 = £ □·□□.

$1 = £ _____

Question 2 (25 marks)

(a) John, Mary and Eileen bought a ticket in a draw. The ticket cost €50. John paid €25, Mary paid €15 and Eileen paid €10. The ticket won a prize of €20 000. The prize is divided in proportion to how much each paid. How much prize money does each person receive?

(b) Assuming that the Earth is a sphere of radius 6378 km:
    (i) Find the length of the equator, correct to the nearest km.

Equator
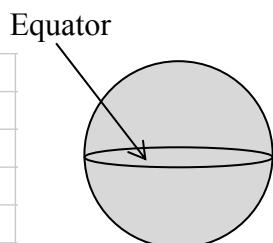
    (ii) Find the volume of the Earth in the form $a \times 10^n$, where $1 \le a < 10$ and $n \in \mathbb{N}$. Give the value of a correct to three decimal places.

(c) The mass of the Earth is $5 \cdot 97 \times 10^{24}$ kg. The mass of the Sun is $1 \cdot 99 \times 10^{30}$ kg. How many times greater than the mass of the Earth is the mass of Sun? Give your answer correct to the nearest whole number.

Question 3 (25 marks)

(a) Simplify $3(4 - 5x) - 2(5 - 6x)$.
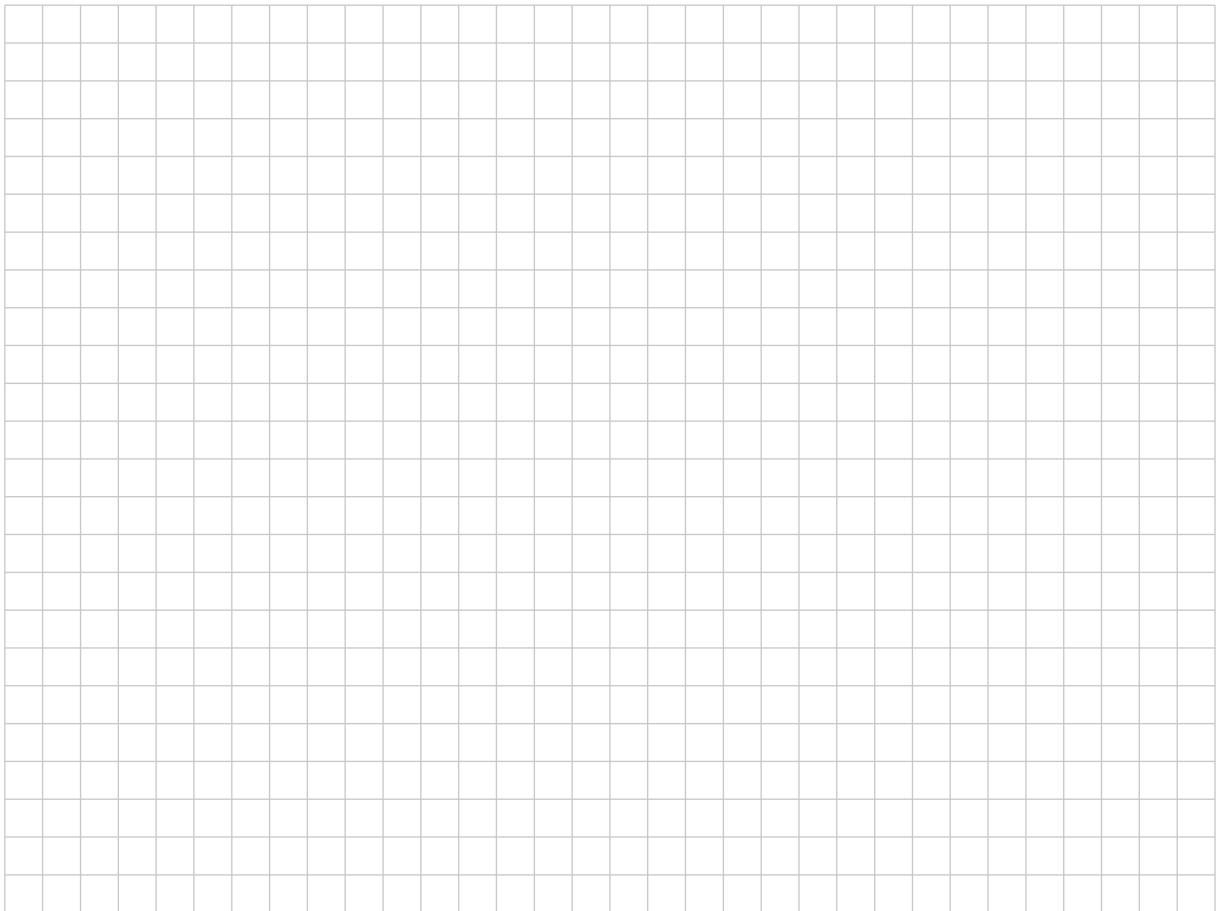
(b) List all the values of x that satisfy the inequality $2 - 3x \geq -6, \ x \in \mathbb{N}$.

