2021 HSC Mathematics Extension 1 Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	C
2	B
3	D
4	A
5	B
6	A
7	D
8	C
9	A
10	C

Section II

Question 11 (a)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\left(\underset{\sim}{i}+6\underset{\sim}{j}\right)+\left(2\underset{\sim}{i}-7\underset{\sim}{j}\right)=3\underset{\sim}{i}-\underset{\sim}{j}$$

Question 11 (b)

Criteria	Marks
• Provides correct solution	2
• Expands using the binomial expansion, or equivalent merit	1

Sample answer:

$$\begin{aligned}(2a-b)^4 &= (2a)^4-4(2a)^3b+6(2a)^2b^2-4(2a)b^3+b^4\\ &= 16a^4-32a^3b+24a^2b^2-8ab^3+b^4\end{aligned}$$

Question 11 (c)

Criteria	Marks
• Provides correct solution	3
• Obtains correct primitive in terms of u	2
• Obtains the integrand in terms of u, or equivalent merit	1

Sample answer:

$$\int x\sqrt{x+1}\ dx \qquad\qquad u = x+1 \quad \therefore x = u-1, \quad du = dx$$

$$= \int (u-1)\sqrt{u}\ du$$

$$= \int u^{\frac{3}{2}} - u^{\frac{1}{2}}\ du$$

$$= \frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}} + c$$

$$= \frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} + c$$

Question 11 (d)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$^{10}C_5 \times {}^{8}C_3 = 14\ 112$

Question 11 (e)

Criteria	Marks
• Provides correct solution	2
• Uses the chain rule $\frac{dv}{dt} = \frac{dv}{dr} \times \frac{dr}{dt}$ OR • Obtains $\frac{dv}{dr}$ from the volume of a sphere	1

Sample answer:

$$\frac{dr}{dr} = 0.2 \text{ mm/s} \quad \text{and} \quad \frac{dv}{dt} = \frac{dv}{dr} \cdot \frac{dr}{dt}$$

$$v = \frac{4}{3}\pi r^3 \qquad \therefore \frac{dv}{dr} = 4\pi^2 \text{ and } \frac{dv}{dt} = 4\pi^2 . \frac{dr}{dt}$$

$$\frac{dv}{dt} = 4\pi \times (0.6)^2 \times (0.2) \simeq 0.9 \text{ mm}^3\text{/s} \qquad \text{(1 decimal place)}$$

Question 11 (f)

Criteria	Marks
• Provides correct solution	2
• Integrates to obtain an arcsin function, or equivalent merit	1

Sample answer:

$$\int_0^{\sqrt{3}} \frac{1}{\sqrt{4 - x^2}}\, dx = \left[\sin^{-1}\left(\frac{x}{2}\right)\right]_0^{\sqrt{3}}$$

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1} 0$$

$$= \frac{\pi}{3} - 0$$

$$= \frac{\pi}{3}$$

Question 11 (g)

Criteria	Marks
• Provides correct solution	3
• Finds all three possible values of $\sin x$ OR • Finds at least two possible values of x, or equivalent merit	2
• Notes that $\sin x = -1$ is a solution of the polynomial, or equivalent merit OR • Takes a factor of $\sin^2 x$ OR $2\sin^2 x$ out of first two terms	1

Sample answer:

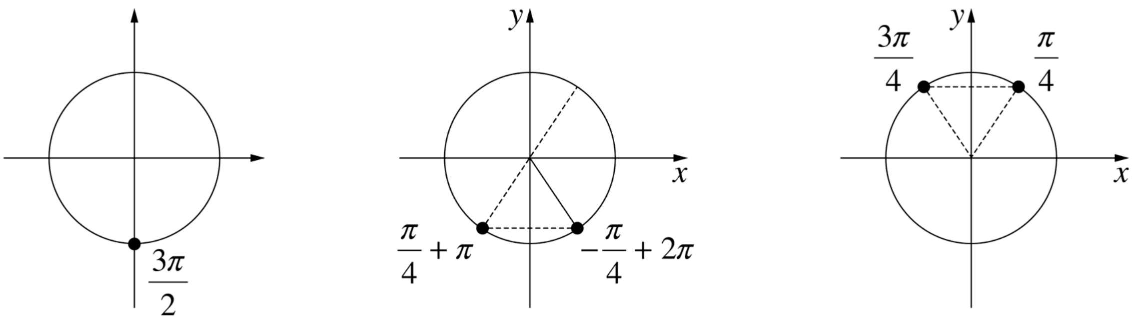
$$2\sin^3 x + 2\sin^2 x - \sin x - 1 = 0$$

$$2\sin^2 x(\sin x + 1) - (\sin x + 1) = 0$$

$$(\sin x + 1)\left(2\sin^2 x - 1\right) = 0$$

$$\sin x = -1 \qquad \text{or} \qquad \sin^2 x = \frac{1}{2}$$

$$\sin x = -1 \qquad \text{or} \qquad \sin x = -\frac{1}{\sqrt{2}} \qquad \text{or} \qquad \sin x = \frac{1}{\sqrt{2}}$$



The solutions in $[0, 2\pi]$ are: $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$.

