Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2008

MATHEMATICS – ORDINARY LEVEL

PAPER 1 (300 marks)

FRIDAY, 6 JUNE – MORNING, 9:30 to 12:00

Attempt **SIX QUESTIONS** (50 marks each).

WARNING: Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

1. **(a)** John works from 09:00 hours to 13:00 hours and again from 14:00 hours to 17:30 hours. He is paid €18·50 per hour.
Find his total pay for the day.

(b) Alice frequently travels from her home to Cork, a distance of 85 km. The journey usually takes 1 hour 15 minutes.

(i) Find her average speed in kilometres per hour for the journey.

(ii) On a day of very heavy rain her average speed on a 28 km section of the journey was reduced to 35 km/h.
How long did this section of the journey take on that day?

(iii) How much longer did the total journey take on that day, if she completed the rest of the journey at her usual average speed?
Give your answer correct to the nearest minute.

(c) A retailer buys an item for €73. She wants to apply a mark-up of 40% of the cost price of the item. She must then add VAT at 21% to this amount to find the price that she would need to charge the customer.

(i) Find this price, correct to the nearest cent.

The retailer adjusts the price charged to the customer so that it is 1 cent less than a multiple of €10, while keeping the mark-up as close as possible to 40%.

(ii) Using this adjusted price, calculate the actual percentage mark-up achieved, correct to the nearest percent.

2. **(a)** Simplify $3(4x+5)-2(6x+4)$.

(b) **(i)** Solve $x^2-4x+1=0$.
Write your solutions in the form $a \pm \sqrt{b}$, where $a, b \in \mathbf{N}$.

(ii) Find the value of x for which

$$\frac{5^x}{3}=\frac{5^6}{75}.$$

(c) **(i)** Factorise x^2+4x+4.

(ii) Simplify $\sqrt{x^2+4x+4} + \sqrt{x^2+2x+1}$, given that $x \geq 0$.

(iii) Given that $x \geq 0$, solve for x

$$\sqrt{x^2+4x+4} + \sqrt{x^2+2x+1} = x^2.$$

3. **(a)** Given that $a(x+5)=8$, express x in terms of a.

(b) **(i)** Solve for x and y

$$x-y=1$$
$$x^2+y^2=25.$$

(ii) Hence, find the two possible values of $x-y^2$.

(c) **(i)** Let $f(x)=x^2+bx+c$, $x \in \mathbf{R}$.
The graph of the function f intersects the y-axis at 3 and the x-axis at -1.
Find the value of b and the value of c.

(ii) The lengths of the sides of an isosceles triangle are $\sqrt{x^2+1}$, $\sqrt{x^2+1}$ and $2x$.
Taking $2x$ as the base, find the perpendicular height of the triangle.

4. **(a)** Let $u=3-4i$, where $i^2=-1$.
Plot on an Argand diagram

(i) u

(ii) $u+5i$.

(b) Let $w=2+5i$.

(i) Express w^2 in the form $x+yi$, where $x, y \in \mathbf{R}$.

(ii) Verify that $|w^2|=|w|^2$.

(c) Let $z=6-4i$.

(i) Find the real number k such that

$$k(z+\bar{z})=24$$

where $\bar{z}$ is the complex conjugate of z.

(ii) Find the real numbers s and t such that

$$\frac{s+ti}{4+3i}=z.$$

5. **(a)** Find the eleventh term of the arithmetic sequence 5, 14, 23 …

(b) The *n*th term of a geometric sequence is

$$T_n = \frac{3^n}{27}.$$

(i) Find *a*, the first term.

(ii) Find *r*, the common ratio.

(iii) The *k*th term of the sequence is 243. Find *k*.

(c) The sum of the first *n* terms of an arithmetic series is given by $S_n = n^2 - 16n$.

(i) Use S_1 and S_2 to find the first term and the common difference.

(ii) Find T_n, the *n*th term of the series.

(iii) Find the values of $n \in \mathbf{N}$ for which $S_n = -63$.

6. **(a)** Let $g(x) = 2x - 5$, where $x \in \mathbf{R}$.
Find the value of *x* for which $g(x) = 19$.

(b) Differentiate $3x^2 + 5$ with respect to *x* from first principles.

(c) Let $f(x) = \frac{x^2 - x}{1 - x^3}$, $x \in \mathbf{R}$, $x \neq 1$.

(i) Find $f'(x)$, the derivative of $f(x)$.

(ii) Show that the tangent to the curve $y = f(x)$ at the point (0, 0) makes an angle of $135°$ with the positive sense of the *x*-axis.

7. **(a)** Differentiate with respect to x

(i) x^7

(ii) $5x - 3x^4$.

(b) **(i)** Differentiate $(1+3x)(4-x^2)$ with respect to x.

(ii) Given that $y = (3x^2 - 4x)^8$, find $\frac{dy}{dx}$ when $x = 1$.

(c) A distress flare is tested by firing it vertically upwards from the top of a tower. The height, h metres, of the flare above the ground is given by

$$h = 20 + 90t - 5t^2$$

where t is the time in seconds from the instant the flare is fired.
The flare is designed to explode 7 seconds after firing.

(i) Find the height above the ground at which the flare explodes.

(ii) Find the speed of the flare at the instant it explodes.

(iii) If the flare failed to explode, find the greatest height above the ground it would reach before falling back down.

8. Let $f(x) = x^3 - 9x^2 + 24x - 18$, where $x \in \mathbf{R}$.

(i) Find $f(1)$ and $f(5)$.

(ii) Find $f'(x)$, the derivative of $f(x)$.

(iii) Find the co-ordinates of the local maximum point and of the local minimum point of the curve $y = f(x)$.

(iv) Draw the graph of the function f in the domain $1 \le x \le 5$.

(v) Use your graph to write down the range of values of x for which $f'(x) < 0$.

(vi) The line $y = -3x + c$ is a tangent to the curve $y = f(x)$. Find the value of c.

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