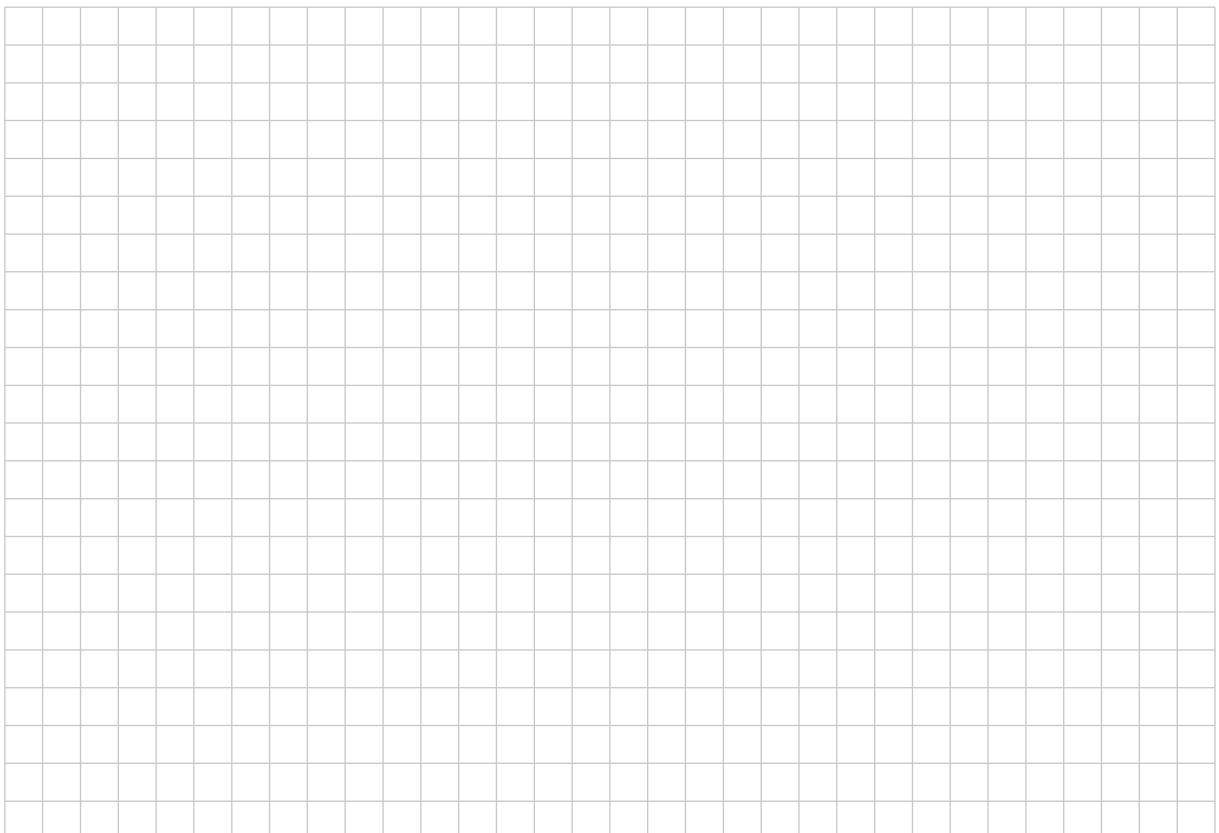(c) $g(x)$ is a function and $(2 - 3x) \times g(x) = 15x^2 - 22x + 8$, for all $x \in \mathbb{R}$. Find $g(x)$.
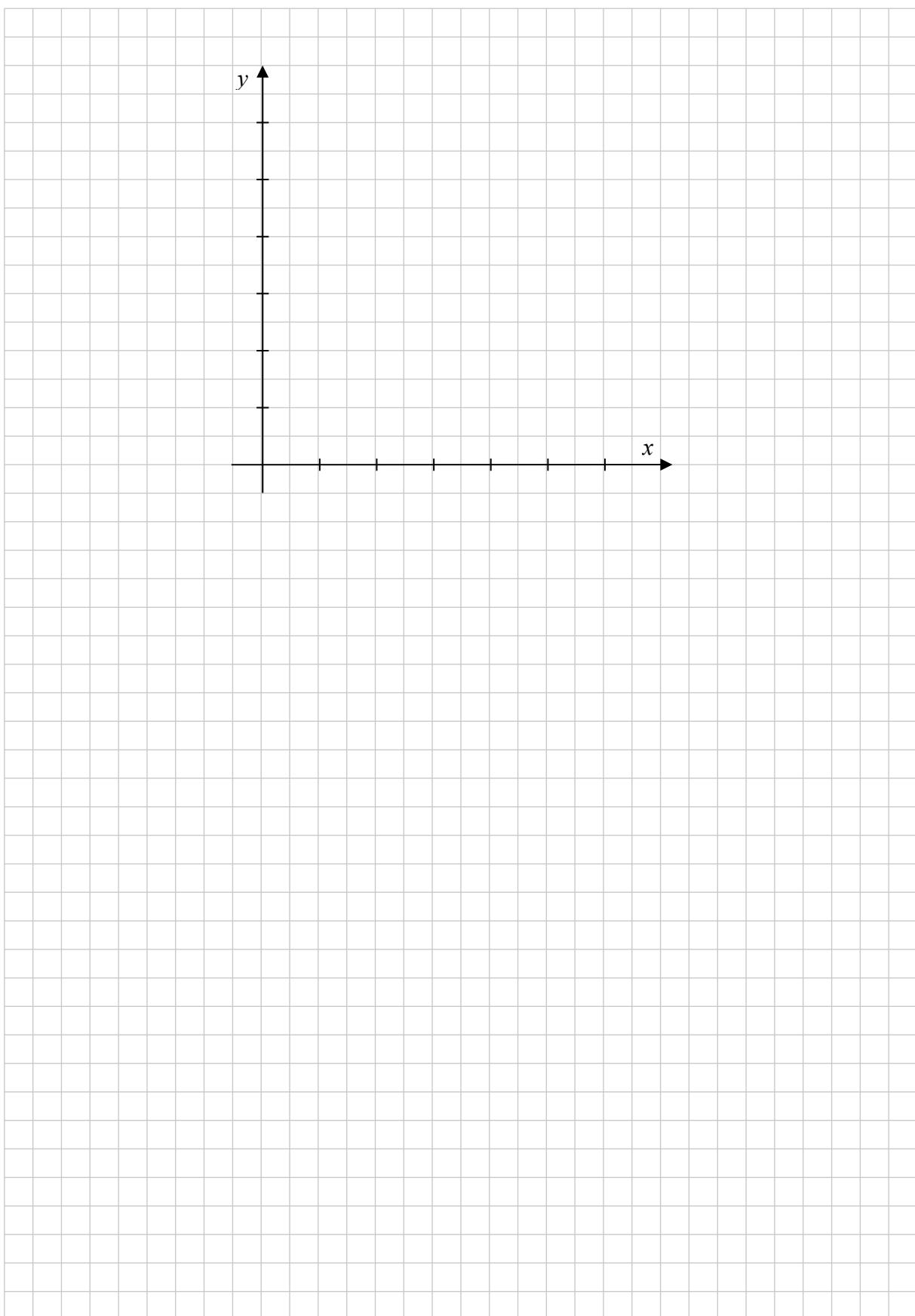
Question 4 **(25 marks)**

