Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2013

Mathematics
(Project Maths – Phase 2)

Paper 2

Higher Level

Monday 10 June Morning 9:30 – 12:00

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer **all nine** questions, as follows:

In Section A, answer

Questions 1 to 5 and

either Question 6A **or** Question 6B.

In Section B, answer Questions 7 to 9.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A — Concepts and Skills — 150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) Explain each of the following terms:

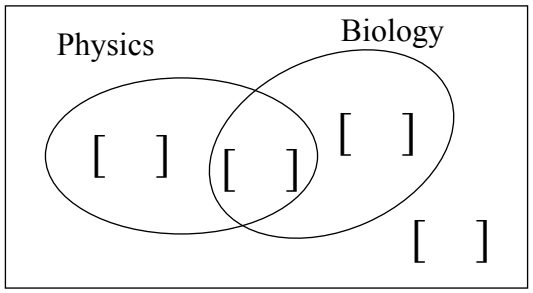
(i) Sample space

(ii) Mutually exclusive events

(iii) Independent events.

(b) In a class of 30 students, 20 study Physics, 6 study Biology and 4 study both Physics and Biology.

(i) Represent the information on the Venn Diagram.



A student is selected at random from this class.
The events E and F are:

E: The student studies Physics

F: The student studies Biology.

(ii) By calculating probabilities, investigate if the events E and F are independent.

Question 2 **(25 marks)**

(a) A random variable X follows a normal distribution with mean 60 and standard deviation 5.

(i) Find $P(X \leq 68)$.

(ii) Find $P(52 \leq X \leq 68)$.

(b) The heights of a certain type of plant, when ready to harvest, are known to be normally distributed, with a mean of μ. A company tests the effects of three different growth hormones on this type of plant. The three hormones were used on a different large sample of the crop. After applying each hormone, it was found that the heights of the plants in the samples were still normally distributed at harvest time.

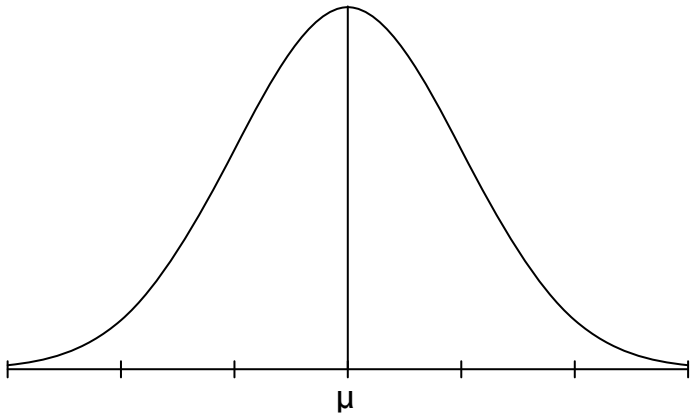
The diagrams A, B and C, on the next page, show the expected distribution of the heights of the plants, at harvest time, without the use of the hormones.

The effect, on plant growth, of each of the hormones is described on the next page. Sketch, on each diagram, a new distribution to show the effect of the hormone.

Hormone A

The effect of hormone A was to increase the height of all of the plants.

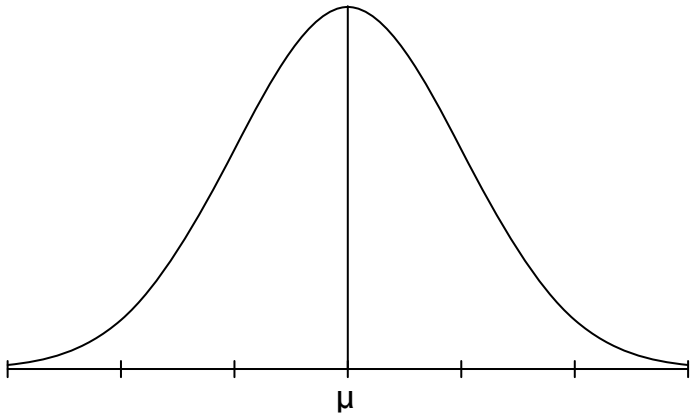
Diagram A



Hormone B

The effect of hormone B was to reduce the number of really small plants and the number of really tall plants. The mean was unchanged.

