BOSTES | BOARD OF STUDIES TEACHING & EDUCATIONAL STANDARDS NSW

2016 HSC Mathematics Extension 1 Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	D
2	B
3	A
4	C
5	A
6	D
7	C
8	B
9	A
10	A

Section II

Question 11 (a)

Criteria	Marks
• Provides correct solution	2
• Obtains $x = y^3 - 2$, or equivalent merit	1

Sample answer:

$$y = x^3 - 2$$

Interchange x and y

$$x = y^3 - 2$$

$$\sqrt[3]{x+2} = y$$

$$\therefore \quad f^{-1}(x) = \sqrt[3]{x+2}$$

Question 11 (b)

Criteria	Marks
• Provides correct solution	3
• Obtains correct integral in terms of u only, or equivalent merit	2
• Attempts to relate du and dx, or equivalent merit	1

Sample answer:

Let $u = x - 4$

$$\frac{du}{dx} = 1 \quad \Rightarrow \quad du = dx$$

$$\therefore \quad \int x\sqrt{x-4}\,dx = \int (u+4)u^{\frac{1}{2}}\,du$$

$$= \int \left(u^{\frac{3}{2}} + 4u^{\frac{1}{2}}\right)du$$

$$= \frac{2}{5}u^{\frac{5}{2}} + \frac{8}{3}u^{\frac{3}{2}} + C$$

$$= \frac{2}{5}(x-4)^{\frac{5}{2}} + \frac{8}{3}(x-4)^{\frac{3}{2}} + C$$

$$= \frac{2}{5}\sqrt{(x-4)^5} + \frac{8}{3}\sqrt{(x-4)^3} + C$$

Question 11 (c)

Criteria	Marks
• Provides correct solution	2
• Obtains $\frac{k}{a^2+(bx)^2}$, or equivalent merit	1

Sample answer:

$$\frac{d}{dx}\left(3\tan^{-1}(2x)\right) = 3\cdot\frac{2}{1+(2x)^2}$$

$$= \frac{6}{1+4x^2}$$

Question 11 (d)

Criteria	Marks
• Provides correct solution	2
• Observes $2\sin x\cos x = \sin 2x$, or equivalent merit	1

Sample answer:

Method 1

$$\lim_{x\to 0}\left(\frac{2\sin x\cos x}{3x}\right) = \frac{2}{3}\lim_{x\to 0}\left(\frac{\sin x}{x}\right)\cdot\lim_{x\to 0}\left(\frac{\cos x}{x}\right)$$

$$= \underline{\underline{\frac{2}{3}}}$$

Method 2

$$\lim_{x\to 0}\left(\frac{2\sin x\cos x}{3x}\right) = \frac{2}{3}\lim_{x\to 0}\left(\frac{\sin 2x}{2x}\right)$$

$$= \underline{\underline{\frac{2}{3}}}$$

Question 11 (e)

Criteria	Marks
• Provides correct solution	3
• Finds the three boundary values for x, or equivalent merit	2
• Attempts to write as a single fraction, or equivalent merit	1

Sample answer:

$$\frac{3}{2x+5} - x > 0$$

$$\frac{3}{2x+5} > x$$

$$(2x+5)^{\not 2} \times \frac{3}{\cancel{(2x+5)}} > x(2x+5)^2 \qquad \left(x \neq -\frac{5}{2}\right)$$

$$3(2x+5) > x(2x+5)^2$$

$$x(2x+5)^2 - 3(2x+5) < 0$$

$$(2x+5)(2x^2+5x-3) < 0$$

$$(2x+5)(2x-1)(x+3) < 0$$

$$x < -3 \text{ or } -\frac{5}{2} < x < \frac{1}{2}$$

Question 11 (e) (continued)

Method 2

$$\frac{3}{2x+5} - x > 0$$

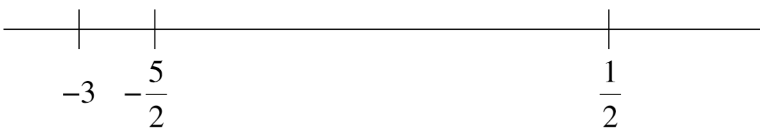
Solve $\quad \dfrac{3}{2x+5} - x = 0 \qquad$ for $x \neq -\dfrac{5}{2}$

$$3 - x(2x+5) = 0$$

$$2x^2 + 5x - 3 = 0$$

$$(2x-1)(x+3) = 0$$

$$x = \frac{1}{2}, \quad x = -3$$



Test	$x = -4$	$x = 0$	$x = 1$
	$\dfrac{3}{-3} + 4 > 0$	$\dfrac{3}{5} > 0$	$\dfrac{3}{7} - 1 < 0$
	✓	✓	✗