(a) Solve the equation $-x^2 + 6x - 4 = 0$. Give each solution correct to one decimal place.

(b) Find the co-ordinates of the turning point of the function $f(x) = -x^2 + 6x - 4, \ x \in \mathbb{R}$.

(c) Use your answers to parts **(a)** and **(b)** above to sketch the curve $y = f(x)$.
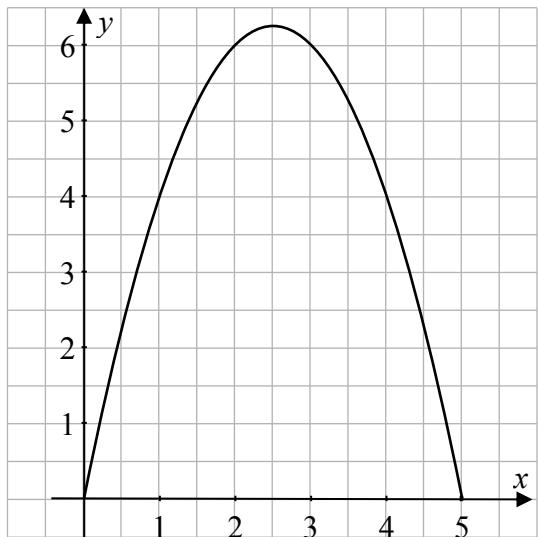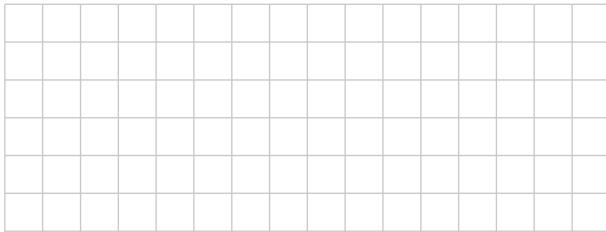Show your scale on both axes.

Question 5 (25 marks)

The diagram shows the graph of the function
$f(x) = 5x - x^2$ in the domain $0 \le x \le 5$, $x \in \mathbb{R}$.

(a) The function g is $g(x) = x + 3$, $x \in \mathbb{R}$.
The points $A(1, k)$ and B are the points
of intersection of f and g.
Find the co-ordinates of A and of B.

(b) The points $O(0, 0)$ and $C(5, 0)$ are on the graph of f.
  (i) Draw the quadrilateral $OCBA$ on the diagram above.