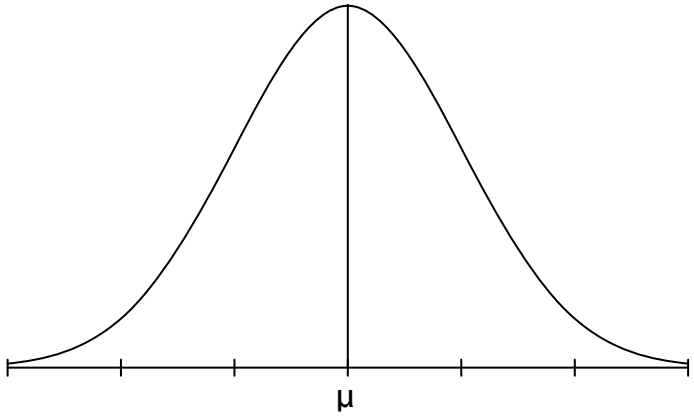
Diagram B



Hormone C

The effect of hormone C was to increase the number of small plants and the number of tall plants. The mean was unchanged.

Diagram C



Question 3 **(25 marks)**

The equations of six lines are given:

Line	Equation
h	$x = 3 - y$
i	$2x - 4y = 3$
k	$y = -\frac{1}{4}(2x - 7)$
l	$4x - 2y - 5 = 0$
m	$x + \sqrt{3}y - 10 = 0$
n	$\sqrt{3}x + y - 10 = 0$

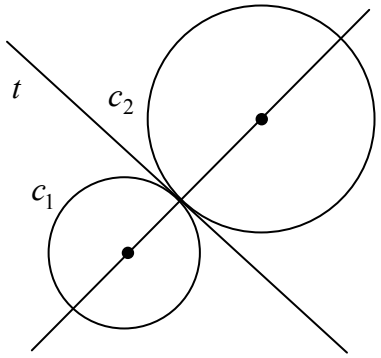
(a) Complete the table below by matching each description given to one or more of the lines.

Description	**Line(s)**
A line with a slope of 2.	
A line which intersects the y-axis at $(0, -2\frac{1}{2})$.	
A line which makes equal intercepts on the axes.	
A line which makes an angle of $150°$ with the positive sense of the x-axis.	
Two lines which are perpendicular to each other.	

(b) Find the acute angle between the lines m and n.

Question 4 **(25 marks)**

The circles c_1 and c_2 touch externally as shown.



(a) Complete the following table:

Circle	Centre	Radius	Equation
c_1	$(-3, -2)$	2	
c_2			$x^2 + y^2 - 2x - 2y - 7 = 0$

(b) **(i)** Find the co-ordinates of the point of contact of c_1 and c_2.

(ii) Hence, or otherwise, find the equation of the tangent, *t*, common to c_1 and c_2.

Question 5 **(25 marks)**

(a) In a triangle ABC, the lengths of the sides are a, b and c. Using a formula for the area of a triangle, or otherwise, prove that

$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}.$$

(b) In a triangle XYZ, $|XY| = 5$ cm, $|XZ| = 3$ cm and $|\angle XYZ| = 27°$.

(i) Find the two possible values of $|\angle XZY|$. Give your answers correct to the nearest degree.

(ii) Draw a sketch of the triangle XYZ, showing the two possible positions of the point Z.

(c) In the case that $|\angle XZY| < 90°$, write down $|\angle ZXY|$, and hence find the area of the triangle XYZ, correct to the nearest integer.

$|\angle ZXY| =$

Question 6 **(25 marks)**

Answer **either** 6A **or** 6B.

Question 6A

(a) Complete each of the following statements.

(i) The circumcentre of a triangle is the point of intersection of _____

_____

(ii) The incentre of a triangle is the point of intersection of _____

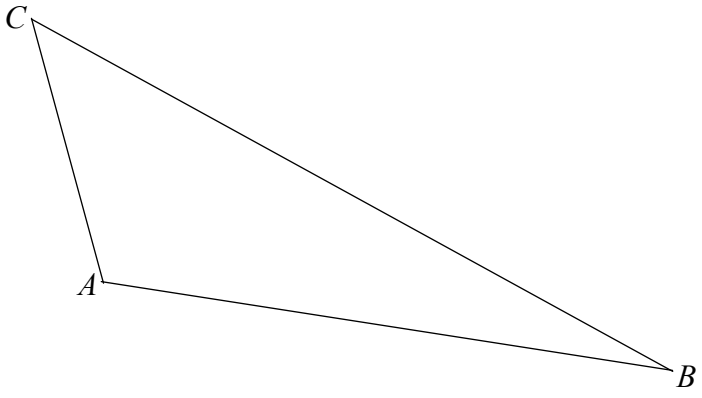
_____

(iii) The centroid of a triangle is the point of intersection of _____

_____

(b) In an equilateral triangle, the circumcentre, the incentre and the centroid are all in the same place. Explain why this is the case.