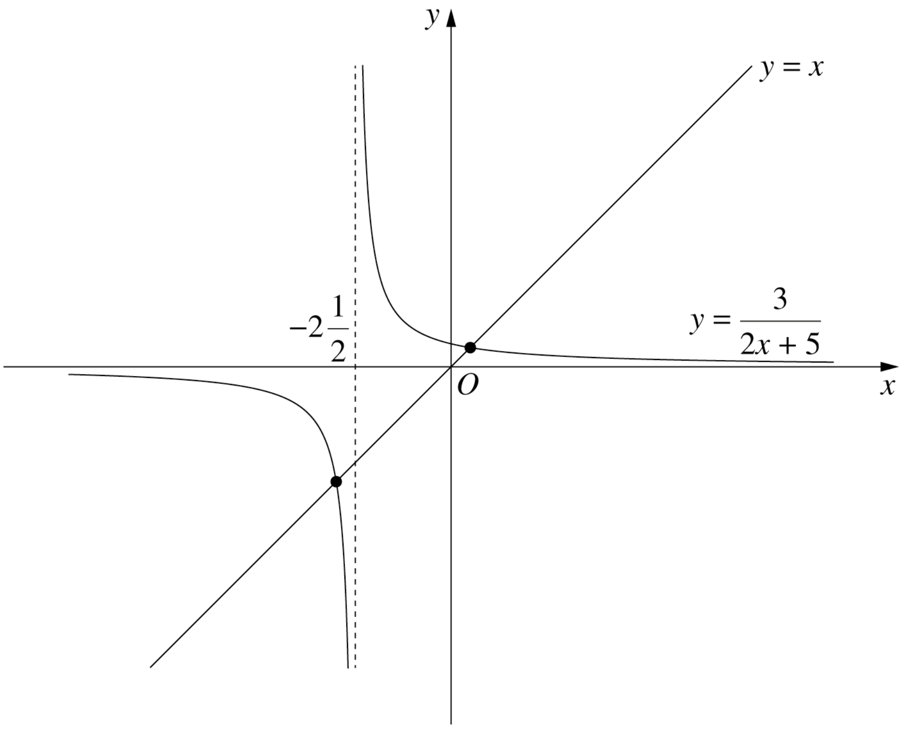
$x < -3$ or $-\dfrac{5}{2} < x < \dfrac{1}{2}$

Question 11 (e) (continued)

Method 3

$$\frac{3}{2x+5} - x > 0$$

$$\frac{3}{2x+5} > x$$



$$3 = 2x^2 + 5x$$

$$2x^2 + 5x - 3 = 0$$

$$(2x-1)(x+3) = 0$$

$$x = \frac{1}{2},\ -3$$

Solution: $x < -3$ or $-2\frac{1}{2} < x < \frac{1}{2}$

Question 11 (f) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$P(S) = \frac{3}{5}$

The possibilities are SNN, NNS, NSN

$$\therefore P(1S + 2N) = \left(\frac{3}{5} \times \frac{2}{5} \times \frac{2}{5}\right) \times 3$$

$$= \frac{36}{125}$$

Question 11 (f) (ii)

Criteria	Marks
• Provides correct solution	2
• Attempts to use complementary events, or equivalent merit	1

Sample answer:

Method 1

$$P(\text{at least 2 bullseyes}) = {}^6C_2\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)^4 + {}^6C_3\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^3 + {}^6C_4\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^2$$

$$+ {}^6C_5\left(\frac{3}{5}\right)^5\left(\frac{2}{5}\right)^1 + {}^6C_6\left(\frac{3}{5}\right)^6\left(\frac{2}{5}\right)^0$$

$$= \frac{432}{3125} + \frac{864}{3125} + \frac{972}{3125} + \frac{2916}{15625} + \frac{729}{15625}$$

$$= \frac{2997}{3125}$$

Method 2

$$P(\text{at least 2 bullseyes}) = 1 - \left[\left(\frac{2}{5}\right)^6 + {}^6C_1\left(\frac{3}{5}\right)\left(\frac{2}{5}\right)^5\right]$$

$$= \frac{2997}{3125}$$

Question 12 (a) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

By similar triangles,

$$\frac{r}{5} = \frac{h}{20}$$

$$\therefore \quad r = \frac{h}{4}$$

Question 12 (a) (ii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$v = \frac{1}{3}\pi r^2 h$$

$$v = \frac{1}{3}\pi \left(\frac{h}{4}\right)^2 h$$

$$v = \frac{\pi}{48}h^3$$

$$\therefore \quad \frac{dv}{dh} = \frac{3\pi h^2}{48}$$

$$\Rightarrow \frac{dv}{dh} = \frac{\pi}{16}h^2$$

Question 12 (a) (iii)

