NSW GOVERNMENT

NSW Education Standards Authority

Centre Number

Student Number

2024 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 2

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks: 100

Section I – 10 marks (pages 2–5)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 6–14)

- Attempt Questions 11–16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 Which of the following vectors is perpendicular to $3\underset{\sim}{i} + 2\underset{\sim}{j} - 5\underset{\sim}{k}$?

A. $-\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$

B. $\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$

C. $-2\underset{\sim}{i} + 3\underset{\sim}{j} + \underset{\sim}{k}$

D. $3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$

2 Consider the following statement written in the formal language of proof

$$\forall\, \theta \in \left(\frac{\pi}{2}, \pi\right)\ \exists\, \phi \in \left(\pi, \frac{3\pi}{2}\right);\ \sin\theta = -\cos\phi.$$

Which of the following best represents this statement?

A. There exists a θ in the second quadrant such that for all ϕ in the third quadrant $\sin\theta = -\cos\phi$.

B. There exists a ϕ in the third quadrant such that for all θ in the second quadrant $\sin\theta = -\cos\phi$.

C. For all θ in the second quadrant there exists a ϕ in the third quadrant such that $\sin\theta = -\cos\phi$.

D. For all ϕ in the third quadrant there exists a θ in the second quadrant such that $\sin\theta = -\cos\phi$.

3 Consider the statement:

'If a polygon is a square, then it is a rectangle.'

Which of the following is the converse of the statement above?

A. If a polygon is a rectangle, then it is a square.

B. If a polygon is a rectangle, then it is not a square.

C. If a polygon is not a rectangle, then it is not a square.

D. If a polygon is not a square, then it is not a rectangle.

4 A monic polynomial, $f(x)$, of degree 3 with real coefficients has 3 and $2 + i$ as two of its roots.

Which of the following could be $f(x)$?

A. $f(x) = x^3 - 7x^2 - 17x + 15$

B. $f(x) = x^3 - 7x^2 + 17x - 15$

C. $f(x) = x^3 + 7x^2 - 17x + 15$

D. $f(x) = x^3 + 7x^2 + 17x - 15$

5 A particle is moving in simple harmonic motion with period 10 seconds and an amplitude of 8 m. The particle starts at the central point of motion and is initially moving to the left with a speed of $V \text{ m s}^{-1}$, where $V > 0$.

What will be the position and velocity of the particle after 7.5 seconds?

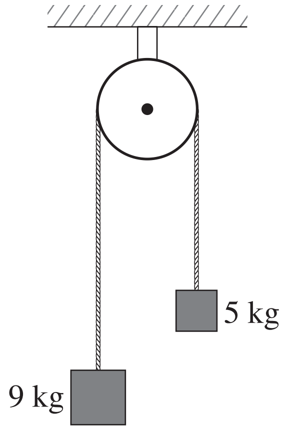
A. At the central point of motion with a velocity of $V \text{ m s}^{-1}$

B. At the central point of motion with a velocity of $-V \text{ m s}^{-1}$

C. 8 m to the left of the central point of motion with a velocity of 0 m s^{-1}

D. 8 m to the right of the central point of motion with a velocity of 0 m s^{-1}

6 A light string passes over a smooth pulley. Attached to the ends of the string are masses of 9 kg and 5 kg, as shown.



The acceleration due to gravity is $g\ \mathrm{m\,s^{-2}}$.

What is the acceleration of the 9 kg mass?

A. $\frac{2}{7}g$

B. $1g$

C. $\frac{7}{2}g$

D. $4g$

7 It is given that $|z - 1 + i| = 2$.

What is the maximum possible value of $|z|$?

A. $\sqrt{2}$

B. $\sqrt{10}$

C. $2 + \sqrt{2}$

D. $2 - \sqrt{2}$

8 Which of the following is equal to $e^{\bar{z}}$, where $z = x + iy$ with x and y real numbers?

A. $\overline{e^{z}}$

B. e^{-z}

C. $e^{2x}e^{z}$

D. $e^{-2x}e^{z}$

9 Consider the solutions of the equation $z^4 = -9$.

What is the product of all of the solutions that have a positive principal argument?

A. 3

B. -3

C. $3i$

D. $-3i$

10 Three unit vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$, in 3 dimensions, are to be chosen so that $\underset{\sim}{a} \perp \underset{\sim}{b}$, $\underset{\sim}{b} \perp \underset{\sim}{c}$ and the angle θ between $\underset{\sim}{a}$ and $\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c}$ is as small as possible.

What is the value of $\cos\theta$?

