Coimisiún na Scrúduithe Stáit
State Examinations Commission

Junior Certificate Examination 2014

Mathematics
(Project Maths – Phase 3)

Paper 2

Higher Level

Model Solutions – Paper 2

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Question 1   **20 Marks**

Pauline flips a fair coin 3 times, and records the outcomes.
She writes *H* for each head and *T* for each tail.

(i)   Complete the table below to show all of the possible outcomes.
        Two outcomes have already been filled in for you.

H H H	*T H H*
H H T	*T H T*
H T H	*T T H*
H T T	*T T T*

(ii)   Find the probability of getting two heads and one tail.

$$\Pr\left(2\,H,\,1\,T\right)=\frac{3}{8}$$

(iii)   Jamie says: "You have the same probability of getting three heads as you do of getting two heads and one tail."
       Do you agree with Jamie? Give a reason for your answer.

        Answer: No

        Reason: $\Pr\left(3\,H\right)=\dfrac{1}{8}$ but $\Pr\left(2\,H,1\,T\right)=\dfrac{3}{8}$

Or:
        Reason: There is only 1 way to get three heads. There are 3 ways to get two heads and one tail.

(iv)   Max says: "You have the same probability of getting *H H H* as you do of getting *H T H*."
       Do you agree with Max? Give a reason for your answer.

        Answer: Yes

        Reason: $\Pr\left(H\,H\,H\right)=\dfrac{1}{8}$ and $\Pr\left(H\,T\,H\right)=\dfrac{1}{8}$
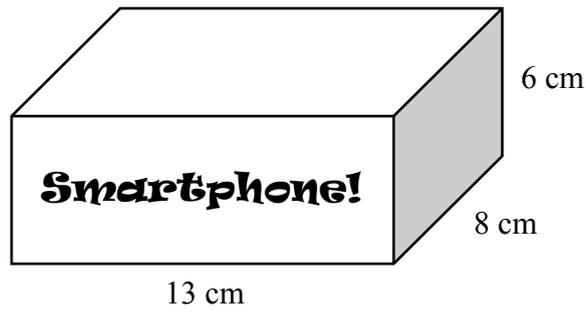
Or:
        Reason: There is only one way to get each event.
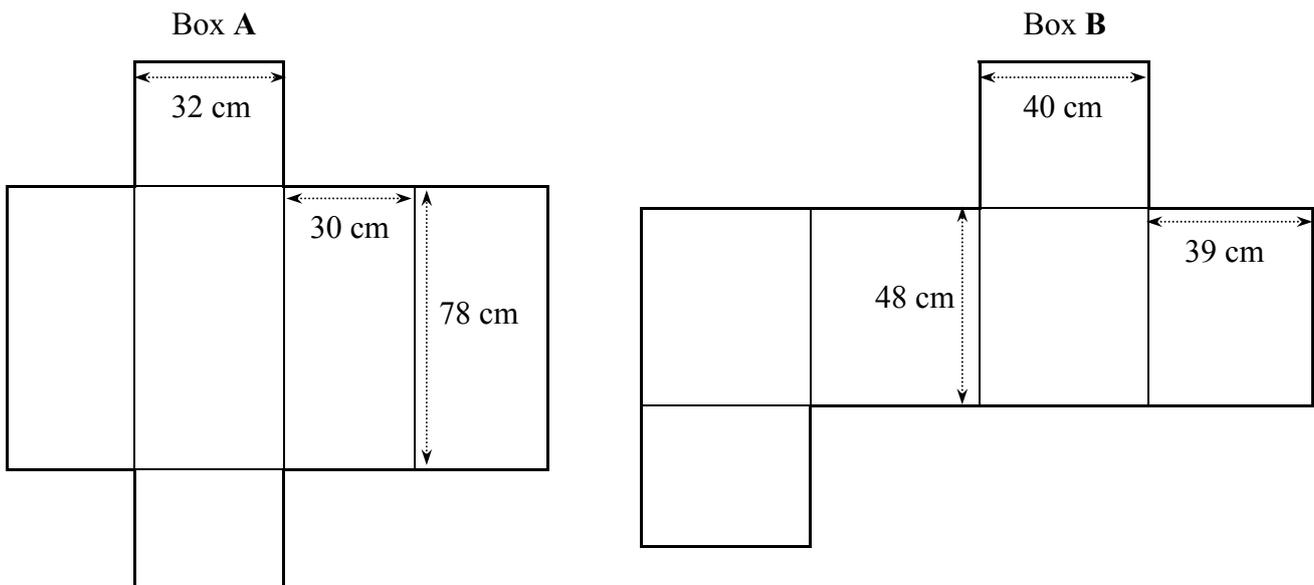
Question 2 35 Marks

The box for an individual mobile phone is 13 cm long, 8 cm wide, and 6 cm high, as shown.

(i) Find the volume of an individual mobile phone box.

Volume $= 13 \times 8 \times 6 = 624$ cm^3

These individual mobile phone boxes will be shipped in a large rectangular box.
Below are diagrams of the nets of two large boxes that could be used, Box **A** and Box **B**.

Box **A**

Box **B**

(ii) Show that Box **A** and Box **B** have the same volume.

Box **A**:	Box **B**:
Volume $= 32 \times 30 \times 78 = 74\ 880$ cm^3	Volume $= 48 \times 40 \times 39 = 74\ 880$ cm^3

(iii) What is the largest number of individual mobile phone boxes that will fit in each large box?

> Box **A**: $32 \div 8 = 4$; $30 \div 6 = 5$; $78 \div 13 = 6$; so Box **A** can be filled completely.
>
> Box **B**: $48 \div 6 = 8$; $40 \div 8 = 5$; $39 \div 13 = 3$; so Box **B** can be filled completely.
>
> Total: $4 \times 5 \times 6 = 120$ individual phone boxes.

(iv) Find the surface area of each large box.

Box **A**:	Box **B**:
Surface Area $= 2(32 \times 30 + 32 \times 78 + 30 \times 78)$ $= 11\ 592$ cm^2.	Surface Area $= 2(48 \times 40 + 48 \times 39 + 40 \times 39)$ $= 10\ 704$ cm^2.

(v) The large boxes are made from cardboard. The cardboard costs €0·67 per m^2. The cardboard just covers the net of a box. Find the cost of the box that uses the least amount of cardboard.

> *Use Box **B**. The cost is given per m^2, so convert surface area to m^2 (or cost to per cm^2).*
>
> $\quad 1$ cm $= 0\cdot01$ m, so 1 cm$^2 = 0\cdot01^2$ m$^2 = 0\cdot0001$ m^2.
>
> $\quad$ Surface area $= 10\ 704$ cm$^2 = 10\ 704 \times 0\cdot0001$ m$^2 = 1\cdot0704$ m^2.
>
> $\quad$ Cost of box $= €1\cdot0704 \times 0\cdot67$
>
> $\qquad\qquad\qquad = €0\cdot717168$
>
> $\qquad\qquad\qquad = €0\cdot72$, to the nearest cent.