Criteria	Marks
• Provides correct solution	2
• Attempts to use $\frac{dA}{dt}$, or equivalent merit	1

Sample answer:

Show $\frac{dh}{dt} = \frac{-0.32}{\pi h}$

$$A = \pi r^2$$

$$r = \frac{h}{4}$$

$$A = \frac{\pi h^2}{16}$$

$$\frac{dA}{dh} = \frac{\pi h}{8}$$

$$\frac{dA}{dt} = \frac{\pi h}{8} \times \frac{dh}{dt} = -0.04$$

$$\therefore \frac{dh}{dt} = \frac{-0.32}{\pi h}$$

Question 12 (a) (iv)

Criteria	Marks
• Provides correct solution	2
• Attempts to use the chain rule, or equivalent merit	1

Sample answer:

$$\frac{dv}{dh} = \frac{\pi}{16}h^2$$

$$\frac{dh}{dt} = \frac{-0.32}{\pi h}$$

$$\frac{dv}{dt} = \frac{dv}{dh} \cdot \frac{dh}{dt}$$

$$= \frac{\cancel{\pi} h^{\cancel{2}}}{16} \cdot \frac{-0.32}{\cancel{\pi}\cancel{h}}$$

$$= -0.02h$$

When $h = 10$, $\frac{dv}{dt} = -0.2$

$\therefore$ the volume decreases at a rate of 0.2 $\text{cm}^3\,\text{s}^{-1}$

Question 12 (b) (i)

Criteria	Marks
• Provides correct solution	3
• Obtains $\frac{dx}{dt} = k(500 - x)$, or equivalent merit	2
• Obtains $\frac{dx}{dt} = ky$ or $y = 500 - x$, or equivalent merit	1

Sample answer:

$$x + y = 500$$
$$y = 500 - x$$

$$\frac{dx}{dt} \alpha y$$

$$\therefore \quad \frac{dx}{dt} = ky \qquad \text{where } k \text{ is a constant}$$

$$\Rightarrow \quad \frac{dx}{dt} = k(500 - x)$$

When $x = 0$, $\frac{dx}{dt} = 2$

$$\therefore \quad 2 = k(500 - 0)$$

$$\Rightarrow \quad k = \underline{\underline{0.004}}$$

$$\therefore \quad \frac{dx}{dt} = \underline{\underline{0.004(500 - x)}}$$

Question 12 (b) (ii)

Criteria	Marks
• Provides correct solution	2
• Differentiates the given expression, or equivalent merit	1

Sample answer:

Method 1

$$x = 500 - Ae^{-0.004t}$$

$$\frac{dx}{dt} = A \times 0.004e^{-0.004t}$$

$$= 0.004\left(Ae^{-0.004t}\right)$$

$$= 0.004(500 - x) \quad \text{as required.}$$

Now, $t = 0$, $\frac{dx}{dt} = 2$, $x = 0$

$$\therefore \quad 0 = 500 - Ae^{-0.004(0)}$$

$$A = 500$$

Method 2

$$\frac{dx}{dt} = 0.004(500 - x)$$

$$\int \frac{dx}{500 - x} = \int 0.004\,dt$$

$$-\ln(500 - x) = 0.004t + C$$

$$500 - x = Ae^{-0.004t} \quad \text{where } A \text{ is a constant.}$$

$$\therefore \quad x = \underline{\underline{500 - Ae^{-0.004t}}}$$

When $t = 0$, $x = 0$, $\Rightarrow$ $A = \underline{\underline{500}}$

$$\therefore \quad x = \underline{\underline{500\left(1 - e^{-0.004t}\right)}}$$

Question 12 (c) (i)

Criteria	Marks
• Provides correct solution	2
• Relates the slopes of both curves at α, or equivalent merit	1