A. 0

B. $\dfrac{1}{\sqrt{3}}$

C. $\dfrac{1}{\sqrt{2}}$

D. $\dfrac{2}{\sqrt{5}}$

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (16 marks) Use the Question 11 Writing Booklet

(a) Find $\int xe^x \, dx$. **2**

(b) Let $z = 2 + 3i$ and $w = 1 - 5i$.

(i) Find $z + \bar{w}$. **1**

(ii) Find z^2. **1**

(c) Find the angle between the two vectors $\underset{\sim}{u} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix} 4 \\ -4 \\ 7 \end{pmatrix}$, giving your answer in radians, correct to 1 decimal place. **2**

(d) Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \frac{1}{\sin\theta + 1} \, d\theta$. **3**

(e) (i) Write the number $\sqrt{3} + i$ in modulus-argument form. **2**

(ii) Hence, or otherwise, write $\left(\sqrt{3} + i\right)^7$ in exact Cartesian form. **2**

(f) Sketch the region defined by $|z| < 3$ and $0 \le \arg(z - i) \le \dfrac{\pi}{2}$. **3**

Question 12 (14 marks) Use the Question 12 Writing Booklet

(a) The vector $\underset{\sim}{a}$ is $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and the vector $\underset{\sim}{b}$ is $\begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix}$.

(i) Find $\dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{\underset{\sim}{b} \cdot \underset{\sim}{b}} \underset{\sim}{b}$. **1**

(ii) Show that $\underset{\sim}{a} - \dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{\underset{\sim}{b} \cdot \underset{\sim}{b}} \underset{\sim}{b}$ is perpendicular to $\underset{\sim}{b}$. **2**

(b) Use partial fractions to find **3**

$$\int \frac{3x^2 + 2x + 1}{(x-1)(x^2+1)}\, dx.$$

(c) Consider the equation $|z| = z + 8 + 12i$, where $z = a + bi$ is a complex number and a, b are real numbers.

(i) Explain why $b = -12$. **1**

(ii) Hence, or otherwise, find z. **2**

(d) Explain why there is no integer n such that $(n+1)^{41} - 79n^{40} = 2$. **2**

(e) The line ℓ passes through the points $A(3, 5, -4)$ and $B(7, 0, 2)$.

(i) Find a vector equation of the line ℓ. **1**

(ii) Determine, giving reasons, whether the point $C(10, 5, -2)$ lies on the line ℓ. **2**

Question 13 (16 marks) Use the Question 13 Writing Booklet

(a) The point A has position vector $8\underset{\sim}{i} - 6\underset{\sim}{j} + 5\underset{\sim}{k}$. The line ℓ has vector equation

$$x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k} = t\left(\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}\right).$$

The point B lies on ℓ and has position vector $p\underset{\sim}{i} + p\underset{\sim}{j} + 2p\underset{\sim}{k}$.

(i) Show that $\left|AB\right|^2 = 6p^2 - 24p + 125$. **1**

(ii) Hence, or otherwise, determine the shortest distance between the point A and the line ℓ. **2**

(b) A particle is moving in simple harmonic motion, described by $\ddot{x} = -4(x+1)$. **3**

When the particle passes through the origin, the speed of the particle is 4 m s^{-1}.

What distance does the particle travel during a full period of its motion?

(c) A particle of unit mass moves horizontally in a straight line. It experiences a resistive force proportional to v^2, where $v \text{ m s}^{-1}$ is the speed of the particle, so that the acceleration is given by $-kv^2$.

Initially the particle is at the origin and has a velocity of 40 m s^{-1} to the right. After the particle has moved 15 m to the right, its velocity is 10 m s^{-1} (to the right).

(i) Show that $v = 40e^{-kx}$. **3**

(ii) Show that $k = \dfrac{\ln 4}{15}$. **1**

(iii) At what time will the particle's velocity be 30 m s^{-1} to the right? **3**

(d) It is known that for all positive real numbers x, y **3**

$$x + y \geq 2\sqrt{xy}. \qquad \text{(Do NOT prove this.)}$$

Show that if a, b, c are positive real numbers with $\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c} = 1$ then $a\sqrt{bc} + b\sqrt{ac} + c\sqrt{ab} \leq abc$.

Question 14 (15 marks) Use the Question 14 Writing Booklet

(a) Prove that if a is any odd integer, then $a^2 - 1$ is divisible by 8. **2**

(b) Use mathematical induction to prove that ${}^{2n}C_n < 2^{2n-2}$, for all integers $n \geq 5$. **3**

(c) For the complex numbers z and w, it is known that $\arg\left(\frac{z}{w}\right) = -\frac{\pi}{2}$. **2**

Find $\left|\frac{z-w}{z+w}\right|$.

