2023 HSC Mathematics Extension 1 Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	D
2	A
3	C
4	C
5	B
6	C
7	B
8	A
9	D
10	B

Section II

Question 11 (a)

Criteria	Marks
• Provides correct solution	2
• Obtains *t* in terms of *x*, or equivalent merit	1

Sample answer:

$$x = 1 + 3t$$

$$\therefore \quad t = \frac{x-1}{3}$$

$$\therefore \quad y = 4\left(\frac{x-1}{3}\right)$$

$$y = \frac{4}{3}x - \frac{4}{3}$$

Question 11 (b)

Criteria	Marks
• Provides correct solution	2
• Divides 10! by a relevant number, or equivalent merit	1

Sample answer:

10 letters, 3 Os and 2 Ns

$$\therefore \quad \frac{10!}{3!2!} = 302\,400$$

Question 11 (c)

Criteria	Marks
• Provides correct solution	3
• Obtains two equations for a and b, or equivalent merit	2
• Observes that $P(-1) = 0$ or $P(2) = -18$, or equivalent merit	1

Sample answer:

$P(-1) = 0$ by factor theorem

$$\begin{aligned}(-1)^3 + a(-1)^2 + b(-1) - 12 &= 0\\ -1 + a - b - 12 &= 0\\ a - b &= 13 \qquad (1)\end{aligned}$$

$P(2) = -18$ by remainder theorem

$$\begin{aligned}2^3 + a(2)^2 + b(2) - 12 &= -18\\ 8 + 4a + 2b - 12 &= -18\\ 4a + 2b &= -14\\ 2a + b &= -7 \qquad (2)\end{aligned}$$

(1) + (2): $3a = 6$
$a = 2$

Using this in (1) $2 - b = 13$
$b = -11$

Question 11 (d)

Criteria	Marks
• Provides correct solution	2
• Attempts to rewrite the integrand in a suitable form, or equivalent merit	1

Sample answer:

$$\begin{aligned}\int \frac{1}{\sqrt{4-9x^2}}\,dx &= \frac{1}{3}\int \frac{3}{\sqrt{2^2-(3x)^2}}\,dx\\ &= \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right) + C\end{aligned}$$

Question 11 (e)

Criteria	Marks
• Provides correct solution	3
• Obtains an equation or graph involving a single trigonometric function OR • Obtains a quadratic equation in *t*, and attempts to solve it OR • equivalent merit	2
• Uses the *t*-substitution, attempts to write $\cos\theta + \sin\theta$ as a single trigonometric function, or equivalent merit	1

Sample answer:

Let $t = \tan\dfrac{\theta}{2}$

$$\therefore \quad \frac{1-t^2}{1+t^2} + \frac{2t}{1+t^2} = 1$$

$$\begin{aligned} 1-t^2+2t &= 1+t^2 \\ 2t^2-2t &= 0 \\ 2t(t-1) &= 0 \end{aligned}$$

$\therefore \quad t = 0$ or $t = 1$

$\tan\dfrac{\theta}{2} = 0$

$\dfrac{\theta}{2} = 0,\ \pi,\ \cancel{2\pi},\ \ldots$

$\theta = 0,\ 2\pi$

$\tan\dfrac{\theta}{2} = 1$

$\dfrac{\theta}{2} = \dfrac{\pi}{4},\ \cancel{\dfrac{5\pi}{4}},\ \ldots$

$\theta = \dfrac{\pi}{2}$

$$\therefore \quad \theta = 0,\ \frac{\pi}{2},\ 2\pi$$

Question 11 (e) (continued)

<u>Alternative solution</u>

$$\cos\theta + \sin\theta = 1 \qquad \theta \in [0, 2\pi]$$

Let $\cos\theta + \sin\theta = R\sin(\theta + \alpha)$

$$= R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$$

$$\therefore \quad \left.\begin{aligned} R\sin\alpha &= 1 \\ R\cos\alpha &= 1 \end{aligned}\right\}$$

$$\therefore \quad R = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\tan\alpha = 1 \qquad \therefore \quad \alpha = \frac{\pi}{4}$$

$$\therefore \quad \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right) = 1$$

$$\sin\left(\theta + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\theta + \frac{\pi}{4} = \frac{\pi}{4},\ \frac{3\pi}{4},\ \frac{9\pi}{4},\ \ldots$$

$$\theta = 0,\ \frac{\pi}{2},\ 2\pi$$

Question 11 (f) (i)