Sample answer:

$$y = \tan x$$

$$\therefore \quad \frac{dy}{dx} = \sec^2 x$$

At $x = \alpha$, $\dfrac{dy}{dx} = \sec^2 \alpha$

$$y = \cos x$$

$$\therefore \quad \frac{dy}{dx} = -\sin x$$

At $x = \alpha$, $\dfrac{dy}{dx} = -\sin \alpha$

$$\begin{aligned} \text{Now product of gradients} &= \sec^2 \alpha \times (-\sin \alpha) \\ &= -\frac{\sin \alpha}{\cos \alpha} \cdot \frac{1}{\cos \alpha} \\ &= -\frac{\tan \alpha}{\cos \alpha} \\ &= -1 \quad \text{since } \tan \alpha = \cos \alpha \text{ at the point of intersection} \end{aligned}$$

$\therefore$ Tangents to the curves are perpendicular at $x = \alpha$.

Question 12 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Finds a relevant function to use Newton's method, or equivalent merit	1

Sample answer:

Let $f(x) = \tan x - \cos x$

$$f'(x) = \sec^2 x + \sin x$$

Using Newton's method to solve $\tan\alpha = \cos\alpha$, with $x_0 = 1$.

$$\begin{aligned}
x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\
&= 1 - \frac{f(1)}{f'(1)} \\
&= 1 - \frac{\tan 1 - \cos 1}{\sec^2 1 + \sin 1} \\
&\approx 1 - \frac{1.0171054}{4.2669898} \\
&\approx 1 - 0.238366 \\
&= \underline{\underline{0.76}} \quad \text{(2 decimal places)}
\end{aligned}$$

Question 13 (a) (i)

Criteria	Marks
• Provides correct explanation	2
• Explains why the amplitude is 4, or equivalent merit	1

Sample answer:

From reference sheet

$x = b + a\cos(nt + \alpha)$

High tide = maximum = 9

Low tide = minimum = 1

$\therefore\ a = \dfrac{9-1}{2} = 4$

Period, $T = \dfrac{2\pi}{n}$; $\quad \therefore\ \dfrac{25}{2} = \dfrac{2\pi}{n}$

$\Rightarrow \quad n = \dfrac{4\pi}{25}$

$t = 0$ at the high tide so $b = 5$ and $\alpha = 0$

$\therefore\ x = 5 + 4\cos\left(\dfrac{4\pi t}{25}\right)$

Question 13 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Attempts to find t when $\ddot{x} = 0$, or equivalent merit	1

Sample answer:

$$x = 5 + 4\cos\left(\frac{4\pi t}{25}\right)$$

$$\frac{dx}{dt} = -4 \cdot \frac{4\pi}{25}\sin\left(\frac{4\pi t}{25}\right)$$

$$\frac{d^2x}{dt^2} = \frac{-16\pi}{25} \cdot \frac{4\pi}{25}\cos\left(\frac{4\pi t}{25}\right)$$

Fastest when $\frac{d^2x}{dt^2} = 0$

Increasing fastest when

$$\cos\left(\frac{4\pi t}{25}\right) = 0$$

$$\therefore \ \frac{4\pi t}{25} = \frac{\pi}{2}, \quad \frac{3\pi}{2}, \quad \cdots$$

$$t = \frac{25}{8}, \quad \frac{75}{8}, \quad \cdots$$

Tide is going *out* when $t = \frac{25}{8}$

$\therefore$ Increasing when $t = \underline{\underline{\frac{75}{8}}} = 9\frac{3}{8}$.

So $9\frac{3}{8}$ hours after 2 am makes the time $11{:}22\frac{1}{2}$ am (11:22:30 am)

(or $11\frac{3}{8}$ hours after midnight).

Question 13 (b) (i)

Criteria	Marks
• Provides correct solution	2
• Finds $\dot{y}$, or equivalent merit	1

Sample answer:

$x = ut\cos\theta \qquad y = ut\sin\theta - 5t^2$

$\dot{x} = u\cos\theta \qquad \dot{y} = u\sin\theta - 10t$

Maximum height if $\dot{y} = 0 \quad \Rightarrow \quad u\sin\theta - 10t = 0$

$$u\sin\theta = 10t$$

$$t = \frac{u\sin\theta}{10}$$

Substitute into $y = ut\sin\theta - 5t^2$

$$\therefore \quad y = u\sin\theta \cdot \frac{u\sin\theta}{10} - 5\left(\frac{u\sin\theta}{10}\right)^2$$

$$= \frac{u^2\sin^2\theta}{10} - \frac{u^2\sin^2\theta}{20}$$

$\therefore$ Maximum height $= \dfrac{u^2\sin^2\theta}{20}$

Question 13 (b) (ii)