(d) The following argument attempts to prove that $0 = 1$. **2**

We evaluate $\int \frac{1}{x}\,dx$ using the method of integration by parts.

$$\begin{aligned}\int \frac{1}{x}\,dx &= \int \frac{1}{x} \times 1\,dx \\ &= \frac{1}{x} \times x - \int -\frac{1}{x^2}x\,dx \\ &= 1 + \int \frac{1}{x}\,dx\end{aligned}$$

So we have

$$\int \frac{1}{x}\,dx = 1 + \int \frac{1}{x}\,dx$$

We may now subtract $\int \frac{1}{x}\,dx$ from both sides to show that $0 = 1$.

Explain what is wrong with this argument.

Question 14 continues on page 10

Question 14 (continued)

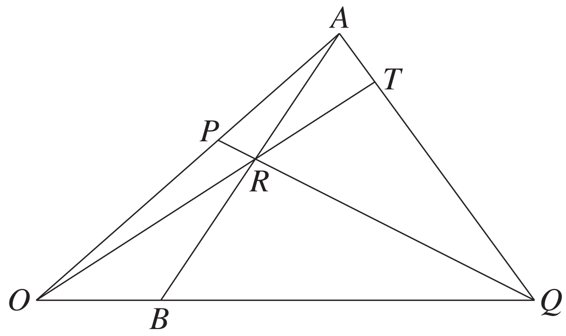
(e) The diagram shows triangle OQA.

The point P lies on OA so that $OP : OA = 3 : 5$.

The point B lies on OQ so that $OB : OQ = 1 : 3$.

The point R is the intersection of AB and PQ.

The point T is chosen on AQ so that O, R and T are collinear.



Let $\underset{\sim}{a} = \overrightarrow{OA}$, $\underset{\sim}{b} = \overrightarrow{OB}$ and $\overrightarrow{PR} = k\overrightarrow{PQ}$ where k is a real number.

(i) Show that $\overrightarrow{OR} = \frac{3}{5}(1 - k)\underset{\sim}{a} + 3k\underset{\sim}{b}$. **2**

Writing $\overrightarrow{AR} = h\overrightarrow{AB}$, where h is a real number, it can be shown that $\overrightarrow{OR} = (1 - h)\underset{\sim}{a} + h\underset{\sim}{b}$. (Do NOT prove this.)

(ii) Show that $k = \frac{1}{6}$. **2**

(iii) Find $\overrightarrow{OT}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. **2**

End of Question 14

Question 15 (15 marks) Use the Question 15 Writing Booklet

(a) Consider the three vectors $\underset{\sim}{a} = \overrightarrow{OA}$, $\underset{\sim}{b} = \overrightarrow{OB}$ and $\underset{\sim}{c} = \overrightarrow{OC}$, where O is the origin and the points A, B and C are all different from each other and the origin.

The point M is the point such that $\frac{1}{2}\left(\underset{\sim}{a} + \underset{\sim}{b}\right) = \overrightarrow{OM}$.

(i) Show that M lies on the line passing through A and B. **1**

(ii) The point G is the point such that $\frac{1}{3}\left(\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c}\right) = \overrightarrow{OG}$. **2**

Show that G lies on the line passing through M and C, and lies between M and C.

(iii) The complex numbers x, w and z are all different and all have modulus 1. **2**

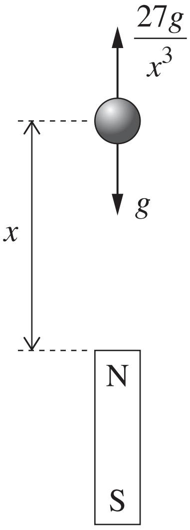
Using part (ii), or otherwise, show that $\frac{1}{3}(x + w + z)$ is never a cube root of xwz.

(b) Let $I_n = \int_0^a x^{n+\frac{1}{2}} (a - x)^{\frac{1}{2}}\, dx$, where $n \geq 0$. **3**

Show that $(2n + 4)I_n = a(2n + 1)I_{n-1}$, for $n > 0$.

Question 15 continues on page 12

(c) A bar magnet is held vertically. An object that is repelled by the magnet is to be dropped from directly above the magnet and will maintain a vertical trajectory. Let x be the distance of the object above the magnet.



The object is subject to acceleration due to gravity, g, and an acceleration due to the magnet $\frac{27g}{x^3}$, so that the total acceleration of the object is given by

$$a = \frac{27g}{x^3} - g.$$

The object is released from rest at $x = 6$.