Criteria	Marks
• Provides correct solution	2
• Obtains p = 0.3, or equivalent merit	1

Sample answer:

$P(\text{born overseas}) = 0.3$

Let $p = 0.3 \quad \therefore\ 1 - p = 0.7$

$n = 900$

$\therefore$ Distribution of $\hat{p}$ is approximately normal with mean $\mu = 0.3$ and

$$\begin{aligned}\sigma^2 &= \frac{0.3 \times 0.7}{900} \\ &= \frac{0.21}{900} \\ &= \frac{7}{30\,000}\end{aligned}$$

Question 11 (f) (ii)

Criteria	Marks
• Provides correct solution	2
• Obtains the correct z-score, or equivalent merit	1

Sample answer:

$$\begin{aligned}P(\hat{p} \le 0.31) &\approx P\left(Z \le \frac{0.31 - 0.3}{\sqrt{\frac{7}{30\,000}}}\right) \\ &= P(X \le 0.6546\ldots) \\ &\approx P(Z \le 0.65) \\ &= 0.7422 && \text{from table} \\ &= 0.74 && \text{2 decimal places}\end{aligned}$$

Question 12 (a)

Criteria	Marks
• Provides correct solution	3
• Obtains correct anti-derivative in terms of *u*, or equivalent merit	2
• Obtains correct integrand in terms of *u*, or equivalent merit	1

Sample answer:

$$\int_3^4 (x+2)\sqrt{x-3}\,dx \qquad u = x - 3 \quad \begin{cases} x = 4 \rightarrow u = 1 \\ x = 3 \rightarrow u = 0 \end{cases}$$

$$du = dx$$

$$= \int_0^1 (u+5)\sqrt{u}\,du$$

$$= \int_0^1 u^{\frac{3}{2}} + 5u^{\frac{1}{2}}\,du$$

$$= \left[\frac{2}{5}u^{\frac{5}{2}} + \frac{5\times 2}{3}u^{\frac{3}{2}}\right]_0^1$$

$$= \frac{2}{5} + \frac{10}{3}$$

$$= \frac{56}{15}$$

Question 12 (b)

Criteria	Marks
• Provides correct solution	3
• Establishes the inductive step, or equivalent merit	2
• Establishes the base case, or equivalent merit	1

Sample answer:

When $n = 1$

$$\text{LHS} = 1 \times 2 = 2$$

$$\text{RHS} = 2 + (1 - 1)2^{1+1} = 2$$

$$\text{LHS} = \text{RHS}$$

$\therefore$ True when $n = 1$

Assume true when $n = k$

ie $\quad (1 \times 2) + \left(2 \times 2^2\right) + \cdots + \left(k \times 2^k\right) = 2 + (k - 1)2^{k+1}$

Consider $n = k + 1$

$$\begin{aligned} \text{RHS} &= 2 + (k + 1 - 1)2^{k+1+1} \\ &= 2 + k \,.\, 2^{k+2} \end{aligned}$$

$$\begin{aligned} \text{LHS} &= (1 \times 2) + \left(2 \times 2^2\right) + \cdots + \left(k \times 2^k\right) + \left((k+1) \times 2^{k+1}\right) \\ &= 2 + (k - 1)2^{k+1} + (k + 1)2^{k+1} \qquad \text{by assumption} \\ &= 2 + 2^{k+1}(k - 1 + k + 1) \\ &= 2 + 2^{k+1}(2k) \\ &= 2 + k \,.\, 2^{k+2} \\ &= \text{RHS} \end{aligned}$$

Therefore, by mathematical induction, it is true for all integers $n \geq 1$.

Question 12 (c) (i)

Criteria	Marks
• Provides correct answer	2
• Demonstrates some understanding of binomial probability, or equivalent merit	1

Sample answer:

$${}^{5}C_{3}(0.65)^{3}(0.35)^{2}$$

Question 12 (c) (ii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$${}^{5}C_{3}(0.65)^{3}(0.35)^{2} \times (0.6)^{4}$$

Question 12 (d)

Criteria	Marks
• Provides correct solution	2
• Uses given result to combine the first two terms on the left-hand side, or equivalent merit	1