(c) Construct the orthocentre of the triangle *ABC* below. Show all construction lines clearly.



OR

Question 6B

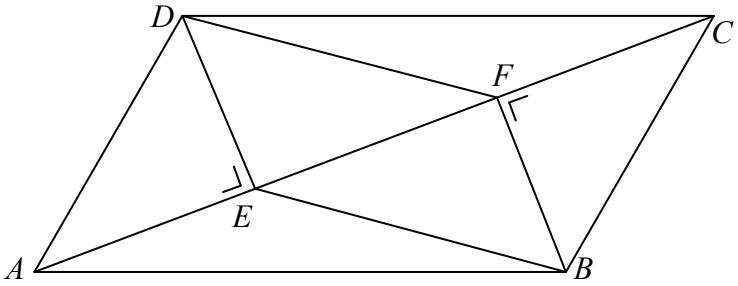
(a) A quadrilateral (four sided figure) has two sides which are parallel and equal in length. Prove that the quadrilateral is a parallelogram.

(b) In the parallelogram *ABCD,*

DE is perpendicular to *AC.*

BF is perpendicular to *AC.*

Prove that *EBFD* is a parallelogram.



Section B — Contexts and Applications — 150 marks

Answer **all three** questions from this section.

Question 7 (75 marks)

Go Fast Airlines provides internal flights in Ireland, short haul flights to Europe and long haul flights to America and Asia. On long haul flights the company sells economy class, business class and executive class tickets. All passengers have a baggage allowance of 20 kg and must pay a cost per kg for any weight over the 20 kg allowance.
Each month the company carries out a survey among 1000 passengers. Some of the results of the survey for May are shown below.

Gender	Male: 479	Female: 521

Previously flown with *Go Fast Airlines*	Yes: 682	No: 318
Would fly again with *Go Fast Airlines*	Yes: 913	No: 87

Passenger Age	Mean age: 42 Median age: 31

Spend on in-flight facilities	Mean spend: €18·65 Median spend: €32·18

Was flight delayed?	Yes	No	Don't Know
	231	748	21

Passenger satisfaction with overall service	Satisfied	Not satisfied	Don't Know
	664	238	98

(a) *Go Fast Airlines* used a **stratified random sample** to conduct the survey.

(i) Explain what is meant by a **stratified random sample**.

(ii) Write down 4 different passenger groups that the company might have included in their sample.

(b) **(i)** What is the probability that a passenger selected at random from this sample

- had his/her flight delayed

Answer: _____

- was not satisfied with the overall service.

Answer: _____

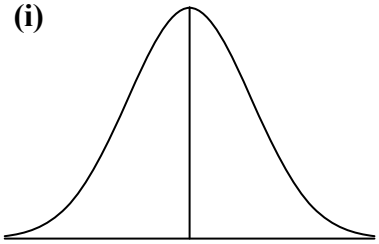
(ii) An employee suggests that the probability of selecting a passenger whose flight was delayed and who was not satisfied with the overall service should be equal to the product of the two probabilities in **(i)** above. Do you agree with the employee?

Answer: _____

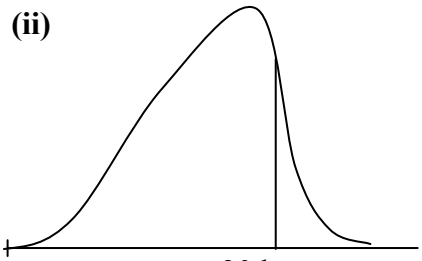
Give a reason for your answer.

(c) Which of the graphs below do you think is most likely to represent the distribution of the weights of passenger baggage?

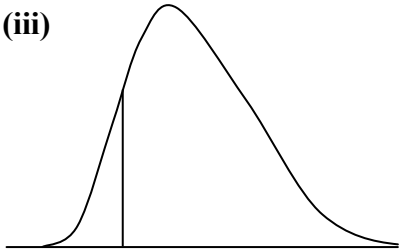
(i)



(ii)



(iii)



Answer: _____

Give a reason for your answer.

(d) **(i)** Draw a sketch of the possible distribution of the ages of the passengers based on the data in the survey.

(ii) Explain your answer.

