# 2011 Specialist Mathematics GA 2: Written examination 1

## GENERAL COMMENTS

The number of students who sat for the 2011 Specialist Maths examination 1 was 3984. Students were required to answer ten short answer questions worth a total of 40 marks and were not allowed to bring any calculators or notes into the examination. In the comments on specific questions in the next section, many common errors are highlighted. These should be brought to the attention of students so that they can develop strategies to avoid them. A particular concern is the need for students to read the questions carefully as responses to several questions indicated that students had not done this.

Another concern that has emerged is the manner in which some students set out their mathematical working. Students are expected to set out their work properly. Students should be reminded that if an assessor is not certain as to what they are attempting to convey, marks cannot be awarded. If an assessor is unable to follow a student's working (or reasoning) full marks cannot be awarded. Equals signs should be placed between quantities that are equal; the working should not appear to be a number of unrelated statements. If there are inconsistencies in the student's working, full marks will not be awarded. For example, if an equals sign is placed between quantities that are not equal, full marks will not be awarded.

Areas of weakness included:

- not reading the question carefully enough. This included not answering the question, proceeding further than required or not giving the answer in the specified form. The latter was common and particularly evident in Questions 1, 2, 3c., 4, 6, 8, 9cii. and 11. In 9cii., many students ignored the word 'hence' and did not use the prescribed method. Students should be reminded that good examination technique includes rereading the question after it has been answered to ensure they have answered what was required and that they have given their answer in the correct form
- algebraic skills. Difficulty with algebra was evident in several questions. The inability to simplify expressions often prevented students from completing a question. Incorrect attempts to factorise, expand and simplify, as well as poor use of brackets, were common.
- showing a given result. This was required in Questions 3a. and 7a. In such questions, the students need to include sufficient relevant working to demonstrate that they know how to derive the result. Students should be reminded that they can use a given value in the remaining part(s) of the question whether they were able to derive it or not
- recognising the need to use the chain rule when differentiating implicitly (Questions 5 and 10)
- recognising the need to use the product rule when differentiating implicitly (Question 10)
- recognising the need to use the chain rule when differentiating (Question 5)
- recognising the need to use the product rule when differentiating (Question 2)
- recognising the method of integration required (Questions 1, 3c., 6 and 11)
- changing the terminals when integrating using substitution (Question 6)
- knowing the exact values for circular functions (Questions 4, 5, 7, 8 and 11)
- consideration of the quadrant for values of circular functions (Questions 4 and 8)
- giving answers in the required form (Questions 2 and 8)
- giving a positive and a negative solution to equations such as $x^2 = 9$ (Questions 8 and 9a.).

Students are expected to be able to apply techniques, routines and processes, involving rational, real and complex arithmetic, without the use of technology and should be able to readily simplify simple arithmetic expressions. Many students found this difficult and missed out on marks as a consequence.

Students need to be reminded that the instruction to 'sketch' (Question 3b.) does not mean that a rough and careless attempt is acceptable. Students should sketch with care and include details such as a reasonable scale, domain, asymptotes and asymptotic behaviour. A smoothly drawn curve is expected.

Many students made algebraic slips at the end of an answer so that the final mark could not be awarded. This was especially unfortunate when they had a correct answer and there was no need for further simplification.

There were several cases where incorrect working fortuitously led to a correct answer. Students should be reminded that in such cases the final answer needs to be supported by relevant and correct working.

# SPECIFIC INFORMATION

**Question 1**

| Marks | 0 | 1 | 2 | 3 | Average |
|---|---|---|---|---|---|
| % | 21 | 5 | 34 | 39 | **1.9** |

$-\frac{2}{3}\log_e|3-x|-\frac{1}{3}\log_e|3+x|$ or $\frac{1}{6}\log_e\left|\frac{3+x}{3-x}\right|-\frac{1}{2}\log_e|9-x^2|$ (other equivalent answers were accepted)