Criteria	Marks
• Provides correct solution	2
• Shows that the maximum height above the projection point is $\frac{45}{4}$, or equivalent merit	1

Sample answer:

Maximum height $= 20 + \dfrac{30^2\sin^2 30}{20}$

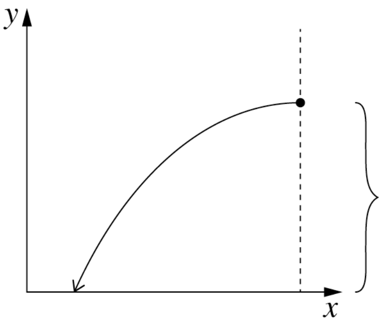
$$= 20 + \frac{90\not{0} \times \frac{1}{4}}{2\not{0}}$$

$$= \frac{125}{4}$$

Question 13 (b) (iii)

Criteria	Marks
• Provides correct solution	2
• Obtains correct equation for y in terms of t, or equivalent merit	1

Sample answer:



Taking left (←) to be positive and origin as start point.

$\dot{x} = 10$ $\qquad \ddot{y} = -10$

$x = 10t$ $\qquad \dot{y} = -10t$ (since $\dot{y}(0) = 0$)

(since $x(0) = 0$) $\quad y = -5t^2$ (since $y(0) = 0$)

$$y = -31.25 \Rightarrow \quad -31.25 = -5t^2$$

$$t^2 = 6.25$$

$$t = \pm 2.5, \quad t > 0$$

$$t = 2.5$$

$\therefore$ Ball hits the ground after 2.5 seconds.

Question 13 (b) (iv)

Criteria	Marks
• Provides correct answer	1

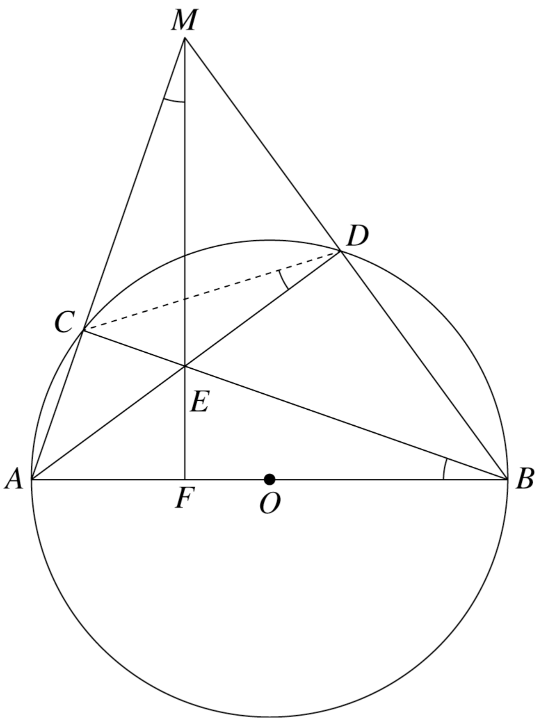
Sample answer:

$t = 2.5 \quad \Rightarrow \quad x = 10(2.5)$

$\qquad\qquad\qquad\; = 25$

$\therefore$ The ball lands 25 m from the base of the wall.

Question 13 (c)



Question 13 (c) (i)

Criteria	Marks
• Provides correct solution	2
• Shows that one of the angles at *C* or *D* is a right angle, or equivalent merit	1

Sample answer:

$\angle ADB = 90°, \quad \angle BCA = 90°$ (∠ in a semicircle)

$\angle MDE = 90°$, (∠ sum, straight line)

$\therefore$ *CMDE* is a cyclic quadrilateral (Exterior $\angle ACB$ equal to opposite interior $\angle MDE$)

Question 13 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Shows $\angle ABC = \angle ADC$, or equivalent merit	1

Sample answer:

Method 1

Construct CD

Let $\angle ABC = x°$

$\therefore\ \angle ADC = x°$ (∠s in the same segment)

Similarly $\angle CME = x°$

$\angle CAB = 90° - x°$ (∠ sum $\triangle ABC$)

$\therefore\ \angle MFA = 180° - \angle CME - \angle CAB$ (∠ sum $\triangle MFA$)

$= 180° - (x°) - (90° - x°)$

$= 90°$

$\therefore\ MF \perp AB$

Method 2

Construct CD as shown in the diagram

$\angle ABC = \angle ADC$ (∠s in the same segment)