(vi) An average of 140 large boxes is produced each month. Find the saving, per annum, if you choose to make the box that uses the least amount of cardboard.

> $\quad$ Cost of Box A $= €(11592 \times 0\cdot0001 \times 0\cdot67)$
>
> $\qquad\qquad\qquad\quad = €0\cdot776664$
>
> $\qquad\qquad\qquad\quad = €0\cdot78$, to the nearest cent.
>
> $\quad$ Saving per annum $= €(0\cdot78 - 0\cdot72) \times 140 \times 12$
>
> $\qquad\qquad\qquad\qquad = €(0\cdot06) \times 1680$
>
> $\qquad\qquad\qquad\qquad = €100\cdot80$.
>
> *Or*:
>
> $\quad$ Difference in area $= (11592 - 10704)$ cm$^2 = 888$ cm$^2 = 0\cdot0888$ m^2.
>
> $\quad$ Saving per annum $= €0\cdot67 \times 0\cdot0888 \times 140 \times 12 = €99\cdot95$.

Question 3   **40 Marks**

All of the students in a class took *IQ Test 1* on the same day. A week later they all took *IQ Test 2*. Their scores on the two IQ tests are shown in the tables below.

IQ Test 1				
86	104	89	105	96
96	103	94	104	119
115	79	97	111	108

IQ Test 2				
83	120	105	111	114
99	111	108	106	97
97	102	94	108	117

(i) Draw a back-to-back stem-and-leaf plot below to display the students' scores.

IQ Test 1							IQ Test 2				
				9	7						
			9	6	8	3					
	7	6	6	4	9	4	7	7	9		
8	5	4	4	3	10	2	5	6	8	8	
		9	5	1	11	1	1	4	7		
					12	0					

Key: 9 | 7 = a score of 97

(ii) Find the range of scores for each IQ test.

Range of *IQ Test 1* $= 119 - 79 = 40$.	Range of *IQ Test 2* $= 120 - 83 = 37$.

(iii) Find the median score for each IQ test.

15 data points in each set, so median is the $\dfrac{15+1}{2} = 8th$ *data point.*	
Median of *IQ Test 1* $= 103$.	Median of *IQ Test 2* $= 106$.

(iv) Find the mean score for each IQ test.

Mean of *IQ Test 1* $= \dfrac{1506}{15} = 100 \cdot 4$.	Mean of *IQ Test 2* $= \dfrac{1572}{15} = 104 \cdot 8$.

(v)   Compare the scores on the two IQ tests. Refer to **at least one** measure of central tendency and **at least one** measure of variability (spread) in your answer.

> In general, the scores in *IQ Test 2* are slightly higher than in *IQ Test 1*, as both the mean and median are higher for *IQ Test 2*.
>
> The scores are slightly more spread out in *IQ Test 1* than in *IQ Test 2*, as the range is bigger for *IQ Test 1*; or The spread of scores is very similar, as the two ranges are almost the same.

(vi)   Marshall says that every student in the class must have done better on *IQ Test 2* than on *IQ Test 1*. Is Marshall correct? Explain your answer.

> Answer: No.
>
> Explanation: The person who got 119 on *IQ Test 1* could have got less, e.g. 94, on *IQ Test 2*.
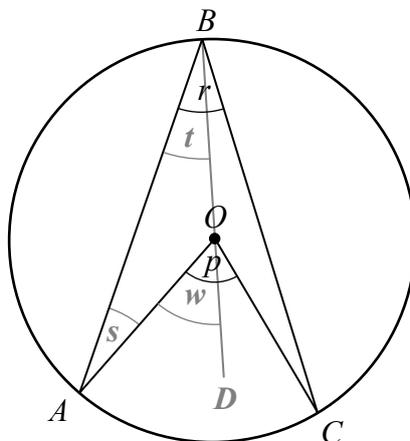>
> *Or*:
>
> Explanation: The maximum score on *IQ Test 1* is greater than the minimum score on *IQ Test 2*.

Question 4 30 Marks

(a) Prove that the angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.

Diagram:

Given: A circle with centre O. Points A, B, and C on the circle. Angles p and r, as shown.

To Prove: $p = 2r$.

Construction: Join B to O, and extend to D. Mark the angles s, t, and w.

Proof:	$|OA| = |OB|$	radii of circle	*Step 1*
	$\therefore s = t$	isosceles triangle	*Step 2*
	$w = s + t$	exterior angle	*Step 3*
	$\therefore w = 2t$		*Step 4*

Similarly, $(p - w) = 2(r - t)$.

So $p = (p - w) + w$
$\quad = 2(r - t) + 2t$
$\quad = 2r$ *Step 5*

(b) *P, Q, R,* and *S* are points on a circle with centre *O*.
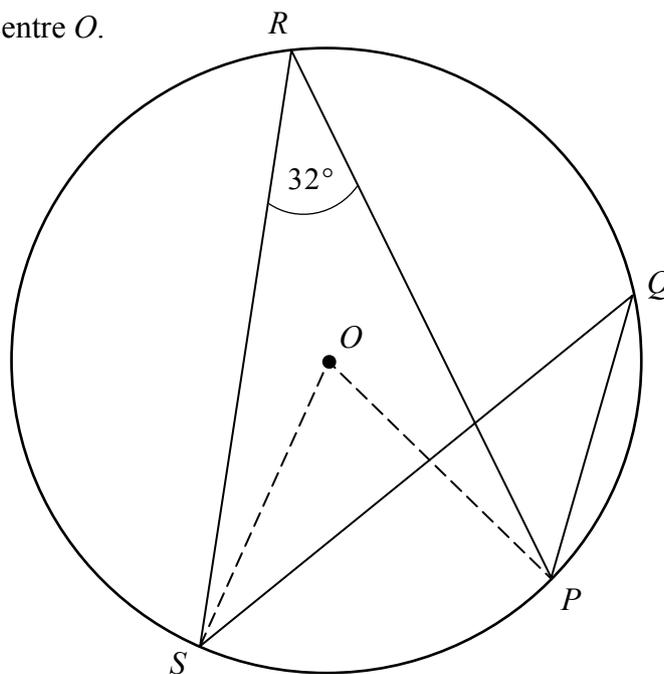$|\angle PRS| = 32°$, as shown.

(i) Find $|\angle SOP|$.

$$|\angle SOP| = 2 \times 32° = 64°.$$

(ii) Find $|\angle SQP|$.