Question 11 (h)

Criteria	Marks
• Provides correct solution	2
• Writes $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{\delta}$ with a common denominator OR • Applies one formula relating roots and coefficients	1

Sample answer:

$$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{\delta}=\frac{\beta\gamma\delta+\alpha\gamma\delta+\alpha\beta\delta+\alpha\beta\gamma}{\alpha\beta\gamma\delta}$$

$x^4-3x+6=0 \qquad \therefore\ a=1,\ b=0,\ c=0,\ d=-3,\ e=6$

$$\begin{aligned}\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\delta\gamma &= \frac{-d}{a} \qquad \text{(coeff of } x\text{)}\\ &= 3\end{aligned}$$

and $\alpha\beta\gamma\delta=\frac{e}{a}$ (constant term)

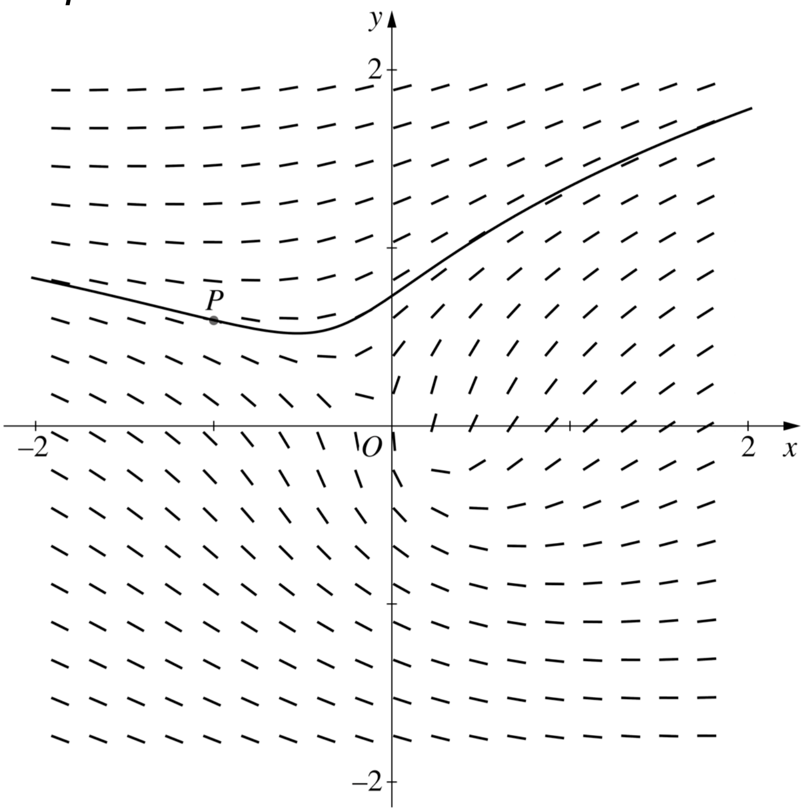
$= 6$

Therefore $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{\delta}=\frac{3}{6}=\frac{1}{2}$.

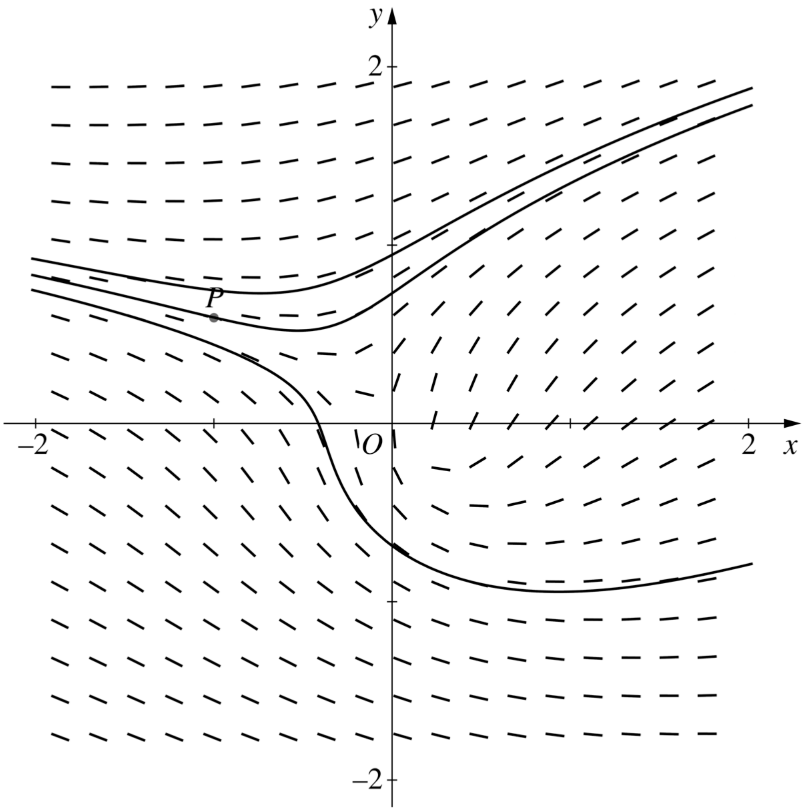
Question 12 (a)

Criteria	Marks
• Provides correct sketch	1

Sample answer:



Answers could include:



Note: An acceptable solution curve does not cross any tangent line in the direction field.