  (ii) Find the area of the quadrilateral $OCBA$.
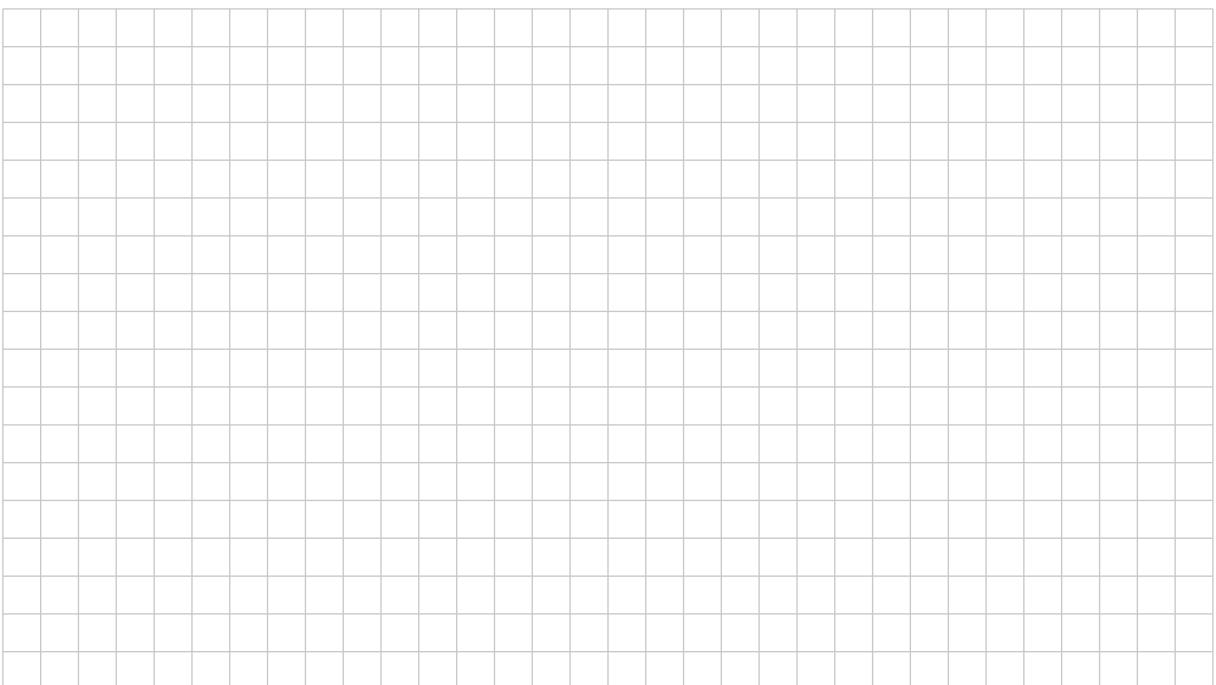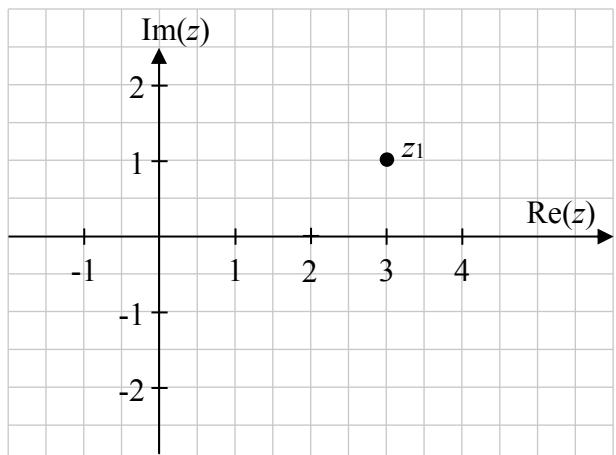
Question 6 (25 marks)

(a) The complex number $z_1 = a + bi$, where $i^2 = -1$, is shown on the Argand diagram below.

(i) Write down the value of a and the value of b.

$a = \underline{\hspace{1.5cm}}$ $b = \underline{\hspace{1.5cm}}$

(ii) The image of z_1 under reflection in the real axis is $z_2 = c + di$. Write down the value of c and the value of d.

$c = \underline{\hspace{1.5cm}}$ $d = \underline{\hspace{1.5cm}}$

(b) **(i)** The angle θ is formed by joining z_1 to $0 + 0i$ to z_2.
Find $\cos\theta$, correct to one decimal place.

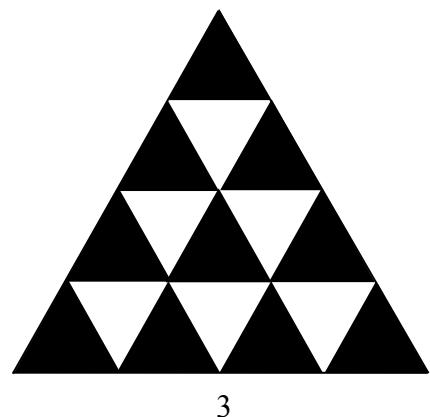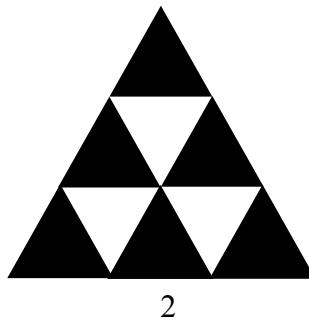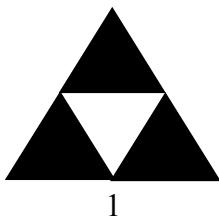**(ii)** Show that $|z_1| \times |z_2| \times \cos\theta = ac + bd$, where $a, b, c,$ and d are the values from part **(a)** above.

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Answer **all three** questions from this section.

Question 7   (70 marks)

The first three patterns in a sequence of patterns are shown below.

(a) Draw the fourth pattern in the sequence.

(b) Complete the table below.