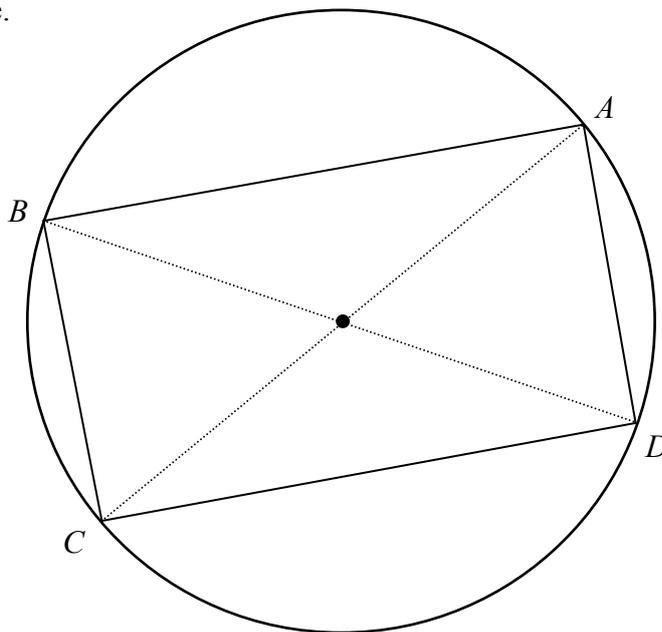$$|\angle SQP| = |\angle SRP| = 32°.$$

(c) *A, B, C,* and *D* are points on a circle, as shown below.
$[AC]$ and $[BD]$ are diameters of the circle.

Prove that *ABCD* is a rectangle.

We just need to prove that the four angles are 90°.

$|\angle BAD| = |\angle BCD| = 90°,$ as $[BD]$ is a diameter.

Similarly, $|\angle CBA| = |\angle CDA| = 90°.$

So *ABCD* is a rectangle.

Question 5   **45 Marks**

Students in a class are investigating spending in their local area. They each carry out a different survey, and display the results.

(a)   John is investigating whether people pay for their weekly shopping with Credit Card, Debit Card, Cash, or Cheque. When people tell him which one of these they usually use, he writes it in a table. His results are shown below.

Credit Card	Debit Card	Debit Card	Cash	Debit Card
Credit Card	Cash	Cash	Credit Card	Debit Card
Debit Card	Debit Card	Cheque	Cash	Cash
Cash	Cash	Debit Card	Cash	Credit Card

(i)   What type of data has John collected? Put a tick (✓) in the correct box below.

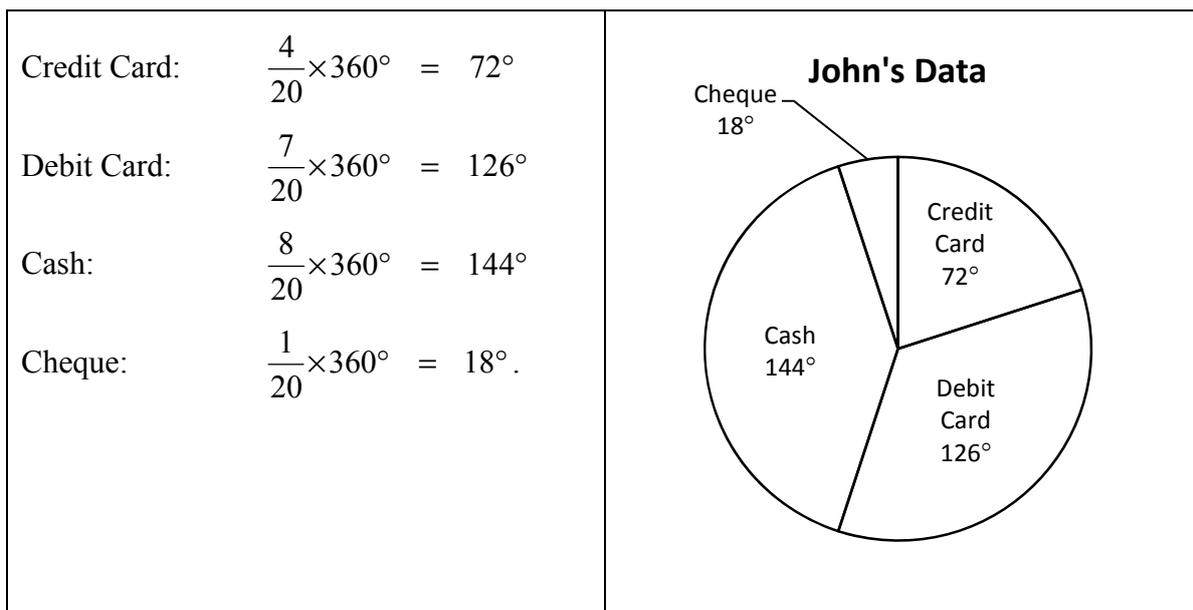| Numerical Continuous	Numerical Discrete	Categorical Nominal	Categorical Ordinal
☐	☐	☑	☐

(ii)   Fill in the frequency table below.

Method of Payment	Credit Card	Debit Card	Cash	Cheque
Frequency	4	7	8	1

(iii)   What is the mode of John's data?   Mode = | Cash |

(iv)   John says that he cannot find the mean of his data. Explain why this is the case.

He cannot add up his values and divide by 20.

(v) Display John's data in a pie chart. Show all of your calculations clearly.

Credit Card: $\dfrac{4}{20} \times 360° = 72°$

Debit Card: $\dfrac{7}{20} \times 360° = 126°$

Cash: $\dfrac{8}{20} \times 360° = 144°$

Cheque: $\dfrac{1}{20} \times 360° = 18°$.

John's Data

Cheque 18°
Credit Card 72°
Cash 144°
Debit Card 126°

(b) Margaret wants to examine if people prefer to do their weekly shopping in *Tesco*, *Dunnes Stores*, *SuperValu*, or *Lidl*. She stands outside her local *Lidl* shop for one day, and asks everyone as they leave the shop where they prefer to do their weekly shopping.

Give one reason why Margaret's data may be biased.

Margaret's data may be biased because her sample is probably not representative. She will probably have a lot more people answering "Lidl" than she should.