Question 12 (b) (i)

Criteria	Marks
• Provides correct solution	3
• Finds the value of k, or equivalent merit	2
• Obtains a solution to the differential equation, that is, $T = 25 + Ae^{kt}$, or equivalent merit OR • Finds the value of A	1

Sample answer:

$$\int \frac{dT}{T-25} = \int k\ dt$$

$$\therefore kt = \ln(T-25) + c$$

$$\therefore T - 25 = Ae^{kt}$$

when $t = 0,\ T = 5$

$$\therefore -20 = A$$

$$\therefore T = 25 - 20e^{kt}$$

when $t = 8,\ T = 10$

$$\therefore 10 = 25 - 20e^{8k}$$

$$-15 = -20e^{8k}$$

$$e^{8k} = \frac{3}{4}$$

$$8k = \ln\left(\frac{3}{4}\right)$$

$$k = \frac{1}{8}\ln\left(\frac{3}{4}\right)$$

when $T = 20$

$$20 = 25 - 20e^{\frac{1}{8}\ln\left(\frac{3}{4}\right)t}$$

$$-5 = -20e^{\frac{1}{8}\ln\left(\frac{3}{4}\right)t}$$

$$\frac{1}{4} = e^{\frac{1}{8}\ln\left(\frac{3}{4}\right)t}$$

$$\frac{1}{8}\ln\left(\frac{3}{4}\right)t = \ln\left(\frac{1}{4}\right)$$

$$\therefore t = \frac{8\ln\left(\frac{1}{4}\right)}{\ln\left(\frac{3}{4}\right)}$$

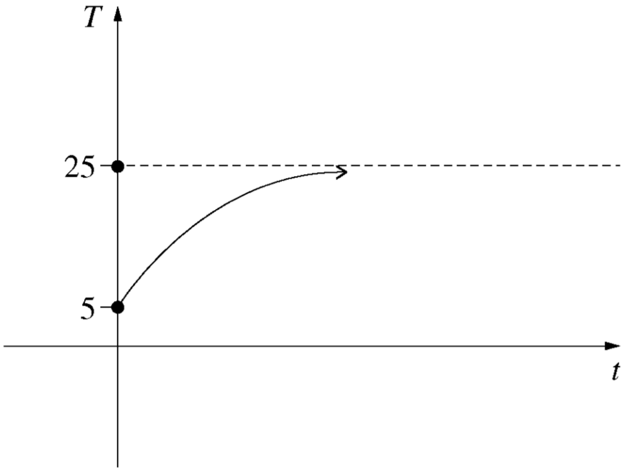
$$= 38.55\ldots$$

$$\simeq 39 \text{ minutes.}$$

Question 12 (b) (ii)

Criteria	Marks
• Provides correct sketch	1

Sample answer:



Question 12 (c)

Criteria	Marks
• Provides correct solution	3
• Proves that *p(k) true* $\Rightarrow p(k+1)$ is true, or equivalent merit	2
• Verifies the initial case, or equivalent merit	1

Sample answer:

When $n = 1$

$$\text{LHS} = \frac{1}{1(1+1)(1+2)} = \frac{1}{1\times2\times3} = \frac{1}{6}$$

$$\text{RHS} = \frac{1}{4} - \frac{1}{2(1+1)(1+2)} = \frac{1}{4} - \frac{1}{2\times2\times3} = \frac{1}{4} - \frac{1}{12} = \frac{1}{6}$$

$\therefore$ statement is true when $n = 1$.

Assume statement is true when $n = k$, some integer $k \geq 1$, that is

$$\frac{1}{1\times2\times3} + \cdots + \frac{1}{k(k+1)(k+2)} = \frac{1}{4} - \frac{1}{2(k+1)(k+2)}$$

Consider $n = k + 1$:

$$\frac{1}{1\times2\times3} + \cdots + \frac{1}{k(k+1)(k+2)} + \frac{1}{(k+1)(k+1+1)(k+1+2)}$$

$$= \frac{1}{4} - \frac{1}{2\big((k+1)+1\big)\big((k+1)+2\big)} = \frac{1}{4} - \frac{1}{2(k+2)(k+3)}$$

$$\begin{aligned}
\text{LHS} &= \frac{1}{1\times2\times3} + \cdots + \frac{1}{k(k+1)(k+2)} + \frac{1}{(k+1)(k+1+1)(k+1+2)} \\
&= \frac{1}{4} - \frac{1}{2(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)} \\
&= \frac{1}{4} - \left\{\frac{1}{2(k+1)(k+2)} - \frac{1}{(k+1)(k+2)(k+3)}\right\} \\
&= \frac{1}{4} - \left\{\frac{k+3-2}{2(k+1)(k+2)(k+3)}\right\} \\
&= \frac{1}{4} - \frac{1}{2\big((k+1)+1\big)\big((k+1)+2\big)} \\
&= \frac{1}{4} - \frac{1}{2(k+2)(k+3)} \\
&= \text{RHS}
\end{aligned}$$

$\therefore$ statement is true for all integers $n \geq 1$, by mathematical induction.

Question 12 (d) (i)

Criteria	Marks
• Provides correct sketch	2
• Sketches the left half of a concave-down parabola OR • Sketches a full concave-down parabola and provides one intercept or the vertex	1

Sample answer:

$y = 4 - \left(1 - \frac{x}{2}\right)^2$ is a parabola.