(i) Show that $v^2 = g\left(\frac{51}{4} - 2x - \frac{27}{x^2}\right)$. **2**

(ii) Find where the object next comes to rest, giving your answer correct to 1 decimal place. **2**

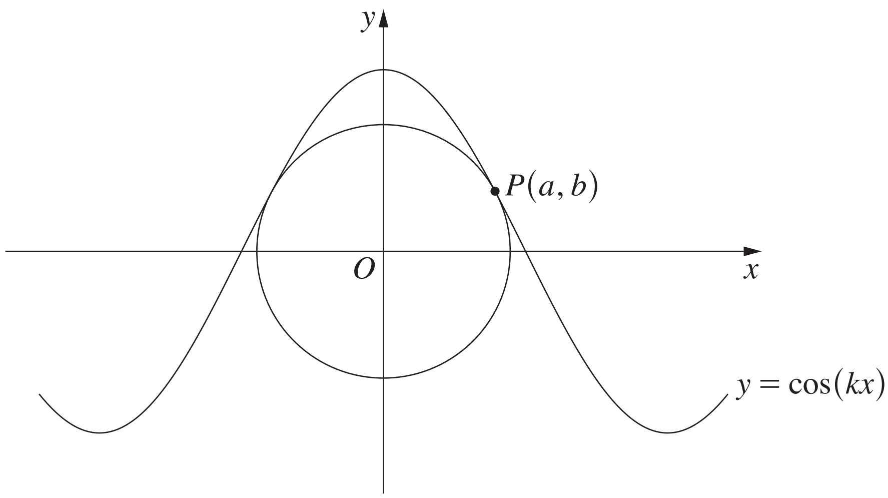
(d) Using a suitable substitution, find **3**

$$\int \frac{2x^2}{\sqrt{2x - x^2}}\, dx.$$

End of Question 15

Question 16 (14 marks) Use the Question 16 Writing Booklet

(a) Consider the function $y = \cos(kx)$, where $k > 0$. The value of k has been chosen so that a circle can be drawn, centred at the origin, which has exactly two points of intersection with the graph of the function and so that the circle is never above the graph of the function. The point $P(a, b)$ is the point of intersection in the first quadrant, so $a > 0$ and $b > 0$, as shown in the diagram. **4**



The vector joining the origin to the point $P(a, b)$ is perpendicular to the tangent to the graph of the function at that point. (Do NOT prove this.)

Show that $k > 1$.

(b) The number $w = e^{\frac{2\pi i}{3}}$ is a complex cube root of unity. The number γ is a cube root of w.

(i) Show that $\gamma + \bar{\gamma}$ is a real root of $z^3 - 3z + 1 = 0$. **3**

(ii) By using part (i) to find the exact value of $\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}\cos\frac{8\pi}{9}$, deduce the value(s) of $\cos\frac{2^n\pi}{9}\cos\frac{2^{n+1}\pi}{9}\cos\frac{2^{n+2}\pi}{9}$ for all integers $n \geq 1$. Justify your answer. **3**

Question 16 continues on page 14

(c) Two particles, A and B, each have mass 1 kg and are in a medium that exerts a resistance to motion equal to kv, where $k > 0$ and v is the velocity of any particle. Both particles maintain vertical trajectories. **4**

The acceleration due to gravity is g m s^{-2}, where $g > 0$.

The two particles are simultaneously projected towards each other with the same speed, v_0 m s^{-1}, where $0 < v_0 < \dfrac{g}{k}$.

The particle A is initially d metres directly above particle B, where $d < \dfrac{2v_0}{k}$.

Find the time taken for the particles to meet.

End of paper

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2024 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}\left[2a + (n - 1)d\right] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, r \neq 1$$

$$S = \frac{a}{1 - r}, |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

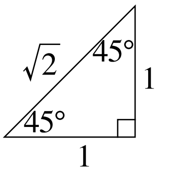
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

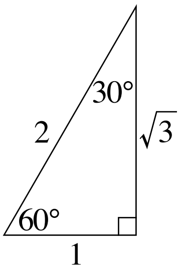
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

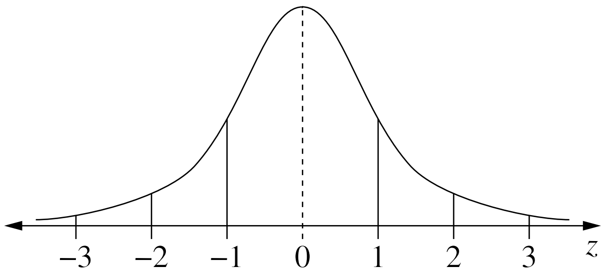
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \quad P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = n f'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a) f'(x) a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a) f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1 - \left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1 - \left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1 + \left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \left[f(x)\right]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + \left[f(x)\right]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_{1}x_{2} + y_{1}y_{2},$$

where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^{n} &= r^{n}(\cos n\theta + i\sin n\theta) \\ &= r^{n}e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^{2}(x-c)$$