Sample answer:

$$\begin{aligned} & {}^{2022}C_{80} + {}^{2022}C_{81} + {}^{2023}C_{1943} \\ &= {}^{2023}C_{81} + {}^{2023}C_{1943} \\ &= {}^{2023}C_{81} + {}^{2023}C_{80} \\ &= {}^{2024}C_{81} \end{aligned}$$

So $p = 2024$ and $q = 81$ is a possible solution

Question 12 (e)

Criteria	Marks
• Provides correct solution	4
• Obtains the volume formed by revolving the hyperbola about the *y*-axis, or equivalent merit	3
• Obtains correct integral expression for the volume formed by revolving the hyperbola about the *y*-axis, or equivalent merit	2
• Writes *x* in terms of y, or recognises that the volume is the sum of two simpler volumes, or equivalent merit	1

Sample answer:

The total volume is the sum of the volume when the given hyperbola is revolved about the y-axis, V_1, and the volume of a cylinder, V_2.

$$y = \frac{60}{x+5}$$

$$x + 5 = \frac{60}{y}$$

$$x = \frac{60}{y} - 5$$

$$V_1 = \pi \int_4^{12} x^2 \, dy$$

$$= \pi \int_4^{12} \left(\frac{60}{y} - 5 \right)^2 dy$$

$$= \pi \int_4^{12} \frac{3600}{y^2} - \frac{600}{y} + 25 \, dy$$

$$= \pi \left[-\frac{3600}{y} - 600 \ln y + 25y \right]_4^{12}$$

$$= \pi \left[\left(-\frac{3600}{12} - 600 \ln 12 + 25 \times 12 \right) - \left(-\frac{3600}{4} - 600 \ln 4 + 25 \times 4 \right) \right]$$

$$= \pi (-600 \ln 12 + 800 + 600 \ln 4)$$

$$= \pi (800 - 600 \ln 3)$$

$$V_2 = \pi \times 10^2 \times 4$$

$$= 400\pi$$

Total = $V_1 + V_2$

$\therefore$ Total = $\pi(1200 - 600 \ln 3)$ units^3

Question 13 (a) (i)

Criteria	Marks
• Provides correct solution	2
• Obtains $\frac{dV}{dh}$, or equivalent merit	1

Sample answer:

$$V = \pi\left(Rh^2 - \frac{h^3}{3}\right)$$

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$

So $$\frac{dV}{dt} = \pi\left(2Rh - h^2\right)\frac{dh}{dt}$$

$$= \pi h(2R - h)\frac{dh}{dt}$$

But we are given

$$\frac{dV}{dt} = k(2R - h)$$

So $$\pi h(2R - h)\frac{dh}{dt} = k(2R - h)$$

$$\pi h\frac{dh}{dt} = k \qquad \text{as } 0 \le h \le R \text{ so } 2R - h \ne 0$$

$$\frac{dh}{dt} = \frac{k}{\pi h}$$

Question 13 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Separates the variables from the differential equation in part (i), or equivalent merit	1

Sample answer:

$$\frac{dh}{dt} = \frac{k}{\pi h}$$

When $t = 0,\ h = 0$
$t = T,\ h = R$

So $$\int_0^R \pi h\, dh = \int_0^T k\, dt$$

$$\left[\frac{\pi h^2}{2}\right]_0^R = \left[kt\right]_0^T$$

$$\frac{\pi R^2}{2} - 0 = kT - 0$$

$$T = \frac{\pi R^2}{2k}$$

Question 13 (a) (iii)

Criteria	Marks
• Provides correct solution	3
• Finds correct expression for $\frac{dh}{dt}$, or equivalent merit	2
• Models the situation to find the new differential equation $\frac{dV}{dt} = -kh$, or equivalent merit	1

Sample answer:

When the tank is full the inflow of water stops. This means that the rate of change in volume is only dependent on *h*.

ie $$-kh = \frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$

So $$-kh = \pi h(2R-h)\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{-k}{\pi(2R-h)}$$

When $t = 0, \quad h = R$
$t = T_E, \quad h = 0$

$$\int_R^0 \pi(2R-h)\,dh = \int_0^{T_E} -k\,dt$$

$$\left[\pi\left(2Rh - \frac{h^2}{2}\right)\right]_R^0 = -kT_E$$

$$-\pi\left(2R^2 - \frac{R^2}{2}\right)^2 = -kT_E$$

$$\pi\frac{3R^2}{2} = kT_E$$

$$T_E = 3\frac{\pi R^2}{2k} = 3T$$

So the tank takes 3 times as long to empty.