This question was quite well done. Most students recognised that partial fractions were required. Some changed the entire fraction into partial fractions immediately, while others split the original fraction first, using substitution on part of it. The latter method led to the correct answer but was inefficient as partial fractions were still required. Typical errors seen included using denominators of $9-x$ and $9+x$ in the partial fractions, integrating $\frac{1}{3-x}$ to give $\ln|3-x|$ (missing the negative sign) and omitting modulus signs. Some students first changed the denominator to $x^2-9$ and many of these subsequently made sign errors. A few students justified removing modulus signs due to $x \in R\setminus\{-3, 3\}$. There was often an incorrect attempt at simplification of the two terms in the answer at the end (and missing out on the final mark); for example $-\frac{2}{3}\ln|3-x|-\frac{1}{3}\ln|3+x|=-\left(\frac{2}{3}\ln|3-x|-\frac{1}{3}\ln|3+x|\right)$. A small number of students produced answers involving $\arctan(x)$.

**Question 2**

| Marks | 0 | 1 | 2 | 3 | Average |
|---|---|---|---|---|---|
| % | 32 | 5 | 15 | 49 | **1.8** |

Students had to show the given result $k=3$.

This question was very well done by students who used the product rule for differentiation; however, a significant number of students did not. There were many algebraic errors. Some students used $\frac{dy}{dx}=kxe^{kx}$, others attempted to integrate the differential equation twice, while a few used $y=e^{2x}(15x+6)$. A few students thought that $\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$.

A number of students failed to substitute *y* despite being able to substitute the derivatives. Some were unable to simplify $k=\frac{15x+6}{5x+2}$.

**Question 3a.**

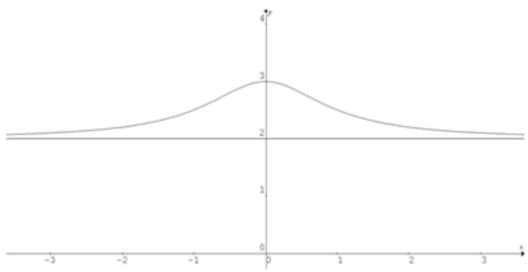
| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 9 | 91 | **0.9** |

Students had to show the given result $\frac{2x^2+3}{x^2+1}=2+\frac{1}{x^2+1}$.

This question was generally very well done. However, as often happens in a 'show that' question, some students were unable to do the relevant algebra yet somehow still managed to give the result stated.

**Question 3b.**

| Marks | 0 | 1 | 2 | 3 | Average |
|---|---|---|---|---|---|
| % | 26 | 7 | 18 | 49 | **1.9** |



Students were required to label the asymptote $y = 2$ and the turning point (0, 3).

This question was well done by many students, but there were some strange graphs. A large number of students seemed to interpret the denominator as $x + 1$ or as $(x + 1)(x - 1)$ and consequently had one or two vertical asymptotes. A few students drew the graph of $y = \frac{3}{x^2+1}$. Among those who had the correct idea, typical errors included omitting the label of the intercept or the asymptote, showing the maximum point as a cusp rather than as a turning point and not showing asymptotic behaviour.

**Question 3c.**

| Marks | 0 | 1 | 2 | 3 | Average |
|---|---|---|---|---|---|
| % | 9 | 18 | 18 | 55 | **2.2** |

$$4 + \frac{\pi}{2}$$

Most students handled this question very well, realising that they needed to express the fraction in two parts. Some students used symmetry successfully. Common errors included integrating the fraction to give $\ln(x^2+1)$, sign problems (often caused by lack of brackets) in the evaluation to give either 4 or $\frac{\pi}{2}$ as the answer and the incorrect evaluation of $\tan^{-1}(-1)$ as $\frac{3\pi}{4}$ (ignoring the quadrant). Some found the area between the graph and the asymptote.

**Question 4**

| Marks | 0 | 1 | 2 | 3 | Average |
|---|---|---|---|---|---|
| % | 18 | 20 | 38 | 23 | **1.7** |

$$\text{Arg}(z) = \frac{11\pi}{12}$$

This question was done reasonably well. The majority of students attempted to answer the question in cartesian form (using conjugates) rather than converting to polar form, leaving them with little chance of finding the argument even where they were successful in correctly obtaining $z$ in the form $a + ib$. Some did manage to achieve this either by using a double angle formula or by recognising the value. Many students who used polar form from the beginning made errors in finding arguments for the two complex numbers in the question, often ignoring the quadrant. Of those students who correctly got $z = \sqrt{2}\,\text{cis}\left(-\frac{13\pi}{12}\right)$, some left this as their answer or gave the principal argument as $-\frac{13\pi}{12}$ or occasionally as $-\frac{\pi}{12}$. Many arithmetic errors were seen, often involving signs.