	Number of Black Triangles	Number of White Triangles	Total Number of Small Triangles
Pattern 1	3	1	T_1 = 4
Pattern 2			T_2 =
Pattern 3			T_3 =
Pattern 4			T_4 =
Pattern 5			T_5 =

(c) Show that the numbers of **black** triangles form a quadratic sequence.

(d) **(i)** How many **black** triangles are in the 9th pattern?　　　Answer: _____

(ii) How many **white** triangles are in the 9th pattern?　　　Answer: _____

(iii) How many small triangles, in total, are in the 9th pattern?　Answer: T_9 = _____

(e) Write an expression in n for the total number of triangles in the nth pattern.

Answer: T_n = _____

(f) The number of **black** triangles in the nth pattern is given by the formula $B_n = \dfrac{1}{2}n^2 + \dfrac{3}{2}n + c.$

Find the value of c.

(g) Use your answers to parts **(e)** and **(f)** above to find a formula for the number of **white** triangles in the nth pattern.

(h) One particular pattern has a total of 625 triangles. Find the number of black triangles and the number of white triangles in that pattern.
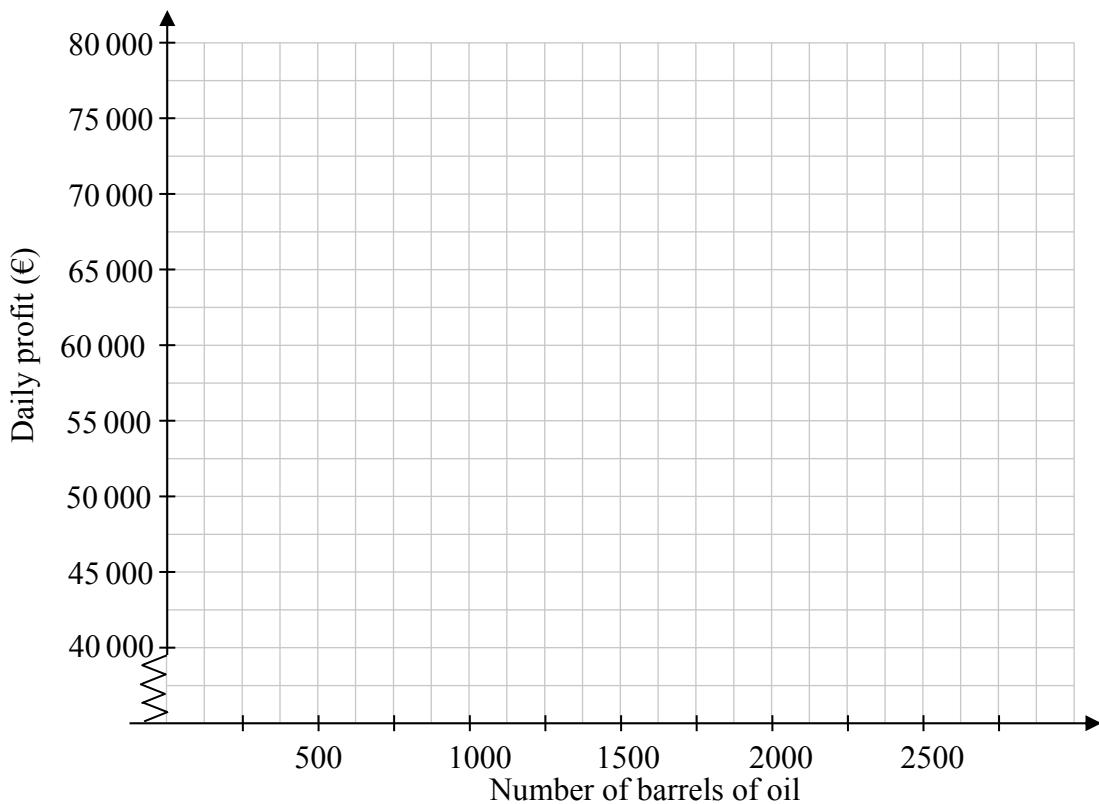
Question 8 (50 marks)

The daily profit of an oil trader is given by the profit function $p = 96x - 0.03x^2$, where p is the daily profit, in euro, and x is the number of barrels of oil traded in a day.

(a) Complete the table below.