Question 13 (b) (i)

Criteria	Marks
• Provides correct proof	2
• Equates *x*-coordinates, or equivalent merit	1

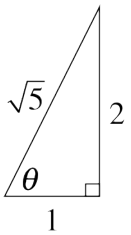
Sample answer:

The particles collide when their x-coordinates are the same at the same time.

$$vt\cos\theta = ut$$

So $u = v\cos\theta$

θ in 1st quad. and $\tan\theta = 2$



So $\cos\theta = \dfrac{1}{\sqrt{5}}$ and $\sin\theta = \dfrac{2}{\sqrt{5}}$

$$u = v\cos\theta$$

$$u = \frac{v}{\sqrt{5}}$$

$$v = \sqrt{5}\,u$$

Question 13 (b) (ii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

Particles collide at time T so y-coordinates are the same.

$$vT\sin\theta - \frac{1}{2}gT^2 = H - \frac{1}{2}gT^2$$

$$\sqrt{5}uT\frac{2}{\sqrt{5}} = H$$

$$T = \frac{H}{2u}$$

Question 13 (b) (iii)

Criteria	Marks
• Provides correct solution	3
• Evaluates the dot product of the velocity vectors and equates to 0, or equivalent merit	2
• Observes that the dot product of the velocity vectors is 0, or equivalent merit	1

Sample answer:

At $t = T$ the vectors $\underset{\sim}{v}_A$ and $\underset{\sim}{v}_B$ are perpendicular, so $\underset{\sim}{v}_A \cdot \underset{\sim}{v}_B = 0$.

$$\begin{pmatrix} v\cos\theta \\ v\sin\theta - gT \end{pmatrix} \cdot \begin{pmatrix} u \\ -gT \end{pmatrix} = 0$$

$$uv\cos\theta - gT(v\sin\theta - gT) = 0$$

$$u\sqrt{5}u\frac{1}{\sqrt{5}} - \sqrt{5}ugT\frac{2}{\sqrt{5}} + g^2T^2 = 0$$

$$u^2 - 2ugT + g^2T^2 = 0$$

$$(u - gT)^2 = 0$$

$$u = gT = \frac{gH}{2u}$$

$$H = \frac{2u^2}{g}$$

Question 13 (b) (iv)

Criteria	Marks
• Provides correct solution	2
• Identifies that $\dot{y} = 0$ for particle *A*, or equivalent merit	1

Sample answer:

Vertex of $y_A(t) = vt\sin\theta - \frac{1}{2}gt^2$ occurs when $\dot{y} = 0$

$$t = \frac{-v\sin\theta}{2\left(-\frac{1}{2}g\right)} \qquad \text{[axis of symmetry]}$$

$$= \frac{\sqrt{5}u\frac{2}{\sqrt{5}}}{g}$$

$$= \frac{2u}{g}$$

Height of particle *A* at that time is

$$y_A = vt\sin\theta - \frac{1}{2}gt^2$$

$$= \sqrt{5}u \times \frac{2u}{g} \times \frac{2}{\sqrt{5}} - \frac{g}{2} \times \frac{4u^2}{g^2}$$

$$= \frac{4u^2}{g} - \frac{2u^2}{g}$$

$$= \frac{2u^2}{g}$$

$$= H$$

Question 14 (a) (i)

Criteria	Marks
• Provides correct explanation	1

Sample answer:

$$f'(x) = 2 + \frac{1}{x} > 0 \text{ for } x > 0$$

f is always increasing.

Therefore since f is one-to-one for $x > 0$, f has an inverse function.

Question 14 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Finds $g'(x)$ or $f'(1)$, or equivalent merit	1

Sample answer:

$f(1) = 2$ therefore $g(2) = 1$

$$g'(x) = \frac{1}{f'(g(x))}$$

So $\quad g'(2) = \dfrac{1}{f'(g(2))} = \dfrac{1}{f'(1)} = \dfrac{1}{3}$

Question 14 (b) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

The points of intersection of the hyperbola and the circle satisfy

$$\begin{cases} y = \dfrac{1}{x} & (1) \\ (x-c)^2 + y^2 = c^2 & (2) \end{cases}$$

Substitute (1) into (2)

$$\begin{aligned} (x-c)^2 + \frac{1}{x^2} &= c^2 \qquad \times x^2 \\ x^2\left(x^2 - 2cx + c^2\right) + 1 &= c^2x^2 \\ x^4 - 2cx^3 + 1 &= 0 \end{aligned}$$