**Question 5**

| Marks | 0 | 1 | 2 | 3 | Average |
|---|---|---|---|---|---|
| % | 21 | 20 | 7 | 51 | **1.9** |

$-\frac{1}{\sqrt{12}} = -\frac{1}{2\sqrt{3}} = -\frac{\sqrt{3}}{6}$

There were several possible approaches to this question. It was generally well done by students who tackled it parametrically. Most students attempted to eliminate the parameter, with varying degrees of success. Those who used implicit differentiation often experienced some success. Students who attempted to express *y* in terms of *x* explicitly were confronted with a more difficult differentiation process. A small percentage of students who used this approach were able to get the correct result. Some put *t* in terms of *x* and then substituted this in to the expression for *y*. This led to a very difficult differentiation, which rarely led to a successful outcome. Typical errors included finding an incorrect value for *t* from the given information, algebraic mistakes by those who attempted to rearrange the equations, including difficulties with expanding brackets and negative sign errors, problems with differentiating implicitly and explicitly, and incorrect simplification of the final answer (missing out on the final mark that would otherwise have been gained); for example, $-\frac{1}{2\sqrt{3}} = -\frac{\sqrt{3}}{4}$.

**Question 6**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 24 | 29 | 47 | **1.3** |

$\sin(e) - \sin(1)$

Most students recognised that an appropriate substitution was $u = e^x$. Those who did not were generally unsuccessful. A few students recognised that $\sin(e^x)$ was likely to have been involved and either by a direct approach or by explicitly differentiating $\sin(e^x)$ were able to get to the answer without the substitution step. Other students tried to use $u = \cos(e^x)$. Many who recognised the appropriate substitution still made errors with the terminals. Many students changed the variable correctly but left the terminals unchanged. It should be emphasised that this is not logically correct, even if changing back to the original variable later enables a correct answer. Typical errors included trying to integrate the two terms in the expression independently (sometimes finding the product of these), giving the integral of cos(*u*) as –sin(*u*) or $\frac{1}{u}\sin(u)$ and 'simplifying' sin(1) as $\frac{\pi}{2}$ or occasionally as 0 and $e^0$ as 0.

**Question 7a.**

| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 38 | 62 | **0.6** |

Students had to show the given result $T_2 = \frac{T_1}{\sqrt{3}}$.

As often happens in a 'show that' question, a number of students were unable to do appropriate working but still managed to 'show' the result. Some of the errors seen were $T_1 = T_2\tan(30^\circ)$, $\frac{T_1}{\sin(30^\circ)} = \frac{T_2}{\sin(60^\circ)}$, $T_1\cos(30^\circ) = mg$, $T_2\cos(60^\circ) = mg$ and $T_1 = \sin(30^\circ)$, $T_2 = \sin(60^\circ)$ (yet in each case still 'finding' that $T_2 = \frac{T_1}{\sqrt{3}}$). In many cases students confused or reversed $30^\circ$ and $60^\circ$ or $T_1$ and $T_2$. Many students did not know the exact values.

**Question 7b.**

| Marks | 0 | 1 | 2 | 3 | Average |
|---|---|---|---|---|---|
| % | 37 | 9 | 13 | 41 | **1.6** |

$m = \frac{196}{g} = 20$ (other equivalent answers were accepted)

Many approaches to this question were seen. Most successful students resolved vertically and used the previous information to lead to the correct answer. A small number of students, realising that the angle between the ropes was a right angle, resolved in the direction of rope 2 to get the answer almost immediately without the need to use that information. Some students correctly applied Lami's Theorem or a triangle of forces. Several students confused or reversed $30^\circ$ and $60^\circ$ or $T_1$ and $T_2$ as well as mixing up lengths with forces. Many students did not know the exact values. Other common errors included writing $T_1 + T_2 = mg$ and omitting $g$ from force equations. A large number of algebraic errors were seen.