(e) **(i)** The company repeatedly asserts that 70% of their customers are satisfied with their overall service Use an hypothesis test at the 5% level of significance to decide whether there is sufficient evidence to conclude that their claim is valid in May. Write the null hypothesis and state your conclusion clearly.

(ii) A manager of the airline says: "If we survey 2000 passengers from June on, we will halve the margin of error in our surveys." Is the manager correct?

Answer: _____

Explain your answer.

(f) The responses of ten individual passengers to the questions on age and in-flight spend are given below.

Age (years)	46	29	37	18	25	75	52	35	40	31
In-flight spend (euro)	30	15	20	0	10	45	25	20	20	30

(i) Draw a scatter plot of the data.

(ii) Calculate the correlation coefficient between passenger age and in-flight spend.

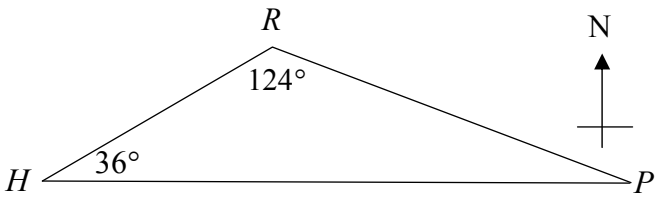
Answer: _____

(iii) What can you conclude from the completed scatter plot and the correlation coefficient?

(iv) Sketch the line of best fit in the completed scatter plot above.

Question 8 **(30 marks)**

(a) A port P is directly east of a port H. To sail from H to P, a ship first sails 80 km, in the direction shown in the diagram, to the point R before turning through an angle of 124° and sailing 110 km directly to P.

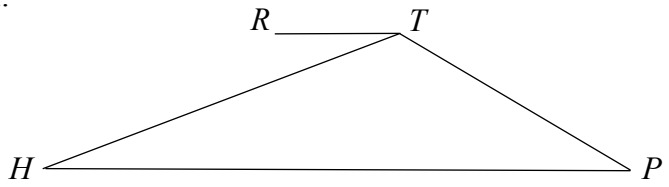


(i) Find the distance from R to HP.

(ii) Calculate $|HP|$.

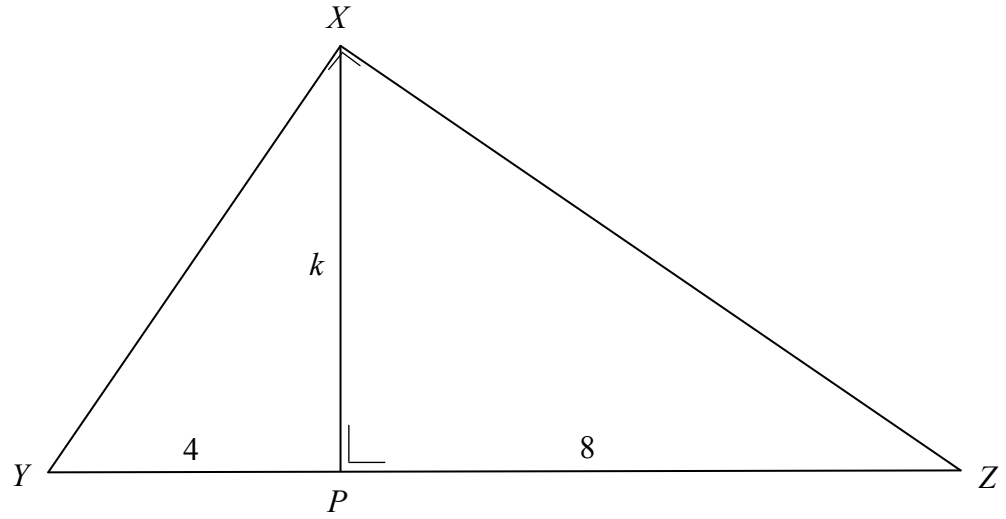
(b) The point T is directly east of the point R.
$|HT| = 110$ km and $|TP| = 80$ km.

Find $|RT|$.