(c) Mary is interested in the amount of money people spend on their weekly shopping.
She surveys people as they leave the local supermarket on a Saturday morning, and displays her results in the two graphs below.
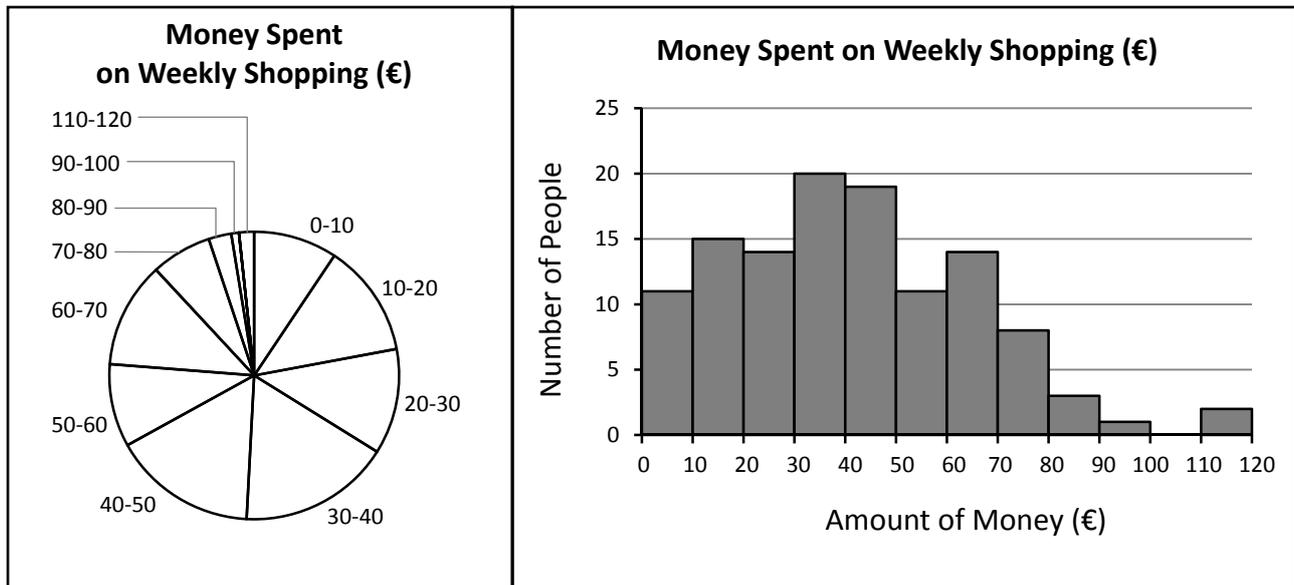
(i) Mary wants to show that about half of her sample spent less than €40 on their weekly shopping. Which graph do you think she should use? Give a reason for your answer.

Answer:	Pie chart.
Reason:	It's easy to see where half the pie chart is (180°).

(ii) Mary wants to show that there were more people in the 30–40 group than in any other. Which graph do you think she should use? Give a reason for your answer.

Answer:	Bar chart / Histogram.
Reason:	It's easy to see which bar is highest.

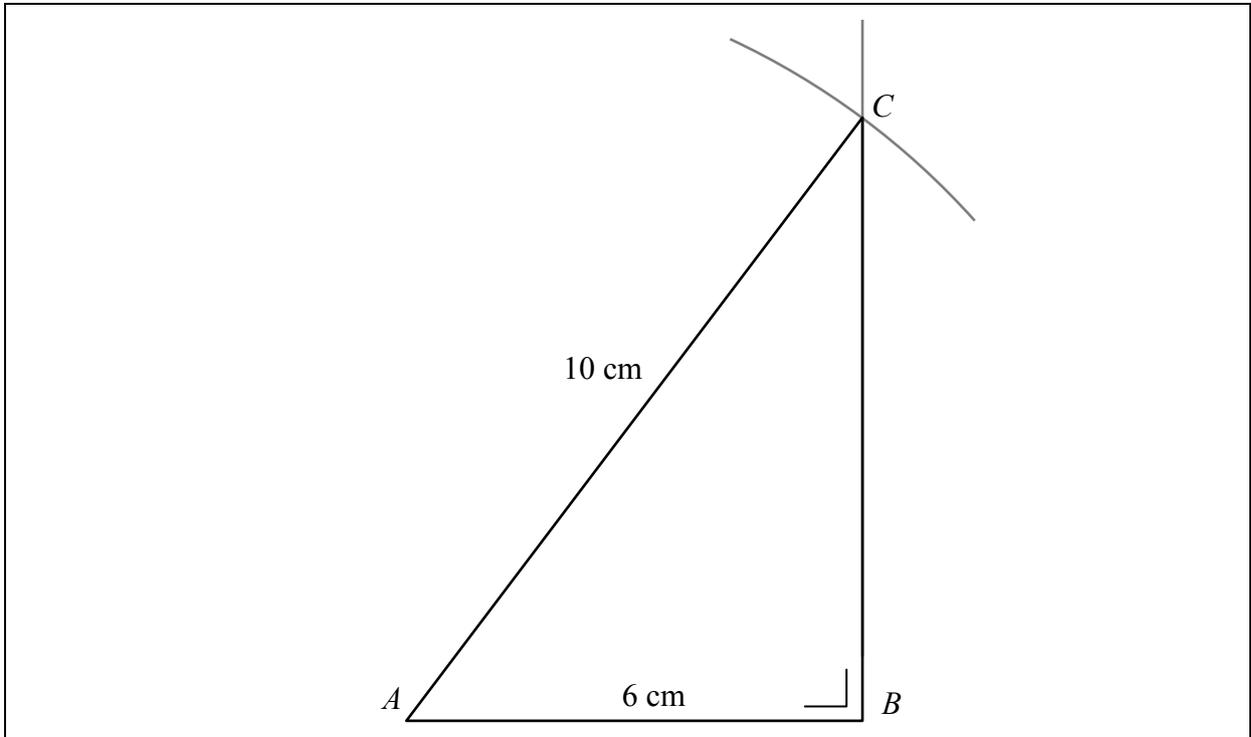**Question 6**   **25 Marks**

(i) Construct a right-angled triangle *ABC,* where:

$$|AB| = 6 \text{ cm}$$
$$|\angle ABC| = 90°$$
$$|AC| = 10 \text{ cm.}$$

Note: *It is also possible to work out the length of the third side,* $[BC]$*, using the Theorem of Pythagoras, and then construct* $[BC]$ *and* $[AC]$*.*

(ii) On your diagram, measure the angle $\angle CAB$. Give your answer correct to the nearest degree.

$$|\angle CAB| = \boxed{\quad 53° \quad}$$

(iii) Let *X* be the whole number you wrote as your answer to **(ii)**.

Use a calculator to find $\cos X$. Give your answer correct to 3 decimal places.

$\cos(53°) = 0 \cdot 6018... = 0 \cdot 602$, correct to three decimal places.

(iv) Jacinta says that $\cos(\angle CAB)$ is exactly $0 \cdot 6$, because $\cos(\angle CAB) = \dfrac{\text{adjacent}}{\text{hypotenuse}}$.

Explain why your answer in **(iii)** is **not** the same as Jacinta's.

They are not the same because $|\angle CAB| = \cos^{-1}\left(\dfrac{6}{10}\right) = 53 \cdot 1301...°$.