Number of barrels traded in a day	x	500	1000	1500	2000	2500
Daily profit (€)	p	40 500				
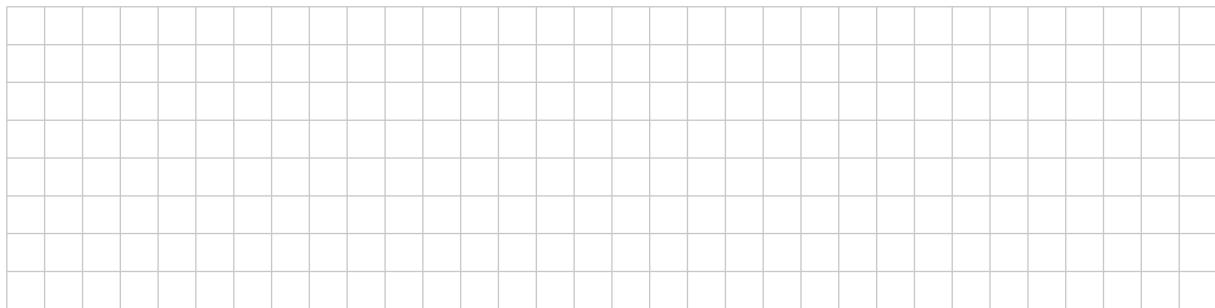
(b) Draw the graph of the trader's profit function on the axes below for $500 \leq x \leq 2500,\ x \in \mathbb{R}$.
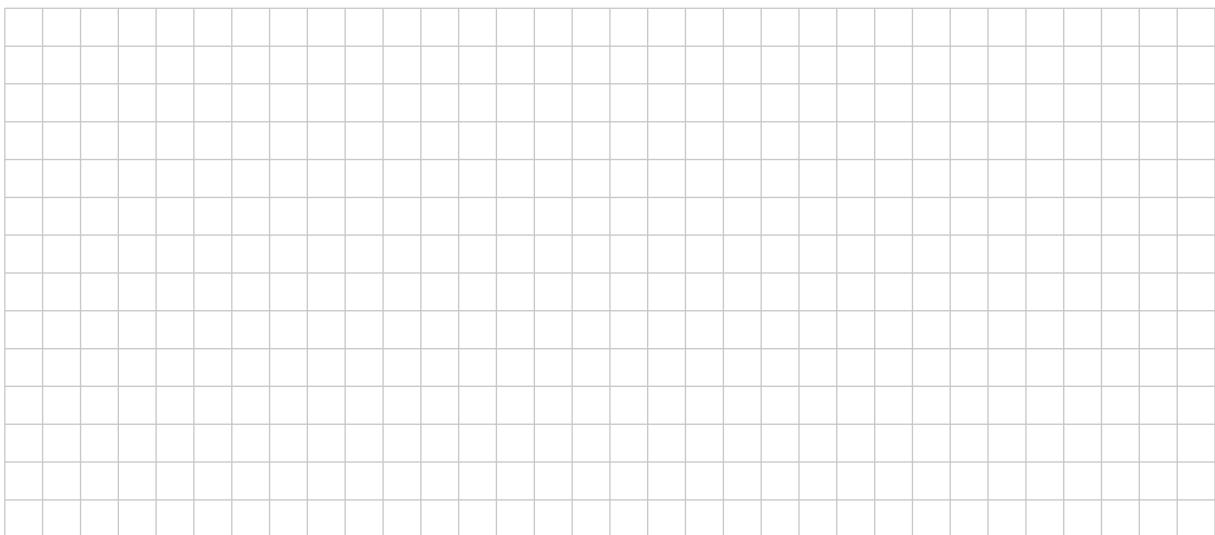

(c) Use your graph to estimate:

(i) The daily profit when 1750 barrels are traded.   Answer: _____

(ii) The numbers of barrels traded when the daily profit is €60 000.

Answer: _____ or _____

(d) **(i)** Use calculus to find the number of barrels of oil traded that will earn the maximum daily profit.
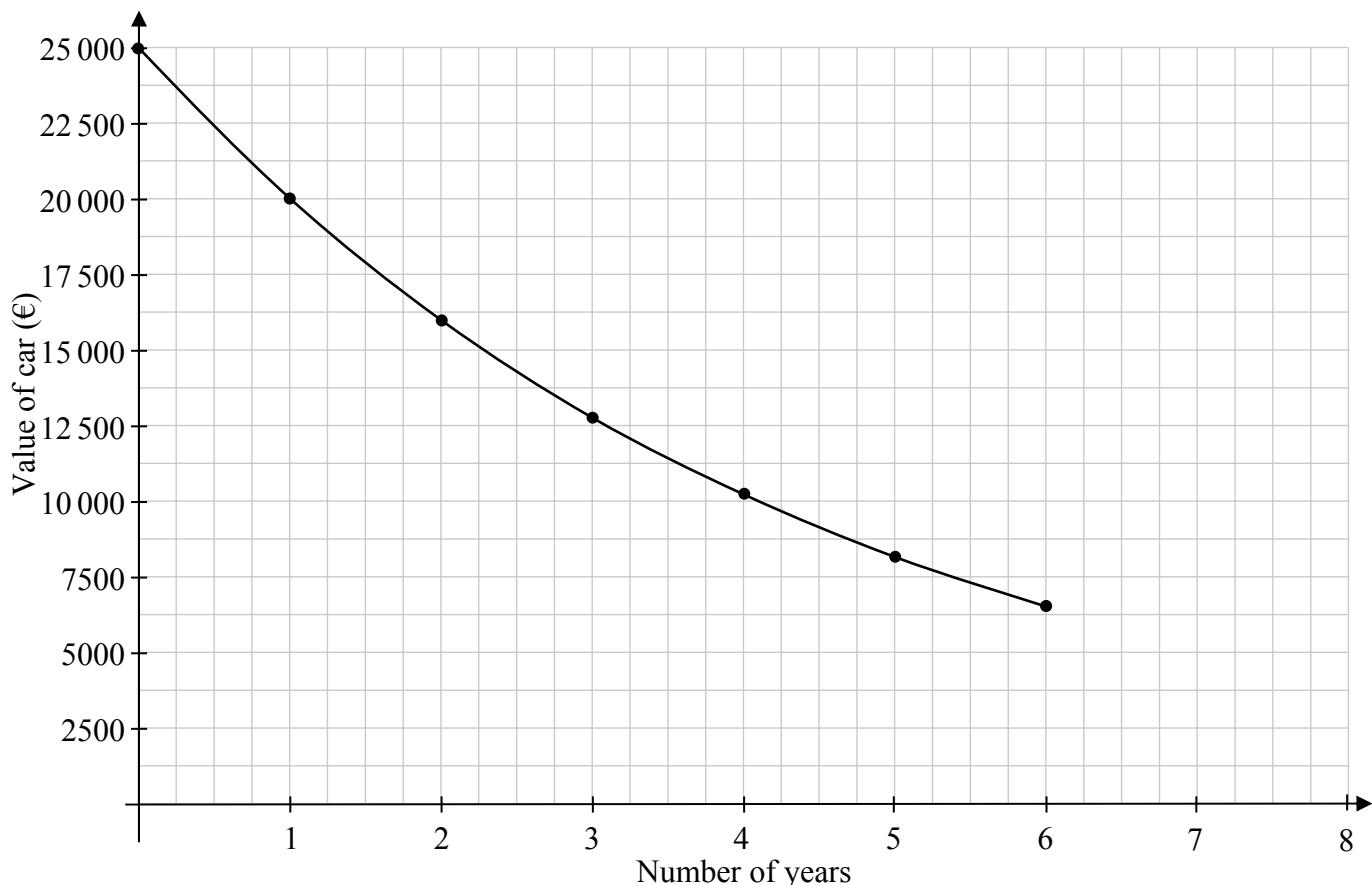
(ii) Find this maximum profit.