$\angle ADC = \angle EMC$ (∠s in the same segment as $CDME$ are concyclic points)

$\triangle CEM\ |||\ \triangle FEB$ (equiangular)

$\therefore\ \angle MCE = \angle BFE = 90°$ (corresponding ∠s)

Question 14 (a) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

R.T.P. $4n^3 + 18n^2 + 23n + 9$

$= (n+1)(4n^2 + 14n + 9)$

$$\begin{aligned}\text{RHS} &= n(4n^2 + 14n + 9) + 1(4n^2 + 14n + 9)\\ &= 4n^3 + 14n^2 + 9n + 4n^2 + 14n + 9\\ &= 4n^3 + 18n^2 + 23n + 9\\ &= \text{LHS}\end{aligned}$$

<u>OR</u>

$$\begin{aligned}\text{LHS} &= 4n^3 + 4n^2 + 14n^2 + 14n + 9n + 9\\ &= 4n^2(n+1) + 14n(n+1) + 9(n+1)\\ &= (n+1)(4n^2 + 14n + 9)\\ &= \text{RHS}\end{aligned}$$

Question 14 (a) (ii)

Criteria	Marks
• Provides correct proof	3
• Establishes induction step, or equivalent merit	2
• Establishes initial case, or equivalent merit	1

Sample answer:

Let $T_n = (2n-1)(2n+1)$, $S_n = \frac{1}{3}n\left(4n^2+6n-1\right)$

R.T.P. $1\times 3 + 3\times 5 + 5\times 7 + \cdots + (2n-1)(2n+1) = \frac{1}{3}n\left(4n^2+6n-1\right)$

When $n = 1$, LHS $= 1\times 3$ $= 3$ RHS $= \frac{1}{3}(1)\left(4(1)^2+6(1)-1\right)$ $= \frac{1}{3}\times 9$ $= 3$

$\therefore$ True for $n=1$

Assume true for some integer k, ie $S_k = \frac{1}{3}k\left(4k^2+6k-1\right)$

and attempt to show it is also true for $k+1$

ie, try to show $S_{k+1} = \frac{1}{3}\left(k+1\right)\left(4(k+1)^2+6(k+1)-1\right)$

$$= \frac{1}{3}\left(k+1\right)\left(4k^2+8k+4+6k+6-1\right)$$

$$= \frac{1}{3}\left(k+1\right)\left(4k^2+14k+9\right)$$

Now $S_{k+1} = S_k + T_{k+1}$

$$= \frac{1}{3}k\left(4k^2+6k-1\right) + \left(2(k+1)-1\right)\left(2(k+1)+1\right)$$

$$= \frac{1}{3}k\left(4k^2+6k-1\right) + \left(2k+1\right)\left(2k+3\right)$$

$$= \frac{1}{3}\left(4k^3+6k^2-k\right) + 4k^2+8k+3$$

$$= \frac{1}{3}\left(4k^3+6k^2-k+12k^2+24k+9\right)$$

$$= \frac{1}{3}\left(4k^3+18k^2+23k+9\right)$$

$$= \frac{1}{3}\left(k+1\right)\left(4k^2+14k+9\right) \qquad \text{(From part (i))}$$

as required

$\therefore$ Hence the result is true for all positive integers n, by induction.

Question 14 (b) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n$$

Let $x = 1$ then

$$(1+1)^n = \binom{n}{0} + \binom{n}{1}(1) + \binom{n}{2}(1)^2 + \cdots + \binom{n}{n}(1)^n$$

$$\therefore \quad 2^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \cdots + \binom{n}{n}$$

Question 14 (b) (ii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

Differentiate $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n$, with respect to x.

$$n(1+x)^{n-1} = \binom{n}{1} + \binom{n}{2}(2x) + \binom{n}{3}(3x^2) + \cdots + \binom{n}{n}nx^{n-1}$$

and substitute $n = 1$ into this equation

$$n(1+1)^{n-1} = \binom{n}{1} + \binom{n}{2}(2(1)) + \binom{n}{3}(3(1)^2) + \cdots + \binom{n}{n}(n(1)^{n-1})$$

$$\therefore \; n \cdot 2^{n-1} = \binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \cdots + n\binom{n}{n}$$

Question 14 (b) (iii)

Criteria	Marks
• Provides correct solution	2
• Attempts to write the right-hand side in terms of the previous two parts, or equivalent merit	1

Sample answer:

Method 1

$$\sum_{r=1}^{n}\binom{n}{r}(2r-n) = \binom{n}{1}(2-n)+\binom{n}{2}(4-n)+\binom{n}{3}(6-n)+\cdots+\binom{n}{n}(2n-n)$$

$$= 2\binom{n}{1}+4\binom{n}{2}+6\binom{n}{3}+\cdots+2n\binom{n}{n}-n\left[\binom{n}{1}+\binom{n}{2}+\binom{n}{3}+\cdots+\binom{n}{n}\right]$$

$$= 2\left\{\binom{n}{1}+2\binom{n}{2}+3\binom{n}{3}+n\binom{n}{n}-n\left[\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\cdots+\binom{n}{n}\right]-\binom{n}{0}\right\}$$

$$= 2\left(n\cdot 2^{n-1}\right)-n\left(2^n-1\right)$$

$$= n2^n - n2^n + n$$

$$= n$$

Method 2

$$\sum_{r=1}^{n}\binom{n}{r}(2r-n)$$

$$= 2\sum_{r=1}^{n}\binom{n}{r}r - n\sum_{r=1}^{n}\binom{n}{r}$$

$$= 2\sum_{r=0}^{n}\binom{n}{r}r - n\left(\sum_{r=0}^{n}\binom{n}{r}-1\right)$$

$$= 2.n.2^{n-1} - n\left(2^n-1\right)$$

$$= n$$

Question 14 (c) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

From reference sheet, equation of tangent is $y = tx - at^2$

At D on the directrix, $y = -a$

$$\Rightarrow \quad -a = tx - at^2$$

$$tx = at^2 - a$$

$$x = at - \frac{a}{t}$$

$\therefore$ D is $\left(at - \frac{a}{t}, -a\right)$

Question 14 (c) (ii)

Criteria	Marks
• Provides correct solution	3
• Attempts to eliminate the parameter to find the Cartesian equation of $\mathcal{P}_2$, or equivalent merit	2
• Finds coordinates of R, or equivalent merit	1

Sample answer:

From reference sheet, equation of normal is $x + ty = at^3 + 2at$

At R (above D), $x = at - \dfrac{a}{t}$

$$\Rightarrow \quad at - \frac{a}{t} + ty = at^3 + 2at$$

$$ty = at^3 + at + \frac{a}{t}$$

$$y = at^2 + a + \frac{a}{t^2}$$

$$= a\left(t^2 + 1 + \frac{1}{t^2}\right)$$

$$\therefore R \text{ is } \left(a\left(t - \frac{1}{t}\right),\ a\left(t^2 + 1 + \frac{1}{t^2}\right)\right)$$

Method 1

$$x = a\left(t - \frac{1}{t}\right)$$

$$x^2 = a^2\left(t^2 - 2 + \frac{1}{t^2}\right)$$

$$\frac{x^2}{a^2} = t^2 + \frac{1}{t^2} - 2$$

$$\frac{x^2}{a^2} + 2 = t^2 + \frac{1}{t^2}$$

$$y = a\left(\left(t^2 + \frac{1}{t^2}\right) + 1\right)$$

$$= a\left(\frac{x^2}{a^2} + 2 + 1\right)$$

$$y = \frac{x^2}{a} + 3a$$

$$ay = x^2 + 3a^2$$

Method 2

$$\frac{x}{a} = t - \frac{1}{t}$$

$$y = a\left(t^2 + 1 + \frac{1}{t^2}\right)$$

$$= a\left(\left(t - \frac{1}{t}\right)^2 + 3\right)$$

$$= a\left(\frac{x^2}{a^2} + 3\right) = \frac{x^2}{a} + 3a$$

$$\therefore (y - 3a)a = x^2$$

Locus is $x^2 = a(y - 3a)$.

A parabola! Vertex $(0, 3a)$

Question 14 (c) (iii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$x^2 = a(y - 3a)$$