So if *X* is a whole number then $\cos X$ can never be exactly $0 \cdot 6$.
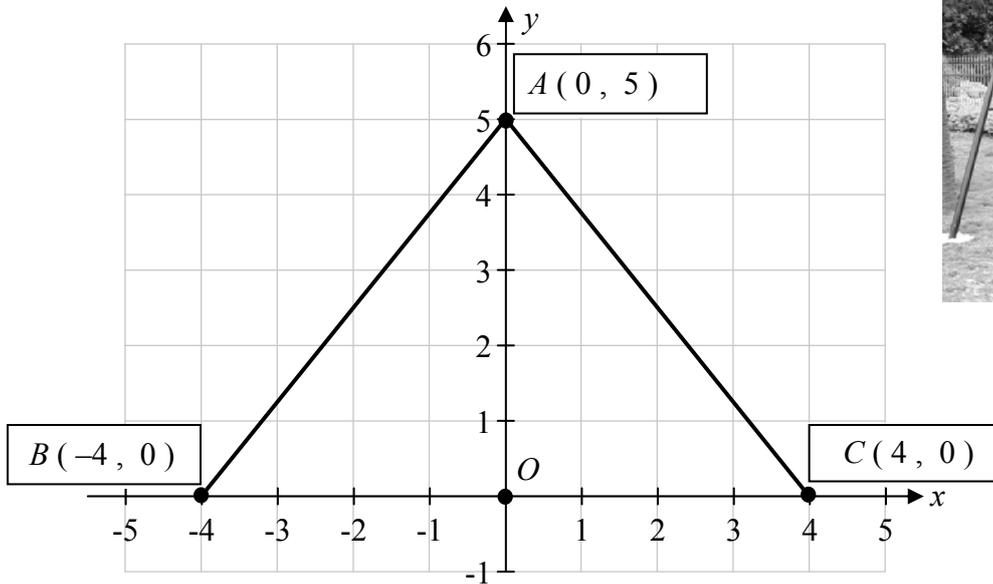
Question 7   **45 Marks**

The diagram below shows part of the frame of a swing on a co-ordinate grid.
Each unit on the grid represents one metre.
The line segments $[AB]$ and $[AC]$ represent metal bars.

(i)   Write the co-ordinates of the points *A*, *B*, and *C* in the spaces provided in the diagram.

(ii)   Find the total length of metal bar needed to make this part of the swing.
Give your answer in metres, correct to one decimal place.

> $|AB| = \sqrt{4^2 + 5^2} = \sqrt{41}.$
>
> Similarly, $|AC| = \sqrt{41}.$
>
> Total length of metal bar $= 2\sqrt{41} = 12 \cdot 80... = 12 \cdot 8$ m, correct to one decimal place.

(iii)   Find the slope of *AB* and the slope of *AC*.

AB:	*AC*:
Slope $= \dfrac{\text{rise}}{\text{run}} = \dfrac{5}{4}$ or $1 \cdot 25.$	Slope $= \dfrac{5-0}{0-4} = -\dfrac{5}{4}$ or $-1 \cdot 25.$

(iv)   Is *AB* perpendicular to *AC*? Give a reason for your answer.

> Answer: No
>
> Reason: Product of slopes $= \dfrac{5}{4} \times -\dfrac{5}{4} = -\dfrac{25}{16} \neq -1.$
>
> *Or*:   Reason: When you invert one slope and change the sign, you don't get the other slope.

(v) Madison draws the scale diagram of the triangle OAB shown on the right.
She marks in the angle X.

Recall that $[AB]$ is a metal bar, which is part of the frame of the swing.

Write down the value of $\tan X$, and hence find the size of the angle X.
Give the size of the angle X correct to two decimal places.

$$\tan X = \frac{5}{4}$$

$$|\angle X| = \tan^{-1}\left(\frac{5}{4}\right) = 51 \cdot 340... = 51 \cdot 34°,$$

correct to two decimal places.
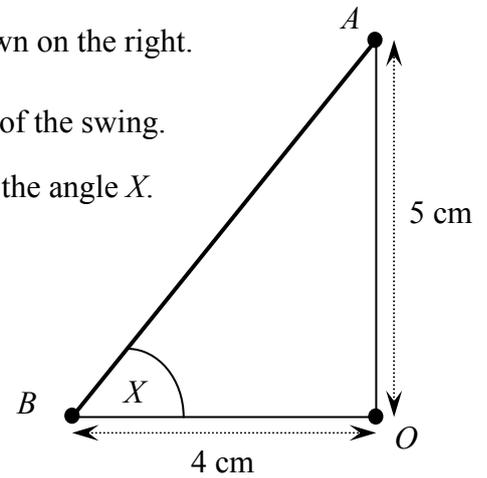
In order to increase the height of the swing, it is decided to increase X by 20%.
The distance $|AB|$ will be kept the same.

(vi) Find the new height of the swing. Give your answer in metres, correct to one decimal place.

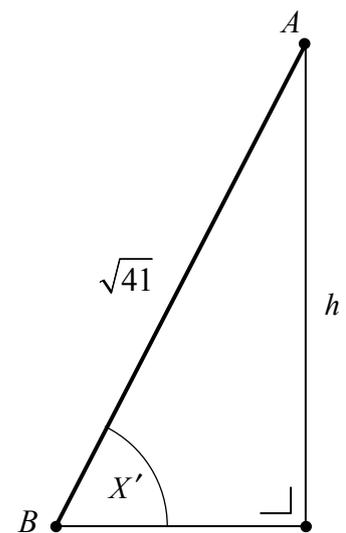Recall from **(ii)** that $|AB| = \sqrt{41}$ m.

Increase $|\angle X|$ by 20% to get $|\angle X'|$:

$$|\angle X'| = 51 \cdot 34 \times 1 \cdot 2 = 61 \cdot 608°$$

From the diagram, $\quad \sin X' \quad = \quad \sin 61 \cdot 608 \quad = \quad \dfrac{h}{\sqrt{41}}.$

$$\Rightarrow \qquad h \quad = \quad \sqrt{41}\sin 61 \cdot 608$$
$$= \quad 5 \cdot 632...$$
$$= \quad 5 \cdot 6 \text{ m},$$

correct to one decimal place.

Question 8   20 Marks

The equation of the line l is $x - 3y - 6 = 0$.

(i)   Find the slope of the line l.