(e) The trader will not make a profit if he trades more than k barrels of oil in a day. Calculate the value of k.

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Question 9   **(30 marks)**

Amanda buys a new car for €25 000. She knows that the value of cars depreciates by a fixed percentage each year. She draws the graph below to represent the value of her car for the next six years.

(a)   **(i)**   Use Amanda's graph to estimate the length of time it will take for the car to depreciate from €15 000 to €10 000.

   Answer: _____

  (ii)   Continue Amanda's graph to find the approximate value of the car after 7·5 years.

   Answer: _____

  (iii)   What name is given to the type of curve Amanda has drawn above?

           Answer: _____

(b) On the day that Amanda bought her car Cathal also bought a new car. His car cost €22 500. Amanda's car will travel 15 km on a litre of fuel. Cathal's car will travel 10 km on a litre of fuel. How many kilometres will the cars have to travel until the cost price of the car plus the cost of fuel are equal, for the two cars? Assume that fuel costs €1·50 per litre over the period.

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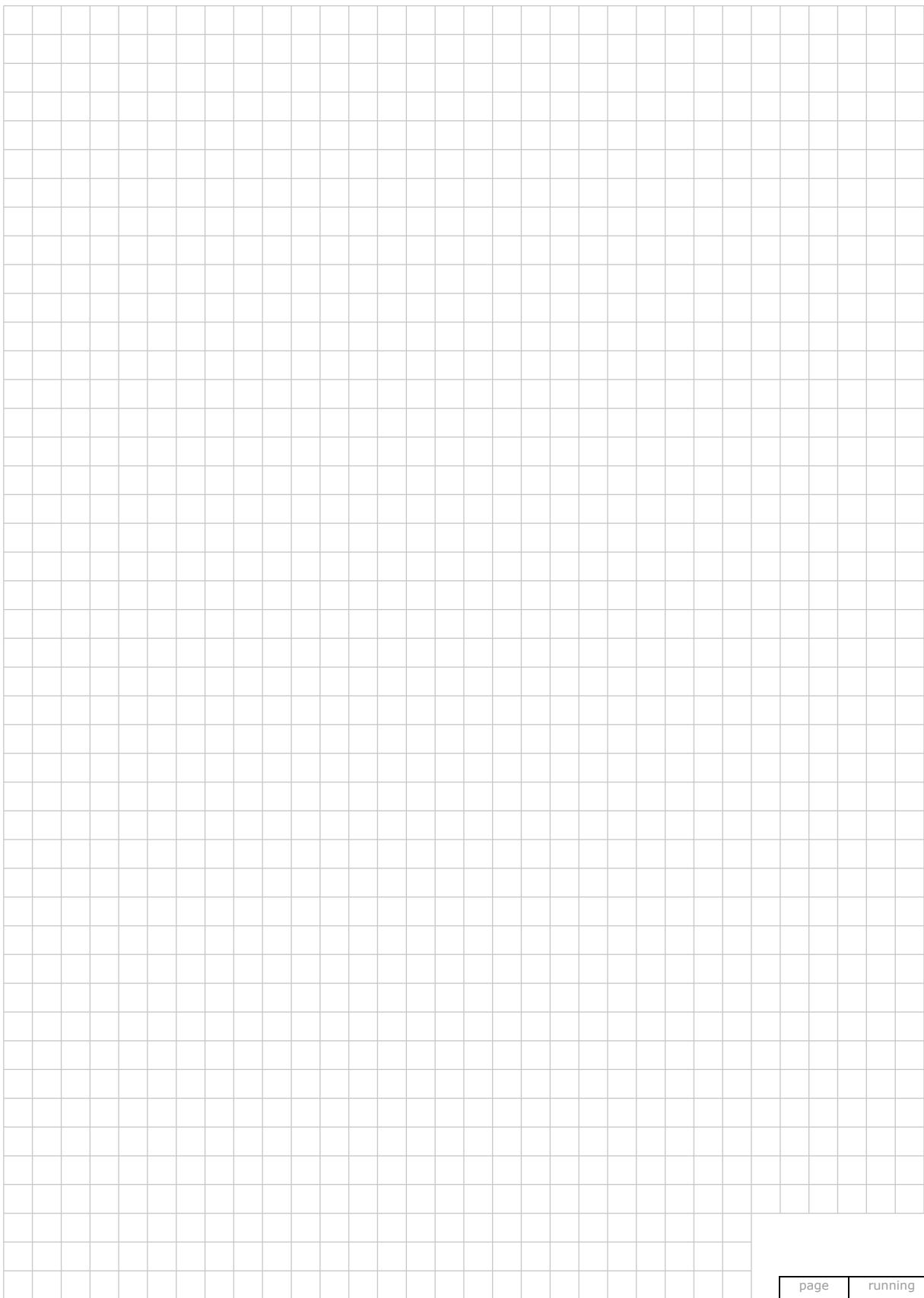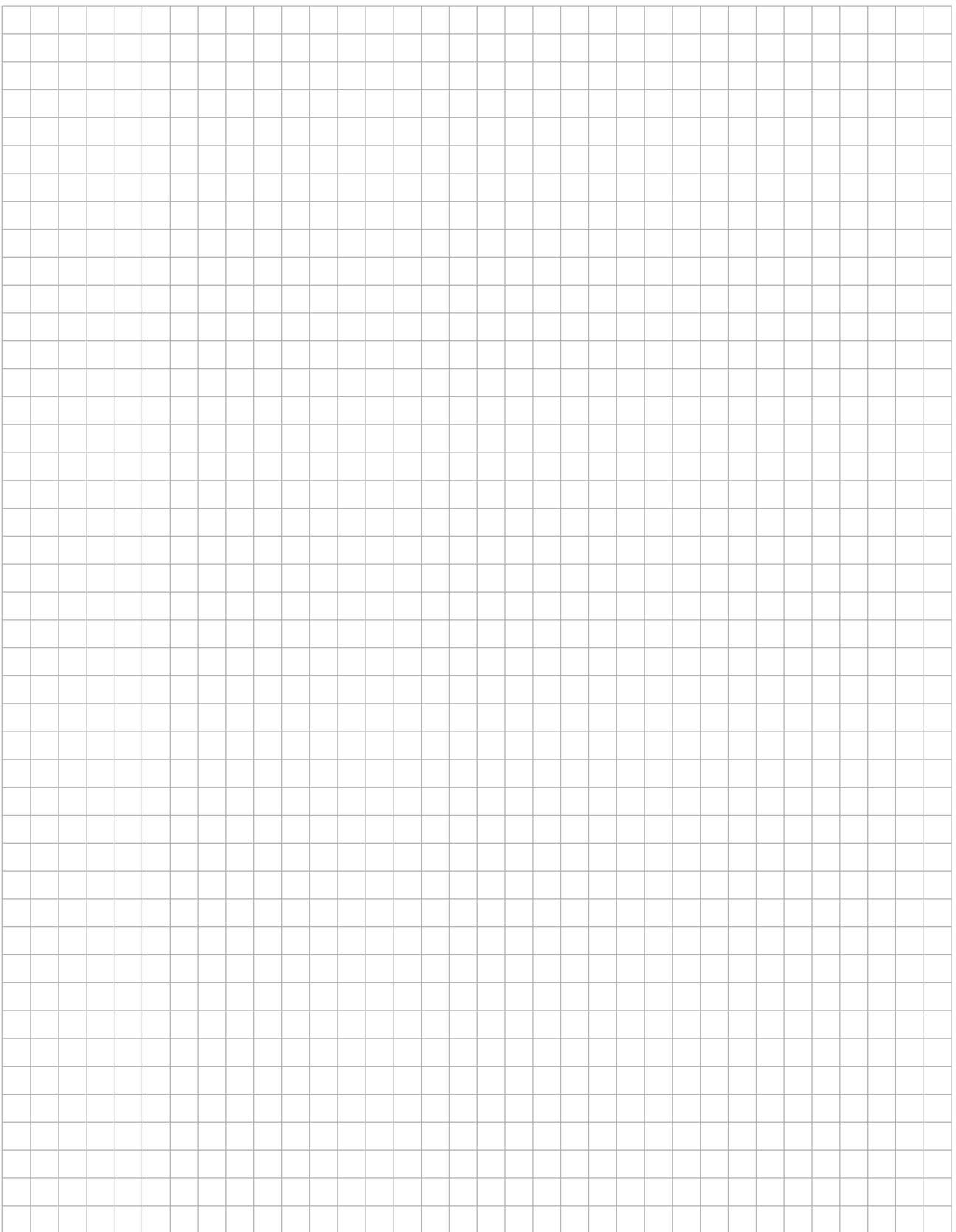
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