Vertex when $\left(1 - \frac{x}{2}\right)^2 = 0 \qquad \therefore x = 2$

$$\begin{aligned} y &= 4 - 0 \\ &= 4 \end{aligned}$$

$\therefore$ vertex $(2, 4)$

y-intercept: $\quad y = 4 - \left(1 - \frac{0}{2}\right)^2$
$= 3$

x-intercept:

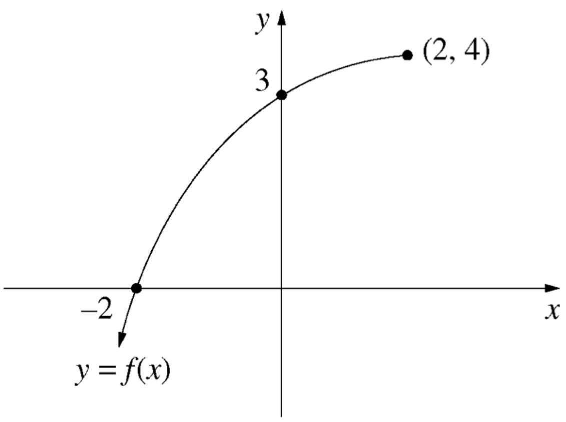
$$\begin{aligned} 4 - \left(1 - \frac{x}{2}\right)^2 &= 0 \\ \left(1 - \frac{x}{2}\right)^2 &= 4 \\ 1 - \frac{x}{2} &= \pm 2 \end{aligned}$$

$$\therefore \quad \frac{x}{2} = 1 + 2 \quad \text{or} \quad \frac{x}{2} = 1 - 2$$

$$= 3 \qquad\qquad = -1$$

$$x = 6 \qquad\qquad x = -2$$

Coefficient of x^2 is negative, so concave down.
Domain $(-\infty, 2]$, so left half only.



Question 12 (d) (ii)

Criteria	Marks
• Provides correct solution	3
• Finds the expression for $f^{-1}(x)$ OR • States the domain and attempts to either – make x the subject, or – find the inverse by switching variables	2
• States the domain $(\infty, 4]$ OR • Attempts to make x the subject OR • Attempts to find the inverse by switching variables	1

Sample answer:

For $f(x)$, $\quad y = 4 - \left(1 - \dfrac{x}{2}\right)^2$, x in the domain $(-\infty, 2]$

For inverse, $\quad x = 4 - \left(1 - \dfrac{y}{2}\right)^2$, y in the domain $(-\infty, 2]$

$$\left(1 - \frac{y}{2}\right)^2 = 4 - x$$

$$\left(\frac{y}{2} - 1\right)^2 = 4 - x$$

$$\frac{y}{2} - 1 = \pm\sqrt{4 - x}$$

$$y = 2 \pm 2\sqrt{4 - x}$$

Range of $f(x)$ is $(-\infty, 4]$, so domain of $f^{-1}(x)$ is $(-\infty, 4]$.

$\therefore\ f^{-1}(x) = 2 - 2\sqrt{4 - x}$ for x in the domain $(-\infty, 4]$.

Alternative solution for Q12 (d) (ii)

$$y = 4 - \left(1 - \frac{x}{2}\right)^2 \quad \text{for} \quad x \le 2$$

For $f^{-1}(x)$, make x the subject:

$$\left(1 - \frac{x}{2}\right)^2 = 4 - y$$

$$x \le 2 \quad \therefore 1 - \frac{x}{2} \ge 0$$

$$\therefore\ 1 - \frac{x}{2} = \sqrt{4 - y}$$

$$\frac{x}{2} = 1 - \sqrt{4 - y}$$

$$x = 2 - 2\sqrt{4 - y}$$

$$\therefore f^{-1}(x) = 2 - 2\sqrt{4 - x}$$

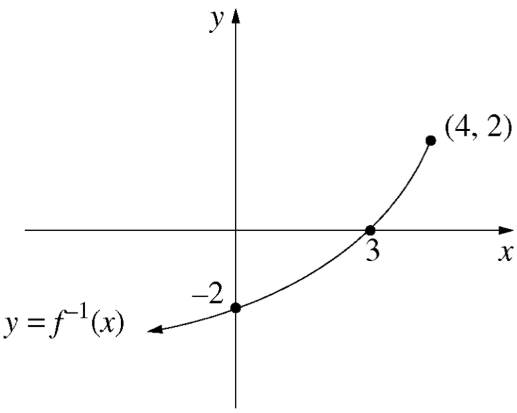
From the graph, range of $f(x)$ is $(-\infty, 4]$

$\therefore$ domain of $f^{-1}(x)$ is $(-\infty, 4]$.