Method 1:	*Method 2*:
$\begin{aligned} -3y &= -x + 6 & \text{\textit{Step 1}} \\ 3y &= x - 6 \\ y &= \frac{1}{3}x - 2 & \text{\textit{Step 2}} \\ \\ \Rightarrow \text{Slope} &= \frac{1}{3} & \text{\textit{Step 3}} \end{aligned}$	$\begin{aligned} \text{Slope} &= -\frac{a}{b} \\ &= -\frac{1}{-3} \\ &= \frac{1}{3} \end{aligned}$

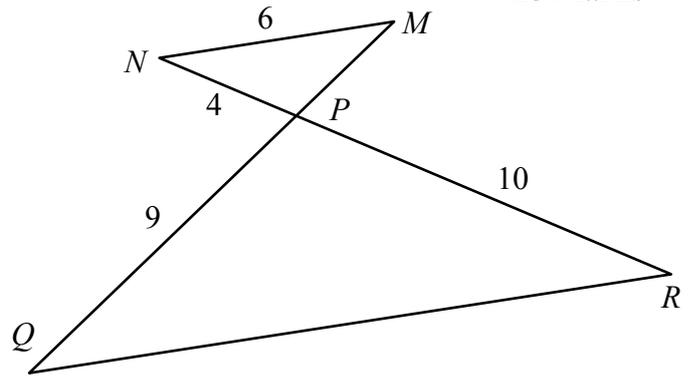**(ii)**   Show that the point $(1, -2)$ is **not** on the line l.

Sub in $(1, -2)$ to l: LHS $= 1 - 3(-2) - 6 = 1 \neq 0 =$ RHS.

Point not on l.

(iii)   The line k passes through $(1, -2)$ and is parallel to the line l.
        Find the equation of the line k.

Slope of $k = \frac{1}{3}$.   Point on $k = (1, -2)$.

Equation of k:	*Or*: Equation of k:
$\begin{aligned} y - (-2) &= \frac{1}{3}(x - 1) \\ \Rightarrow \qquad y &= \frac{x}{3} - \frac{7}{3} \\ \text{or} \qquad x - 3y - 7 &= 0 \end{aligned}$	$\begin{aligned} x - 3y + c &= 0 \\ \Rightarrow \qquad 1 - 3(-2) + c &= 0 \\ \Rightarrow \qquad c &= -7 \\ \Rightarrow \qquad x - 3y - 7 &= 0 \end{aligned}$

Question 9   **25 Marks**

In the diagram below, $|\angle MNP| = |\angle PRQ|$.

(i) Prove that $\triangle MNP$ and $\triangle QRP$ are similar.

> Proof: $|\angle MNP|$ = $|\angle PRQ|$ (given)
>
> $|\angle NPM|$ = $|\angle QPR|$ (vertically opposite)
>
> $|\angle NMP|$ = $|\angle PQR|$ (third angle)
>
> $\Rightarrow$ Triangles are similar.

(ii) Is NM parallel to QR? Give a reason for your answer.

> Answer: Yes
>
> Reason: $|\angle MNP| = |\angle PRQ|$ *or* $|\angle NMP| = |\angle PQR|$ *or* alternate angles are equal.

Given $|MN| = 6,\ |NP| = 4,\ |QP| = 9,$ and $|PR| = 10,$ find:

(iii) $|QR|$

> By similar triangles $\triangle MNP$ and $\triangle QRP$:
>
> $$\frac{|QR|}{6} = \frac{10}{4}$$
>
> $\Rightarrow |QR| = 6 \times \dfrac{10}{4} = 15.$

(iv) $|QM|$.

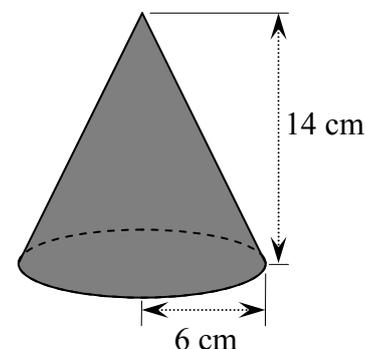| By similar triangles $\triangle MNP$ and $\triangle QRP$:	
$\dfrac{|PM|}{9} = \dfrac{6}{15}$ or $\dfrac{4}{10}$ $\Rightarrow |PM| = \dfrac{18}{5}$ or $3\cdot6$ $\Rightarrow |QM| = 9 + 3\cdot6 = \dfrac{63}{5}$ or $12\cdot6$.	$Or:$ $\dfrac{|PM|}{4} = \dfrac{9}{10}$ $\Rightarrow |PM| = 4\times\dfrac{9}{10} = \dfrac{18}{5}$ or $3\cdot6$ $\Rightarrow |QM| = 9 + 3\cdot6 = \dfrac{63}{5}$ or $12\cdot6$.

Question 10   15 Marks

A solid cone has a radius of 6 cm and a height of 14 cm, as shown.

**(i)** Find the volume of the cone. Give your answer in terms of π.

> Volume $= \dfrac{1}{3}\pi(6)^2 \times 14 = 168\pi$ cm^3

14 cm

6 cm

The shape shown below is a *frustum*. This is made by taking the cone above, cutting it horizontally at a height of 7 cm, and removing the upper portion. The radius of the circular top of the frustum is 3 cm, as shown in the diagram.

(ii) Find the ratio of the volume of the frustum to the volume of the original cone.

Volume of upper (removed) portion: $\dfrac{1}{3}\pi(3)^2 \times 7 = 21\pi$ cm^3 Volume of frustum: $168\pi - 21\pi = 147\pi$ cm^3
$Or:$ Volume of frustum $= \dfrac{1}{3}\pi h\left[R^2 + Rr + r^2\right]$ $= \dfrac{1}{3}\pi \times 7\left(6^2 + 6\times3 + 3^2\right) = 147\pi$ cm^3 Required ratio: $\dfrac{147\pi}{168\pi} = \dfrac{7}{8}$ or   7:8   or $0\cdot875$

3 cm

7 cm

6 cm
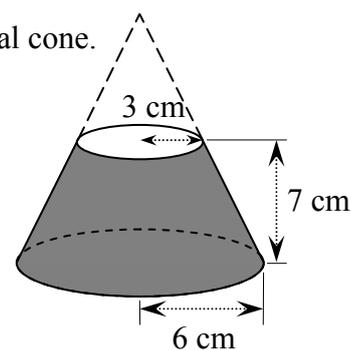
Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect), scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D
No of categories	2	3	4	5
5-mark scale	0, 5	0, 2, 5	0, 2, 3, 5	
10-mark scale			0, 3, 7, 10	0, 2, 5, 7, 10
15-mark scale				0, 4, 7, 11, 15

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)
- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)
- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work, or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may be awarded. Thus, for example, in Scale 10C, 9 marks may be awarded.

Accept a candidate's work in one part of a question for use in subsequent parts of the question, unless this oversimplifies the work involved.