**Question 8**

| Marks | 0 | 1 | 2 | 3 | Average |
|---|---|---|---|---|---|
| % | 29 | 30 | 13 | 27 | **1.4** |

$\left(2, \frac{4}{3}\right), \left(4, \frac{4}{3}\right), \left(8, \frac{4}{3}\right), \left(10, \frac{4}{3}\right)$

Many of the attempts to solve this question were disappointing, given that the techniques used draw on assumed knowledge and skills which would have been developed in Mathematical Methods (CAS). A relatively small percentage of students were able to find the four sets of coordinates. Some students tried various trigonometric identities, occasionally eventually arriving at the correct answer. Many used $\operatorname{cosec}(x) = \frac{1}{\cos(x)}$. A large number of students stated that $\sin^2\left(\frac{\pi x}{6}\right) = \frac{3}{4}$ gives $\sin\left(\frac{\pi x}{6}\right) = \frac{\sqrt{3}}{2}$ (forgetting or explicitly rejecting the negative square root). Other common errors included not knowing the exact values, making various algebraic slips and giving the answers as 2, 4, 8, 10 rather than coordinates, despite this instruction being in bold. Some students gave incorrect $y$ values, despite these values being given.

**Question 9a.**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 5 | 18 | 77 | **1.7** |

$\pm\sqrt{7}$

This question was very well done. Most students were able to reach $m^2 = 7$, but many then gave the answer as $m = \sqrt{7}$ (omitting the negative possibility) or as $m = \pm 7$. Some students simplified $\sqrt{5+m^2}$ to get $\sqrt{5}+m$.

**Question 9b.**

| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 18 | 82 | **0.8** |

$\frac{1}{2}$

Most students were able to complete this question successfully. Sign errors were the main source of problems.

**Question 9ci–ii.**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 11 | 48 | 41 | **1.3** |

$2\underset{\sim}{i}+4\underset{\sim}{j}-5\underset{\sim}{k}, \quad -\frac{5}{2}$

Part i. was generally correctly done, with the bulk of issues being due to sign errors. A reasonable proportion of students were able to link this to part ii. to successfully find *m*. Since this part was a 'hence' question, students who were unable to deduce the value of *m* using their previous answer could not be awarded this mark. Several attempted to do this part from scratch.

**Question 10**

| Marks | 0 | 1 | 2 | 3 | 4 | Average |
|---|---|---|---|---|---|---|
| % | 14 | 4 | 5 | 13 | 64 | **3.1** |

$-\frac{3}{2}$

This question was very well done, suggesting that many students have had a lot of practice at implicit differentiation. Typical errors included not applying the product rule on the left side of the equation, not applying the chain rule on the right side of the equation, making sign errors when transposing and making other algebraic slips. Some students isolated *y* and then attempted to use the quotient rule. This pathway tended to be messy and unsuccessful. A number of students could not simplify $e^0$ or $\log_e 1$, or thought that $e^0 = 0$ or $2e^0 = 1$.

**Question 11**

| Marks | 0 | 1 | 2 | 3 | Average |
|---|---|---|---|---|---|
| % | 21 | 19 | 14 | 46 | **1.9** |

$\frac{\pi^2}{12}-\frac{\pi\sqrt{3}}{8}$ (other equivalent answers were accepted)

Most students were able to write a correct expression for the required volume and many were able to complete the question successfully. When setting up the integral, some left out $\pi$ and some put sin(*x*) rather than $\sin^2(x)$.
Other errors included not transposing or applying the double angle formula correctly, giving the integral of cos(2*x*) as 2sin(2*x*), integrating $\sin^2(x)$ to get $\cos^2(x)$ or $\frac{1}{3}\sin^3(x)$ and incorrect evaluation of the upper terminal. Several students simplified their final answer poorly at the end; for example, $\pi\left(\frac{\pi}{12}-\frac{\sqrt{3}}{8}\right)=\frac{\pi}{12}-\frac{\pi\sqrt{3}}{8}$. Other examples involved erroneously attempting to put the answer on a common denominator or to factorise it. Exact values were again a problem. Some students found the associated area.