Question 12 (d) (iii)

Criteria	Marks
• Provides correct sketch	1

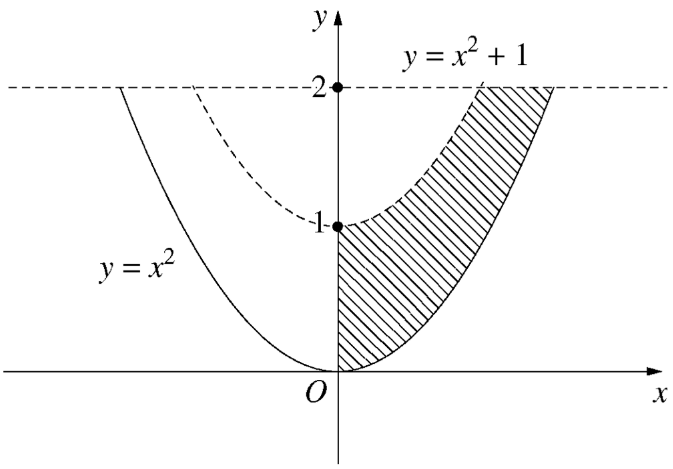
Sample answer:



Question 13 (a)

Criteria	Marks
• Provides correct solution	3
• Evaluates one volume OR • Obtains a correct expression for the volume as a difference of two integrals that are only in terms of y	2
• Obtains an integral for one relevant volume OR • Writes the required volume as the difference of two relevant volumes OR • Finds the limits of integration for both volumes	1

Sample answer:



The volume of the garden sculpture is the difference between the outer volume and the inner one.

$$\text{Outer volume} = \pi \int_0^2 x^2 \, dy$$

$$= \pi \int_0^2 y \, dy \quad \text{since } y = x^2$$

$$\text{Inner volume} = \pi \int_1^2 x^2 \, dy$$

$$= \pi \int_1^2 y - 1 dy \quad \text{since } y = x^2 + 1$$

$$\therefore V = \int_0^2 \pi y \, dy - \int_1^2 \pi (y-1) \, dy$$

$$V = \pi \left[\frac{y^2}{2} \right]_0^2 - \pi \left[\frac{y^2}{2} - y \right]_1^2$$

$$= \pi \left(\frac{4}{2} - 0 \right) - \pi \left(\left[\frac{4}{2} - 2 \right] - \left[\frac{1}{2} - 1 \right] \right)$$

$$= \frac{3\pi}{2}$$

The volume of the sculpture is $\dfrac{3\pi}{2}$ m^3.

Question 13 (b)

Criteria	Marks
• Provides correct solution	4
• Finds the maximum height and the time of flight, or equivalent merit.	3
• Finds the maximum height, or equivalent merit	2
• Finds an expression for the vertical velocity, or equivalent merit	1

Sample answer:

$\underset{\sim}{v}(t) = V\cos\theta\,\underset{\sim}{i} + (-10t + V\sin\theta)\,\underset{\sim}{j}$

The height is greatest when $\dot{y} = 0$ ie when $-10t + V\sin\theta = 0$

$$t = \frac{V\sin\theta}{10} = \frac{12\sin 30°}{10} = \frac{6}{10} = 0.6 \text{ s}$$

$$\therefore y = -5\times(0.6)^2 + 12\times(0.6)\times\sin 30° + 1$$

$$= 2.8 \text{ m}$$

The room is 3 metres high and maximum height reached by the object is 2.8 m so the object will not hit the ceiling.

To know if the object hits the far wall or not, it is enough to determine if the unrestricted horizontal range would be more or less than 10 metres.

$y = 0$ if $-5t^2 + Vt\sin\theta + h = 0$

$-5t^2 + 12\times\frac{1}{2}t + 1 = 0$

$-5t^2 + 6t + 1 = 0$

$\triangle = 6^2 - 4(-5) = 56$

$\therefore\ t = \dfrac{-6+\sqrt{56}}{-10} < 0$ or $\dfrac{-6-\sqrt{56}}{-10} > 0$

We need $t > 0$, so the object, in the absence of a far wall, would hit the floor after $t = \dfrac{6+\sqrt{56}}{10} \simeq 1.348$ s.

Substitute in $x(t)$:

$$x(t) = Vt\cos\theta = 12\times 1.348\cos 30°$$
$$\approx 14 \text{ m}$$

Since the far wall is only 10 m away, the object will hit the wall.

Alternative solution for second part

When $x = 10$ $\quad 12t\cos 30^\circ = 10$

$$6\sqrt{3}t = 10$$

$$t = \frac{10}{6\sqrt{3}} = 0.9622 \text{ seconds}$$

When $t = 0.9622$

$y = -5(0.9622\ldots)^2 + 12(0.9622\ldots)\sin 30^\circ + 1$

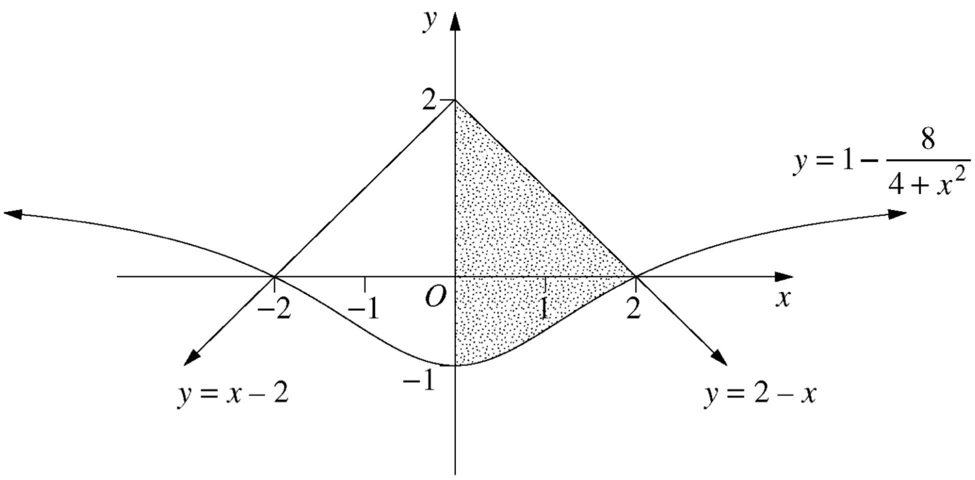
$= 2.144 \text{ m}$

$\therefore$ Ball is still above the floor when $x = 10$ m and so will hit the far wall without hitting the floor.