Question 14 (b) (ii)

Criteria	Marks
• Provides correct solution	3
• Finds the *x*-coordinate of the double root in terms of *c*, or equivalent merit	2
• Recognises that the circle and hyperbola are tangent when the polynomial has a double root and so is tangent to the *x*-axis, or equivalent merit	1

Sample answer:

Let $f(x) = x^4 - 2cx^3 + 1$

The hyperbola and the circle have exactly one point of intersection if there exists a value of x such that $f(x) = 0$ and $f'(x) = 0$.

$$f'(x) = 4x^3 - 6cx^2$$

$$f'(x) = 2x^2(2x - 3c)$$

$$f'(x) = 0 \Leftrightarrow x = 0 \quad \text{or} \quad x = \frac{3c}{2}$$

Note that $f(0) \neq 0$

$$f\left(\frac{3c}{2}\right) = 0 \quad \Leftrightarrow \left(\frac{3c}{2}\right)^4 - 2c\left(\frac{3c}{2}\right)^3 + 1 = 0$$

$$\times 2^4$$

$$\Leftrightarrow 3^4c^4 - 3^3 \times 2^2c^4 + 2^4 = 0$$

$$\Leftrightarrow -27c^4 = -16$$

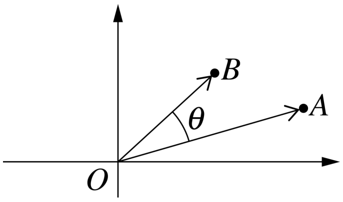
$$\Leftrightarrow c = \frac{2}{\sqrt[4]{27}}$$

Question 14 (c) (i)

Criteria	Marks
• Provides correct solution	3
• Recognise that the dot product of $\overline{OB}$ with the vector perpendicular to $\overline{OA}$, using the given information, is related to the area, or vice versa, or equivalent merit	2
• Attempts to use a vector perpendicular to $\overline{OA}$ or $\overline{OB}$, or equivalent merit	1

Sample answer:

Let θ be the angle between $\overline{OA}$ and $\overline{OB}$



Area of triangle = $\frac{1}{2}ab\sin\theta$, where $a = \left|\overline{OA}\right|$ and $b = \left|\overline{OB}\right|$

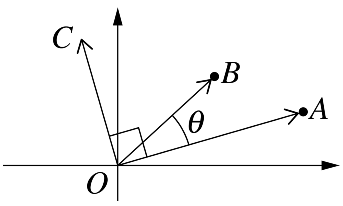
Let $\overrightarrow{OC} = \begin{pmatrix} a_2 \\ -a_1 \end{pmatrix}$ be the vector perpendicular to $\overline{OA}$ using the given information.

We are told that $\overline{OC}$ is perpendicular to $\overline{OA}$ and with the same magnitude.

$$\overrightarrow{OB} \cdot \overrightarrow{OC} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \cdot \begin{pmatrix} a_2 \\ -a_1 \end{pmatrix} = a_2 b_1 - a_1 b_2 \qquad \text{——— (1)}$$

We can also express $\overline{OB} \cdot \overline{OC}$ in terms of θ

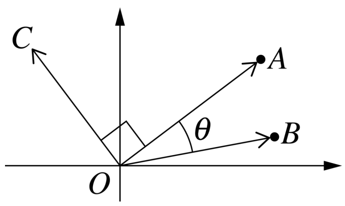
$$\begin{aligned} \overline{OB} \cdot \overline{OC} &= \left|\overline{OB}\right| \cdot \left|\overline{OC}\right| \cos(\angle BOC) \\ &= b \times a \times \cos(\angle BOC) \end{aligned}$$



Case 1:

$\angle BOC = \frac{\pi}{2} - \theta$

so $\overline{OB} \cdot \overline{OC} = ab\sin\theta$



Case 2:

$\angle BOC = \frac{\pi}{2} + \theta$

so $\overline{OB} \cdot \overline{OC} = -ab\sin\theta$

In both cases $\left|\overline{OB} \cdot \overline{OC}\right| = \left|ab\sin\theta\right|$ = 2 × (Area of triangle)