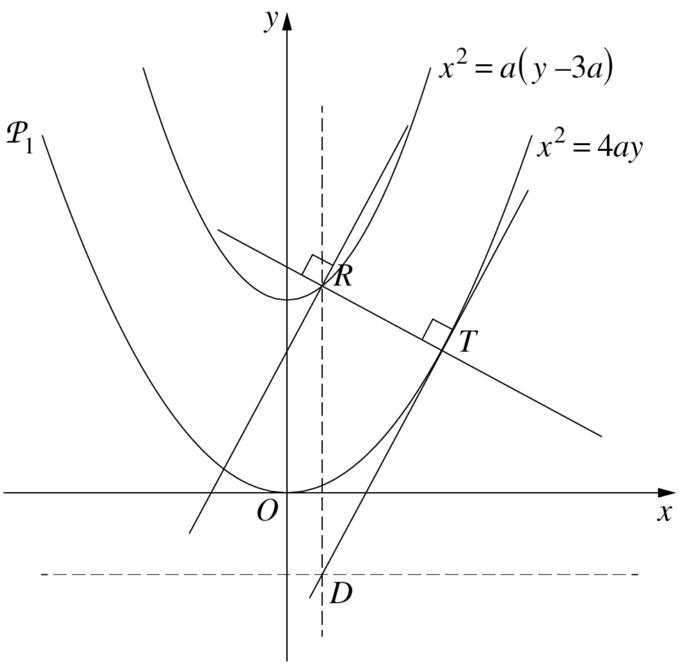
ie $x^2 = 4\left(\frac{a}{4}\right)(y - 3a)$

$\therefore$ Focal length $\frac{a}{4}$

Question 14 (c) (iv)

Criteria	Marks
• Provides correct solution	2
• Finds slope of $\mathcal{P}_2$ at R, or equivalent merit	1

Sample answer:



$T(2at,\ at^2)$

$R\left(at - \frac{a}{t},\ a\left(t^2 + 1 + \frac{1}{t^2}\right)\right)$

A common normal will occur when the tangents at T and R are parallel.

Now gradient at T is t

For R, $\quad \frac{x^2}{a} = y - 3a$

Question 14 (c) (iv) continued

$$y = \frac{x^2}{a} + 3a$$

$$y' = \frac{2x}{a}$$

So $\quad y' = \dfrac{2\left(at - \dfrac{a}{t}\right)}{a}$, since $x = at - \dfrac{a}{t}$,

$$= 2\left(t - \frac{1}{t}\right)$$

So gradient at R is $2t - \dfrac{2}{t}$

For tangents to be parallel, $t = 2t - \dfrac{2}{t}$

$$t^2 = 2t^2 - 2$$

$$2 = t^2$$

$$t = \pm\sqrt{2}$$

2016 HSC Mathematics Extension 1 Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	7.1	H5, HE7
2	1	16.2E	PE3
3	1	5.7E	P3, PE6
4	1	2.10E, 2.9E	PE3
5	1	13.6E	HE6
6	1	15.2E, 5.9E	HE4
7	1	14.3, 14.4E	HE5
8	1	18.1E	PE3
9	1	10.1, 10.4	H6
10	1	16.1 E, 16.3E	PE3

Section II

Question	Marks	Content	Syllabus outcomes
11 (a)	2	15.1E	HE4
11 (b)	3	11.5E	HE6
11 (c)	2	15.5E	HE4
11 (d)	2	5.7E, 13.4E	HE7
11 (e)	3	1.4E	PE3
11 (f) (i)	1	3.2, 18.2E	HE3
11 (f) (ii)	2	18.2E	HE3
12 (a) (i)	1	2.3	P2, PE6
12 (a) (ii)	1	14.1E	HE5
12 (a) (iii)	2	14.1E	HE5
12 (a) (iv)	2	14.1E	HE5
12 (b) (i)	3	14.2E	HE3
12 (b) (ii)	2	14.2E	HE3
12 (c) (i)	2	6.2, 10.7, 13.5	H5, HE4
12 (c) (ii)	2	16.4E	HE7
13 (a) (i)	2	14.4E	HE3
13 (a) (ii)	2	14.4E	HE3

Question	Marks	Content	Syllabus outcomes
13 (b) (i)	2	14.3E	HE3
13 (b) (ii)	2	14.3E	HE3
13 (b) (iii)	2	14.3E	HE3
13 (b) (iv)	1	14.3E	HE3
13 (c) (i)	2	2.10E, 2.7E	PE3
13 (c) (ii)	2	2.10E, 2.8E	PE3
14 (a) (i)	1	1.3	P4, PE6
14 (a) (ii)	3	7.4E	HE2
14 (b) (i)	1	17.1E, 17.3E	HE7
14 (b) (ii)	1	8.7, 17.1E, 17.3E	HE4, HE7
14 (b) (iii)	2	17.1E, 17.3E	HE7
14 (c) (i)	1	9.6E	PE3
14 (c) (ii)	3	9.6E	PE3
14 (c) (iii)	1	9.6E	PE3
14 (c) (iv)	2	10.7, 9.6E	HE7