Question 13 (c)

Criteria	Marks
• Provides correct solution	3
• Obtains the (signed) area OR the area of either side of the y-axis between the x-axis and the curve $y = 1 - \dfrac{8}{4 + x^2}$, or equivalent merit	2
• Finds an integral expression for the area, or equivalent merit OR • Recognises that the total area is twice the area on one side of the y-axis OR • Finds the area of a relevant triangle	1

Sample answer:



By symmetry, area required is double the shaded region above

$$\frac{\text{Area}}{2} = \int_0^2 (2 - x) - \left(1 - \frac{8}{4 + x^2}\right) dx$$

$$= \int_0^2 1 - x + \frac{8}{4 + x^2}\, dx$$

$$= \left[x - \frac{x^2}{2} + \frac{8}{2}\tan^{-1}\left(\frac{x}{2}\right)\right]_0^2$$

$$= \left(2 - 2 + 4\tan^{-1} 1\right) - 0$$

$$= \pi$$

So Area $= 2\pi$

Question 13 (d) (i)

Criteria	Marks
• Provides correct solution	2
• Uses $A = B - d$ and $C = B + d$ in the left hand side and attempts to use a suitable trigonometric identity, or equivalent merit	1

Sample answer:

Given $A = B - d$ and $C = B + d$

$$\therefore \frac{\sin A + \sin C}{\cos A + \cos C} = \frac{\sin(B-d) + \sin(B+d)}{\cos(B-d) + \cos(B+d)}$$

$$= \frac{2\sin B \cos d}{2\cos B \cos d}$$

$$= \tan B$$

Question 13 (d) (ii)

Criteria	Marks
• Provides correct solution	2
• Identifies the value of B in order to use as in part (i), or equivalent merit	1

Sample answer:

$$A = \frac{5\theta}{7}, \quad C = \frac{6\theta}{7} \quad \therefore B = \frac{\frac{5\theta}{7} + \frac{6\theta}{7}}{2} = \frac{11\theta}{14}$$

$$\therefore \text{ LHS} = \tan\frac{11\theta}{14}$$

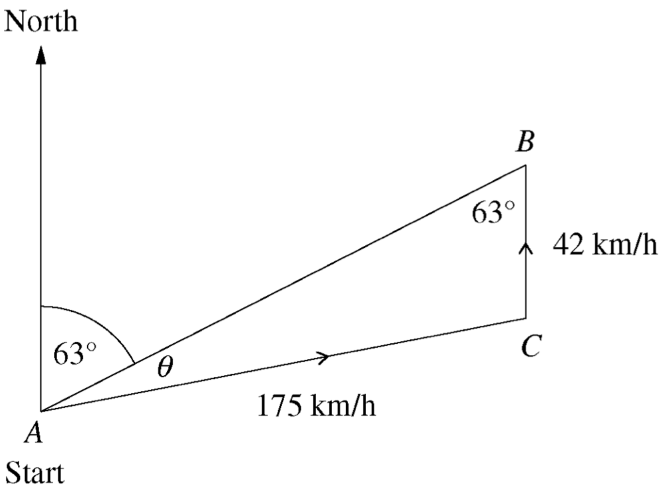
$$\tan\frac{11\theta}{14} = \sqrt{3}$$

$$\theta = \frac{14\pi}{33} \quad \text{or} \quad \frac{56\pi}{33}$$

Question 14 (a)

Criteria	Marks
• Provides correct solution	3
• Attempts to use the sine rule in the correct triangle, or equivalent merit	2
• Sketches a suitable diagram, or equivalent merit	1

Sample answer:



$$\angle ABC = 63°$$

Let $\angle BAC = \theta$

$$\therefore \quad \frac{\sin\theta}{42} = \frac{\sin 63°}{175}$$

$$\sin\theta = 0.2138\ldots$$

$$\theta = 12.34\ldots$$

$$\approx 12°$$

$\therefore$ required bearing $= 63 + 12$

$= 075°$

Question 14 (b)

Criteria	Marks
• Provides correct solution	4
• Uses the given information to obtain two equations in A and C, or equivalent merit	3
• Integrates both sides correctly, or equivalent merit	2
• Attempts to separate the variables in the differential equation, or equivalent merit	1