Summary of mark allocations and scales to be applied

Question 1 (20)

(i)	5B
(ii)	5A
(iii)	5B
(iv)	5B

Question 2 (35)

(i)	5B
(ii)	5C
(iii)	5C
(iv)	10C
(v)	5B
(vi)	5B

Question 3 (40)

(i)	10D
(ii)	5C
(iii)	5B
(iv)	10C
(v)	5C
(vi)	5B

Question 4 (30)

(a)	15D
(b)	5C
(c)	10C

Question 5 (45)

(a)(i)	5A
(a)(ii)	5B
(a)(iii)	5A
(a)(iv)	5A
(a)(v)	10C
(b)	5A
(c)(i)	5B
(c)(ii)	5B

Question 6 (25)

(i)	10C
(ii)	5B
(iii)	5B
(iv)	5A

Question 7 (45)

(i)	5C
(ii)	5C
(iii)	10C
(iv)	5B
(v)	10C
(vi)	10D

Question 8 (20)

(i)	5C
(ii)	5B
(iii)	10C

Question 9 (25)

(i)	5C
(ii)	5B
(iii)	5C
(iv)	10C

Question 10 (15)

(i)	5C
(ii)	10C

Detailed marking notes

Question 1 (20)

(i)	Scale 5B (0, 2, 5)	
	Partial credit:	3 further correct outcomes.
(ii)	Scale 5A (0, 5)	
(iii)	Scale 5B (0, 2, 5)	
	Partial credit:	Correct answer, no reason or incorrect reason given; *or* Indication of one way of getting first outcome and three ways of getting second outcome; *or* 1/8; *or* 3/8.
(iv)	Scale 5B (0, 2, 5)	
	Partial credit:	Correct answer, no reason or incorrect reason given; *or* Indication of one way of getting each outcome; *or* 1/8.

Question 2 (35)

(i)	Scale 5B (0, 2, 5)	
	Partial credit:	Volume = $l \times w \times h$; *or* Volume = $13 \times 8 \times 6$.
(ii)	Scale 5C (0, 2, 3, 5)	
	Low partial credit:	Volume = $l \times w \times h$.
	High partial credit:	Correct volume of Box **A** or Box **B**.
(iii)	Scale 5C (0, 2, 3, 5)	
	Low partial credit:	Division of all dimensions of either box by dimensions of the phone.
	High partial credit:	74 880 / 624.
	Full credit:	Accept 74 880 / 624 = 120 for *Full credit*.
(iv)	Scale 10C (0, 3, 7, 10)	
	Low partial credit:	Surface area = $2[l \times w + l \times h + w \times h]$; *or* 2 sides multiplied.
	High partial credit:	Correct surface area of Box **A** or Box **B**.
(v)	Scale 5B (0, 2, 5)	
	Partial credit:	Any work of merit, e.g. 10 704 × 0·67 = €7 171·68.
	Full credit −1:	Cost for Box **A** correctly found.
(vi)	Scale 5B (0, 2, 5)	
	Partial credit:	Any work of merit.

Question 3 (40)

(i)	Scale 10D (0, 2, 5, 7, 10)	
	Low partial credit:	1 score placed correctly on **either** side.
	Middle partial credit:	One side fully completed.
	High partial credit:	10 or more scores placed correctly on **each** side.
	Full credit –1:	Fully correct graph, but key omitted or incorrect.
	Full credit:	An unordered graph may be accepted for *Full credit*.
(ii)	Scale 5C (0, 2, 3, 5)	
	Low partial credit:	Use of 119 or 79 or 120 or 83.
	High partial credit:	Range of one test correct; *or* One of 119, 79, 120, 83 incorrect; *or* 79 – 119 **and** 83 –120.
	Full credit –1:	Ranges of tests swapped.
(iii)	Scale 5B (0, 2, 5)	
	Partial credit:	One median correct; *or* Indication of the 8th or middle score
(iv)	Scale 10C (0, 3, 7, 10)	
	Low partial credit:	Sum of scores for either test correct; *or* Indication of division by 15; *or* Indication of sum of 15 correct scores for either test.
	High partial credit:	Mean for one test correct; *or* 1506 / 15 and 1572 / 15.
(v)	Scale 5C (0, 2, 3, 5)	
	Low partial credit:	Mention of mean or median or mode or average or range.
	High partial credit:	Comparison using measure of central tendency **or** measure of spread only.
(vi)	Scale 5B (0, 2, 5)	
	Partial credit:	Correct answer, no reason or incorrect reason given.

Question 4 (30)

(a)	Scale 15D (0, 4, 7, 11, 15)	
	Low partial credit:	Diagram.
	Middle partial credit:	Diagram, Given, To Prove and Construction only; *or* More than one step missing in proof.
	High partial credit:	One step missing in proof; *or* Fully correct but with no reason given; *or* gets as far as $w = 2t$ or equivalent.
	Full credit:	Given, Construction, and To Prove may be indicated on diagram. Some steps in Proof may be indicated on diagram.

(b)	Scale 5C (0, 2, 3, 5)	
	Low partial credit:	Indication that the angle at the centre is twice the angle at the circumference; *or* Indication that $|\angle SRP| = |\angle SQP|$.
	High partial credit:	**(i)** or **(ii)** correct.
(c)	Scale 10C (0, 3, 7, 10)	
	Low partial credit:	Work of some merit, e.g. States that the angles in a rectangle are right angles; *or* Indication that the angle in a semi-circle is a right angle; *or* Indication that *AC* and *BD* bisect each other; *or* Indication of isosceles or congruent triangles; *or* Identification of vertically opposite angles. Accept indication that opposite sides or opposite angles are equal (one pair).
	High partial credit:	$|\angle BAD| = |\angle BCD| = |\angle ABC| = |\angle ADC| = 90^{\circ}$ with no reason. This may be indicated on the diagram.

Question 5 (45)

(a)(i)	Scale 5A (0, 5)	
(a)(ii)	Scale 5B (0, 2, 5)	
	Partial credit:	One correct frequency.
	Full credit −1:	Tally or relative frequency given.
(a)(iii)	Scale 5A (0, 5)	
(a)(iv)	Scale 5A (0, 5)	
	Full credit:	Accept "data is not numerical" or "data is categorical" or similar.
(a)(v)	Scale 10C (0, 3, 7, 10)	
	Low partial credit:	One angle or fraction correct; *or* Correct labelled pie chart, no work and no angle shown.
	High partial credit:	Angles correctly calculated and two angles correct in pie chart with labelling; *or* Angles correctly calculated and correct pie chart drawn but no labelling or incorrect labelling; *or* Angles or fractions correct, no work shown and correct pie chart drawn.
	Full credit:	Allow a tolerance of $\pm 2^{\circ}$ in drawing.

(b)	Scale 5A (0, 5)	
(c)(i)	Scale 5B (0, 2, 5)	
	Partial credit:	Correct answer, no reason or incorrect reason given.
	Full credit:	Reason must refer to the diagram.
(c)(ii)	Scale 5B (0, 2, 5)	
	Partial credit:	Correct answer, no reason or incorrect reason given.
	Full credit:	Reason must refer to the diagram.