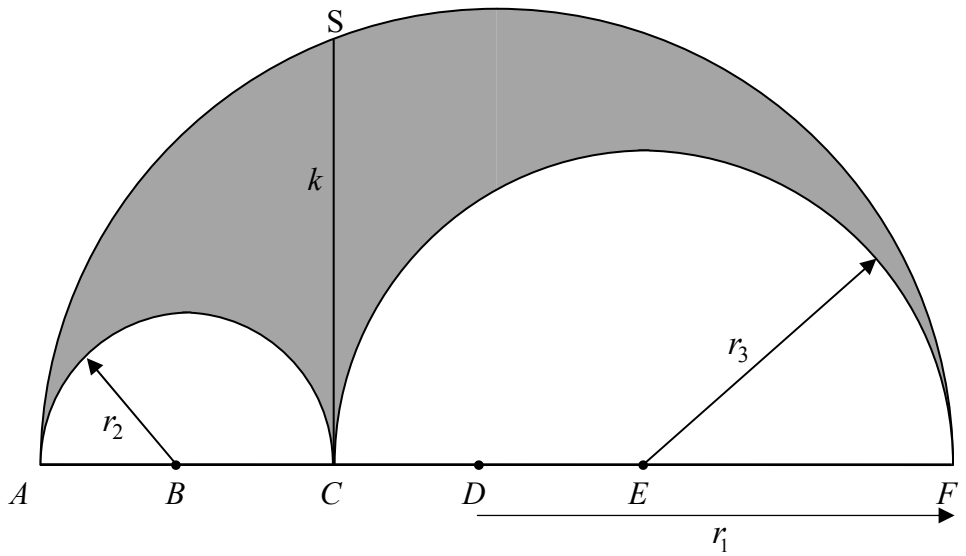


Question 9 **(45 marks)**

(a) The triangle XYZ is right-angled at X and XP is perpendicular to YZ.
$|YP| = 4$, $|PZ| = 8$ and $|PX| = k$. Find the value of k.



(b) The shaded region in the diagram below is called an **arbelos.** It is a plane semicircular region of radius r_1 from which semicircles of radius r_2 and r_3 are removed, as shown. In the diagram $SC \perp AF$ and $|SC| = k$.



(i) Show that, for fixed r_1, the perimeter of the arbelos is independent of the values of r_2 and r_3.

(ii) If $r_2 = 2$ and $r_3 = 4$, show that the area of the arbelos is the same as the area of the circle of diameter k.

(c) To investigate the area of an arbelos, a student fixed the value of r_1 at 6 cm and completed the following table for different values of r_2 and r_3.

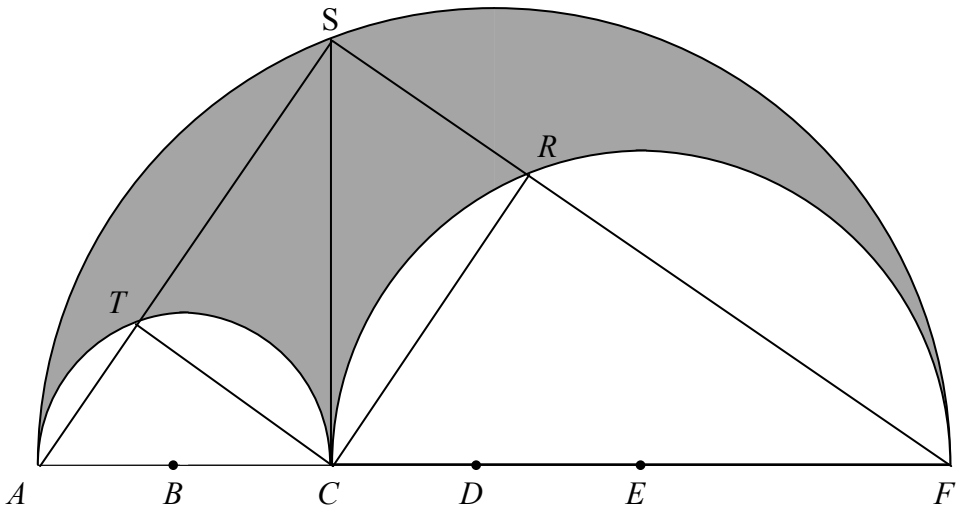
(i) Complete the table.

r_1	r_2	r_3	Area of arbelos
6	1		
6	2		
6	3		
6	4		
6	5		

(ii) In general, for $r_1 = 6$ cm and $r_2 = x$, $0 < x < 6$, $x \in \mathbb{R}$, find an expression in x for the area of the arbelos.

(iii) Hence, or otherwise, find the maximum area of an arbelos that can be formed in a semicircle of radius 6 cm.

(d) *AS* and *FS* cut the two smaller semicircles at *T* and *R* respectively.
Prove that *RSTC* is a rectangle.



You may use this page for extra work.

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