Sample answer:

$$\frac{dP}{dt} = 0.1P\left(\frac{C-P}{C}\right)$$

$$\int \frac{C}{P(C-P)}\,dP = \int 0.1\,dt$$

$$\int \left(\frac{1}{P} + \frac{1}{C-P}\right)dP = \int 0.1\,dt$$

$\ln|P| - \ln|C-P| = 0.1t + k$ where k is a constant

$$\ln\left|\frac{P}{C-P}\right| = 0.1t + k$$

$\frac{P}{C-P} = Ae^{0.1t}$ where $A = e^k$ is a constant

When $t = 0$, $P = 150\,000$ so $\dfrac{150\,000}{C - 150\,000} = A$ ①

When $t = 20$, $P = 600\,000$ so $\dfrac{600\,000}{C - 600\,000} = Ae^2$ ②

Substituting ① into ②: $\dfrac{600\,000}{C - 600\,000} = \dfrac{150\,000}{C - 150\,000}e^2$

Taking the reciprocal of both sides

$$\frac{C - 600\,000}{600\,000} = \frac{C - 150\,000}{150\,000}e^{-2}$$

$$150\,000(C - 600\,000) = 600\,000(C - 150\,000)e^{-2}$$

$$C\left(150\,000 - 600\,000e^{-2}\right) = 150\,000 \times 600\,000\left(1 - e^{-2}\right)$$

$$C = \frac{150\,000 \times 600\,000\left(1 - e^{-2}\right)}{150\,000 - 600\,000e^{-2}} \simeq 1\,131\,121$$

$$\simeq 1\,131\,000$$

Question 14 (c) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

Let $\underset{\sim}{v} = \begin{pmatrix} x \\ y \end{pmatrix}$ be a vector

$$\underset{\sim}{v} \cdot \underset{\sim}{v} = x \times x + y \times y = x^2 + y^2 = |\underset{\sim}{v}|^2$$

Alternative:

$$\underset{\sim}{v} \cdot \underset{\sim}{v} = |\underset{\sim}{v}|.|\underset{\sim}{v}|\cos\theta$$
$$= |\underset{\sim}{v}|^2 \cos\theta$$

where θ is the angle between and $\underset{\sim}{v}$ and $\underset{\sim}{v}$

$\therefore \quad \theta = 0 \quad \text{and} \quad \cos\theta = 1$

$\therefore \quad \underset{\sim}{v} \cdot \underset{\sim}{v} = |\underset{\sim}{v}|^2$

Question 14 (c) (ii)

Criteria	Marks
• Provides correct solution	3
• Equates $\overrightarrow{AC} \cdot \overrightarrow{AC}$ with $\overrightarrow{BD} \cdot \overrightarrow{BD}$, in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$, or equivalent merit	2
• Expresses AC or BD in terms of a and b, or equivalent merit	1

Sample answer:

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \underset{\sim}{a} + \underset{\sim}{b}$

$\overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD} = -\underset{\sim}{a} + k\underset{\sim}{b}$

By part (i) $\left|\overrightarrow{AC}\right| = \left|\overrightarrow{BD}\right|$ implies

$$\overrightarrow{AC} \cdot \overrightarrow{AC} = \overrightarrow{BD} \cdot \overrightarrow{BD}$$

$$\left(\underset{\sim}{a} + \underset{\sim}{b}\right) \cdot \left(\underset{\sim}{a} + \underset{\sim}{b}\right) = \left(-\underset{\sim}{a} + k\underset{\sim}{b}\right) \cdot \left(-\underset{\sim}{a} + k\underset{\sim}{b}\right)$$

$$\cancel{\underset{\sim}{a} \cdot \underset{\sim}{a}} + 2\underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{b} \cdot \underset{\sim}{b} = \cancel{\underset{\sim}{a} \cdot \underset{\sim}{a}} - 2k\underset{\sim}{a} \cdot \underset{\sim}{b} + k^2\underset{\sim}{b} \cdot \underset{\sim}{b}$$

$$2(1+k)\underset{\sim}{a} \cdot \underset{\sim}{b} = \left(k^2 - 1\right)\underset{\sim}{b} \cdot \underset{\sim}{b}$$

$$2(1+k)\underset{\sim}{a} \cdot \underset{\sim}{b} = (k+1)(k-1)\underset{\sim}{b} \cdot \underset{\sim}{b}$$

$$1 + k \neq 0 \quad \text{since } k > 0$$

$$\therefore \quad 2\underset{\sim}{a} \cdot \underset{\sim}{b} = (k-1)\underset{\sim}{b} \cdot \underset{\sim}{b}$$

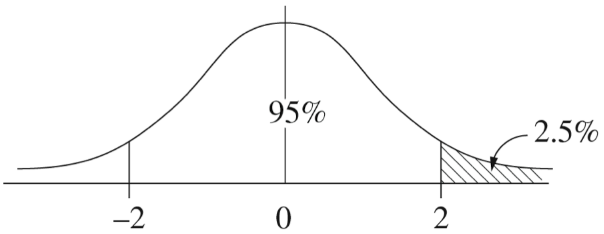
$$\therefore \quad 2\underset{\sim}{a} \cdot \underset{\sim}{b} = (k-1)\left|\underset{\sim}{b}\right|^2$$

$$2\underset{\sim}{a} \cdot \underset{\sim}{b} + (1-k)\left|\underset{\sim}{b}\right|^2 = 0$$

Question 14 (d)

Criteria	Marks
• Provides correct solution	3
• Recognises 2σ is important AND has σ in terms of n OR • Recognises 2σ is important AND has the value of σ OR • Obtains $2 = \frac{\sqrt{n}}{c}$ where c is a constant, or equivalent merit	2
• Finds σ in terms of n OR • Sketches a normal distribution and shades the correct region OR • Writes $P\left(\hat{p} \geq \frac{4}{500}\right) < 0.025$, or explains in words, or equivalent merit	1