Question 6 (25)

(i)	Scale 10C (0, 3, 7, 10)	
	Low partial credit:	One side correctly drawn; *or* Use of Pythagoras theorem; *or* Sketch drawn with given measurements shown; *or* 1 side or angle correctly drawn.
	High partial credit:	Triangle correctly drawn with no construction lines or no work for $|BC|$; *or* Triangle correctly drawn but unlabelled or incorrectly labelled.
	Full credit:	Allow a tolerance of ± 2 mm or $\pm 2°$.
(ii)	Scale 5B (0, 2, 5)	
	Partial credit:	Wrong angle correctly measured; *or* Triangle incorrect and unlabelled but an angle correctly measured.
	Full credit:	Allow a tolerance of $\pm 2°$.
(iii)	Scale 5B (0, 2, 5)	
	Partial credit:	Calculator in incorrect mode.
(iv)	Scale 5A (0, 5)	
	Full credit:	Accept mention of whole number rounding/approximation of candidate's work.

Question 7 (45)

(i)	Scale 5C (0, 2, 3, 5)	
	Low partial credit:	1 point correct; *or* All 3 reversed, i.e. (y, x).
	High partial credit:	2 points correct.
(ii)	Scale 5C (0, 2, 3, 5)	
	Low partial credit:	Some correct substitution into distance formula or Pythagoras theorem; *or* Correct formula from tables; *or* $|AB|$ or $|AC|$ between 6·4 and 6·6 inclusive, without work.
	High partial credit:	$|AB|$ or $|AC|$ found, with work ($\sqrt{41}$ or 6·4); *or* Total length between 12·8 and 13·2, without work.
(iii)	Scale 10C (0, 3, 7, 10)	
	Low partial credit:	Some correct substitution into slope formula; *or* Indication that slope = $\dfrac{\text{rise}}{\text{run}}$; *or* Correct formula from tables.
	High partial credit:	Slope of AB or AC correctly found; *or* Correct substitution into slope formula in both cases.
(iv)	Scale 5B (0, 2, 5)	
	Partial credit:	Indication that the product of slopes of perpendicular lines $= -1$, or of the negative reciprocal of one slope, or that "perpendicular" means lines make 90° angle; *or* Correct answer, no reason or incorrect reason given.
		Note: Reason by measurement alone is not acceptable for *Full credit*.
(v)	Scale 10C (0, 3, 7, 10)	
	Low partial credit:	Some use of Pythagoras Theorem; *or* Correct trigonometric ratio.
	High partial credit:	$X = \tan^{-1}\dfrac{5}{4}$; *or* $\tan X = 4/5$, and $X = 38\cdot66°$; *or* Tan X not found and X correct using sin or cos.
	Full credit –1:	Tan X correct but X correct using sin or cos.

(vi)	Scale 10D (0, 2, 5, 7, 10)	
	Low partial credit:	Mentions 120%; *or* $X' = 61 \cdot 608$; *or* 20% = 10·268; *or* Correct diagram; *or* Correct trigonometric ratio; *or* $\|AB\| = \sqrt{41}$.
	Middle partial credit:	$X' = 61 \cdot 608°$ and correct diagram; *or* $\tan 61 \cdot 608° = \dfrac{h}{4}$.
	High partial credit:	$\sin 61 \cdot 608° = \dfrac{h}{\sqrt{41}}$; *or* $h = 4 \tan 61 \cdot 608° = 7 \cdot 4$.

Question 8 (20)

(i)	Scale 5C (0, 2, 3, 5)	
	Low partial credit:	Some work of merit, e.g. Correct manipulation of line equation; *or* Correct substitution into slope formula; *or* Correct slope formula ($y = mx + c$, Slope = $-a/b$, or similar).
	High partial credit:	First 2 steps correct in Method 1 (as presented in Model Solutions above); *or* 1 error in Method 2.
(ii)	Scale 5B (0, 2, 5)	
	Partial credit:	Some correct substitution into line equation.
	Full credit:	Conclusion needed for *Full credit*.
(iii)	Scale 10C (0, 3, 7, 10)	
	Low partial credit:	Correct slope; *or* Some correct substitution into line formula; *or* Correct formula from tables.
	High partial credit:	Correct slope with some correct substitution into line formula; *or* Correct slope but both *x* and *y* reversed in substitution.
	Full credit:	It is not necessary to write the equation in the form $ax + by + c = 0$ for *Full credit*.

Question 9 (25)

(i)	Scale 5C (0, 2, 3, 5)	
	Low partial credit:	One of the first two pairs of equal angles correctly identified; *or* Wrong angle in first two pairs of angles.
	High partial credit:	First 2 pairs of equal angles correctly identified and no conclusion; *or* Diagram with 3 pairs of equal angles marked.
(ii)	Scale 5B (0, 2, 5)	
	Partial credit:	Correct answer, no reason or incorrect reason given; *or* "Alternate angles are equal" stated.
(iii)	Scale 5C (0, 2, 3, 5)	
	Low partial credit:	One correct relevant ratio; *or* Corresponding sides identified; *or* Indication that corresponding sides are proportional.
	High partial credit:	$\dfrac{|QR|}{6} = \dfrac{10}{4}$, or equivalent
(iv)	Scale 10C (0, 3, 7, 10)	
	Low partial credit:	One correct relevant ratio; *or* Corresponding sides identified; *or* Indication that corresponding sides are proportional; *or* $|QM| = |QP| + |PM|$, or similar.
	High partial credit:	$|PM|$ correctly found, no addition.

Question 10 (15)

(i)	Scale 5C (0, 2, 3, 5)	
	Low partial credit:	Incorrect relevant formula used; *or* Some correct substitution into volume of cone formula; *or* Correct formula from tables.
	High partial credit:	$\dfrac{1}{3} \times \pi \times 36 \times 14$
	Full credit –1:	Answer correct, but not in terms of π.
(ii)	Scale 10C (0, 3, 7, 10)	
	Low partial credit:	Volume of small cone : Volume of large cone = 1 : 8; *or* Incorrect relevant formula used; *or* Some correct substitution into volume of cone or frustum formula; *or* Correct formula from tables.
	High partial credit:	Volume of frustum = 147π ; *or* Volume of small cone : Volume of frustum = 1 : 7.
	Full credit –1:	Ratio reversed; *or* Answer not in simplest form.

Bonus marks for answering through Irish

Bonus marks are applied separately to each paper, as follows:

If the mark achieved is 225 or less, the bonus is 5% of the mark obtained, rounded **down**.
For instance, 198 marks $\times$ 5% = 9·9 $\Rightarrow$ bonus = 9 marks.

If the mark achieved is above 225, the following table applies:

Bunmharc (Mark achieved)	Marc Bónais (Bonus mark)	Bunmharc (Mark achieved)	Marc Bónais (Bonus mark)
226	11	261 – 266	5
227 – 233	10	267 – 273	4
234 – 240	9	274 – 280	3
241 – 246	8	281 – 286	2
247 – 253	7	287 – 293	1
254 – 260	6	294 – 300	0