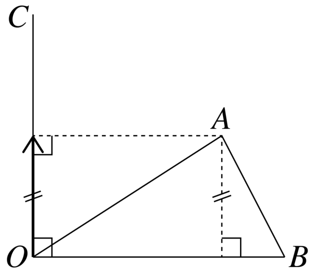
Therefore, area of triangle = $\frac{1}{2}\left|\overrightarrow{OB} \cdot \overrightarrow{OC}\right| = \frac{1}{2}\left|a_2 b_1 - a_1 b_2\right|$ [From (1)]

$$= \frac{1}{2}\left| a_1b_2 - a_2b_1 \right|$$

Question 14 (c) (i) (continued)

Alternative solution

Projection of $\left|\overline{AC}\right|$ onto $\left|\overline{OC}\right|$



Area triangle $= \frac{1}{2}\left|\overrightarrow{OB}\right| \times$ perpendicular height of triangle OAB

$$= \frac{1}{2}\left|\overrightarrow{OB}\right| \times \left|\text{projection of } \overrightarrow{OA} \text{ onto } \overrightarrow{OC}\right|$$

where $\overline{OC} \perp \overline{OB}$ and $\left|\overline{OC}\right| = \left|\overline{OB}\right|$

$$\overrightarrow{OC} = \begin{pmatrix} b_2 \\ -b_1 \end{pmatrix}$$

$$\therefore \quad \left|\overrightarrow{OA} \text{ projected onto } \overrightarrow{OC}\right| = \frac{\left|\overline{OA}\cdot\overline{OC}\right|}{\left|\overrightarrow{OC}\right|}$$

$$= \frac{\left|a_1b_2 - a_2b_1\right|}{\left|\overrightarrow{OB}\right|}$$

$$\therefore \quad \text{Area of triangle } = \frac{1}{2}\left|\overrightarrow{OB}\right| \times \frac{\left|a_1b_2 - a_2b_1\right|}{\left|\overrightarrow{OB}\right|}$$

$$= \frac{1}{2}\left| a_1b_2 - a_2b_1 \right|$$

Question 14 (c) (ii)

Criteria	Marks
• Provides correct solution	4
• Finds the values of *t* at the stationary points of the area, or equivalent merit	3
• Obtains the area of triangle *OPQ* in terms of *t*, or equivalent merit	2
• Obtains the components of $\overline{OP}$ or $\overline{OQ}$ in terms of *t*, or equivalent merit	1

Sample answer:

$$\overrightarrow{OP} = \overrightarrow{OI} + \overrightarrow{IP} = \begin{pmatrix} r + r\cos t \\ r\sin t \end{pmatrix}$$

$$\overrightarrow{OQ} = \overrightarrow{OJ} + \overrightarrow{JQ} = \begin{pmatrix} -R + R\cos 2t \\ R\sin 2t \end{pmatrix}$$

Using part (i), we get that the area of triangle *OPQ* is

$$\begin{aligned} \text{Area of triangle } &= \frac{1}{2}\left|(r + r\cos t)(R\sin 2t) - (r\sin t)(-R + R\cos 2t)\right| \\ &= \frac{1}{2}\left|Rr(1+\cos t)\sin 2t - Rr\sin t(-1+\cos 2t)\right| \\ &= \frac{1}{2}Rr\left|(\sin 2t\cos t - \sin t\cos 2t) + \sin 2t + \sin t\right| \\ &= \frac{1}{2}Rr\left|\sin t + \sin 2t + \sin t\right| \\ \text{Area of triangle } &= \frac{1}{2}Rr\left|\sin 2t + 2\sin t\right| \end{aligned}$$

Let $f(t) = \sin 2t + 2\sin t$, so Area triangle $= Rr|f(t)|$

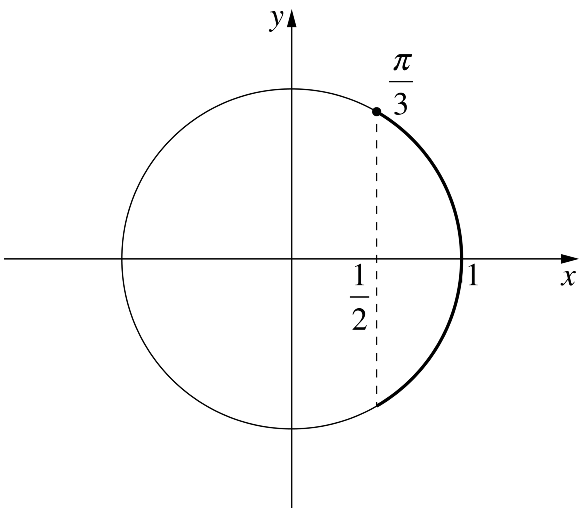
$$\begin{aligned} f'(t) &= 2\cos 2t + 2\cos t \\ &= 2\left(2\cos^2 t + \cos t - 1\right) \\ &= 2(2\cos t - 1)(\cos t + 1) \end{aligned}$$