Sample answer:



A tail with area 2.5% means $\hat{p} = \frac{4}{500}$ is two standard deviations above the mean

so $$\frac{4}{500} = \mu + 2\sigma$$

$$\frac{4}{500} = \frac{3}{500} + 2\sqrt{\frac{\frac{3}{500} \times \left(1 - \frac{3}{500}\right)}{n}}$$

$$\frac{1}{500} = 2\frac{\sqrt{\frac{3}{500}\left(\frac{497}{500}\right)}}{\sqrt{n}}$$

$$\sqrt{n} = 2 \times 500\sqrt{\frac{3}{500} \times \frac{497}{500}}$$

$$= 2 \times \cancel{500}\,\frac{\sqrt{3 \times 497}}{\cancel{500}}$$

$$n = 4 \times 3 \times 497 = 5964$$

The sample size to be chosen so that the chances of shutting down unnecessarily is less than 2.5% is $n = 5964$, ie approximately 6000.

Question 14 (e)

Criteria	Marks
• Provides correct solution	2
• Obtains an expression for the derivative of $g^{-1}(x)$ in terms of x, or equivalent merit	1

Sample answer:

$g(1) = 3 \quad \therefore\ g^{-1}(3) = 1$

Using product rule:

$$f'(x) = g^{-1}(x).1 + x.\frac{d}{dx}\left(g^{-1}(x)\right)$$

Now if $g^{-1}(x) = y$ then $x = g(y)$

$$\therefore \quad \frac{dx}{dy} = g'(y)$$

and $\dfrac{dy}{dx} = \dfrac{1}{g'(y)}$

$$\therefore \quad \frac{d}{dx}\left(g^{-1}(x)\right) = \frac{1}{g'\left(g^{-1}(x)\right)}$$

$$\therefore \quad f'(x) = g^{-1}(x) + \frac{x}{g'\left(g^{-1}(x)\right)}$$

$$\therefore \quad f'(3) = g^{-1}(3) + \frac{3}{g'\left(g^{-1}(3)\right)}$$

$$= 1 + \frac{3}{g'(1)}$$

Now, $g(x) = x^3 + 4x - 2$

$$\therefore \quad g'(x) = 3x^2 + 4$$

$$\therefore \quad f'(3) = 1 + \frac{3}{3(1)^2 + 4}$$

$\therefore$ gradient of tangent $= \dfrac{10}{7}$.

2021 HSC Mathematics Extension 1 Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	ME V1 Introduction to vectors	ME 12–2
2	1	ME C2 Further calculus skills	ME 12–1
3	1	ME F2 Polynomials	ME 11–2
4	1	ME C3 Applications of calculus	ME 12–4
5	1	ME V1 Introduction to vectors	ME 12–2
6	1	ME S1 The binomial distribution	ME 12–5
7	1	ME T3 Trigonometric equations	ME 12–3
8	1	ME F1 Further work with functions	ME 11–2
9	1	ME T1 Inverse trigonometric functions	ME 11–3
10	1	ME A1 Working with combinations	ME 11–5

Section II

Question	Marks	Content	Syllabus outcomes
11 (a)	1	ME V1 Introduction to vectors	ME 12–2
11 (b)	2	ME A1 Working with combinations	ME 11–5
11 (c)	3	ME C2 Further calculus skills	ME 12–4
11 (d)	1	ME C1 Applications of calculus	ME 11–4
11 (e)	2	ME A1 Working with combinations	ME 11–5
11 (f)	2	ME C2 Further calculus skills	ME 12–4
11 (g)	3	ME T3 Trigonometric equations	ME 12–3
11 (h)	2	ME F2 Polynomials	ME 11–2
12 (a)	1	ME C3 Applications of calculus	ME 12–4
12 (b) (i)	3	ME C1 Rates of change	ME 11-4, ME 12–4
12 (b) (ii)	1	ME C1 Rates of change	ME 11–4
12 (c)	3	ME P1 Proof by mathematical induction	ME 12–1
12 (d) (i)	2	ME F1 Further work with functions	ME 11–2
12 (d) (ii)	3	ME F1 Further work with functions	ME 11–1
12 (d) (iii)	1	ME F1 Further work with functions	ME 11–1

Question	Marks	Content	Syllabus outcomes
13 (a)	3	ME C3 Applications of calculus	ME 12–4
13 (b)	4	ME V1 Introduction to vectors	ME 12–2
13 (c)	3	ME C2 Further calculus skills ME C3 Applications of calculus	ME 12–1
13 (d) (i)	2	ME T2 Further trigonometric identities	ME 11–3
13 (d) (ii)	2	ME T3 Trigonometric equations	ME 12–3
14 (a)	3	ME V1 Introduction to vectors	ME 12–2
14 (b)	4	ME C3 Applications of calculus	ME 12–4
14 (c) (i)	1	ME V1 Introduction to vectors	ME 12–2
14 (c) (ii)	3	ME V1 Introduction to vectors	ME 12–2
14 (d)	3	ME S1 The binomial distribution	ME 12–5
14 (e)	2	ME C2 Further calculus skills	ME 12–1