For all t, $\cos t + 1 \geq 0$ therefore $f'(t)$ has the same sign as $2\cos t - 1$

$f(t)$ is odd so $|f(t)|$ is even and we only need to study $|f(t)|$ on $[0, \pi]$
where area of triangle $= \frac{1}{2}Rr|f(t)|$, so $f(t) \geq 0$ on $[0, \pi]$

$f'(t) \geq 0 \quad \Leftrightarrow \quad 2\cos t - 1 \geq 0 \quad \Leftrightarrow \quad \cos t \geq \frac{1}{2}$

which will happen when $-\frac{\pi}{3} \leq t \leq \frac{\pi}{3}$



t	0		$\frac{\pi}{3}$		π
$f'(t)$		$+$	0	$-$	
	0	$\nearrow$		$\searrow$	0

By symmetry, since $|f(t)|$ is even, we see that the area of the triangle is maximum when $t = -\frac{\pi}{3}$ or $t = \frac{\pi}{3}$

2023 HSC Mathematics Extension 1 Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	ME-C1 Rates of Change	ME11-1
2	1	ME-S1 The Binomial Distribution	ME12-5
3	1	ME-C3 Applications of Calculus	ME12-1
4	1	ME-C3 Applications of Calculus	ME12-4
5	1	ME-T1 Inverse Trigonometric Functions	ME11-3
6	1	ME-V1 Introduction to Vectors	ME12-2
7	1	ME-T1 Inverse Trigonometric Functions	ME11-3
8	1	ME-F1 Further Work with Functions	ME11-1
9	1	ME-F1 Further Work with Functions	ME11-1
10	1	ME-A1 Working with Combinatorics	ME11-5

Section II

Question	Marks	Content	Syllabus outcomes
11 (a)	2	ME-F1 Further Work with Functions	ME11-2
11 (b)	2	ME-A1 Working with Combinatorics	ME11-5
11 (c)	3	ME-F2 Polynomials	ME11-2
11 (d)	2	ME-C2 Further Calculus Skills	ME12-1
11 (e)	3	ME-T3 Trigonometric Equations	ME12-3
11 (f) (i)	2	ME-S1 The Binomial Distribution	ME12-5
11 (f) (ii)	2	ME-S1 The Binomial Distribution	ME12-5
12 (a)	3	ME-C2 Further Calculus Skills	ME12-1
12 (b)	3	ME-P1 Proof by Mathematical Induction	ME12-1
12 (c) (i)	2	ME-S1 The Binomial Distribution	ME12-5
12 (c) (ii)	1	ME-S1 The Binomial Distribution	ME12-5
12 (d)	2	ME-A1 Working with Combinatorics	ME11-5
12 (e)	4	ME-C3 Applications of Calculus	ME12-4
13 (a) (i)	2	ME-C1 Rates of Change	ME11-4
13 (a) (ii)	2	ME-C3 Applications of Calculus	ME12-1, ME12-4
13 (a) (iii)	3	ME-C3 Applications of Calculus	ME12-1, ME12-7
13 (b) (i)	2	ME-V1 Introduction to Vectors	ME12-2
13 (b) (ii)	1	ME-V1 Introduction to Vectors	ME12-2
13 (b) (iii)	3	ME-V1 Introduction to Vectors	ME12-2
13 (b) (iv)	2	ME-V1 Introduction to Vectors	ME12-2

Question	Marks	Content	Syllabus outcomes
14 (a) (i)	1	ME-F1 Further Work with Functions	ME11-1
14 (a) (ii)	2	ME-C2 Further Calculus Skills	ME12-1
14 (b) (i)	1	ME-F2 Polynomials	ME11-6
14 (b) (ii)	3	ME-F2 Polynomials	ME11-2, ME11-6
14 (c) (i)	3	ME-V1 Introduction to Vectors	ME12-2
14 (c) (ii)	4	ME-V1 Introduction to Vectors ME-T3 Trigonometric Equations	ME12-3