Centre Number

Student Number

NSW GOVERNMENT

NSW Education Standards Authority

2025 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 2

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page and on the Question 11 Writing Booklet attached

Total marks: 100

Section I – 10 marks (pages 2–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7–14)

- Attempt Questions 11–16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 Points A and B are $(-3, 1)$ and $(1, 4)$ respectively.

Which of the following is a vector equation of the line AB with parameter λ?

A. $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

B. $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

C. $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

D. $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

2 Consider the statement:

$$\exists\, x \in \mathbb{Z}, \text{ such that } x^2 \text{ is odd.}$$

Which of the following is the negation of the statement?

A. $\forall\, x \in \mathbb{Z}, x^2$ is odd

B. $\forall\, x \in \mathbb{Z}, x^2$ is even

C. x^2 is even $\Rightarrow x \in \mathbb{Z}$

D. $\exists\, x \in \mathbb{Z}$, such that x^2 is even

3 What are the square roots of $3 - 4i$?

A. $1 - 2i$ and $-1 + 2i$

B. $1 + 2i$ and $-1 - 2i$

C. $2 - i$ and $-2 + i$

D. $-2 - i$ and $2 + i$

4 A particle in simple harmonic motion has speed v m s^{-1}, given by $v^2 = -x^2 + 2x + 8$ where x is the displacement from the origin in metres.

What is the amplitude of the motion?

A. 1 m

B. 3 m

C. 6 m

D. 9 m

5 Consider the statement:

$$\text{If } x^2 - 2x \geq 0, \text{ then } x \leq 0.$$

Which of the following is the contrapositive of the statement?

A. If $x > 0$, then $x^2 - 2x < 0$.

B. If $x \leq 0$, then $x^2 - 2x \geq 0$.

C. If $x^2 - 2x < 0$, then $x < 0$.

D. If $x^2 - 2x \leq 0$, then $x > 0$.

6 The complex numbers z and w lie on the unit circle. The modulus of $z + w$ is $\frac{3}{2}$.

What is the modulus of $z - w$?

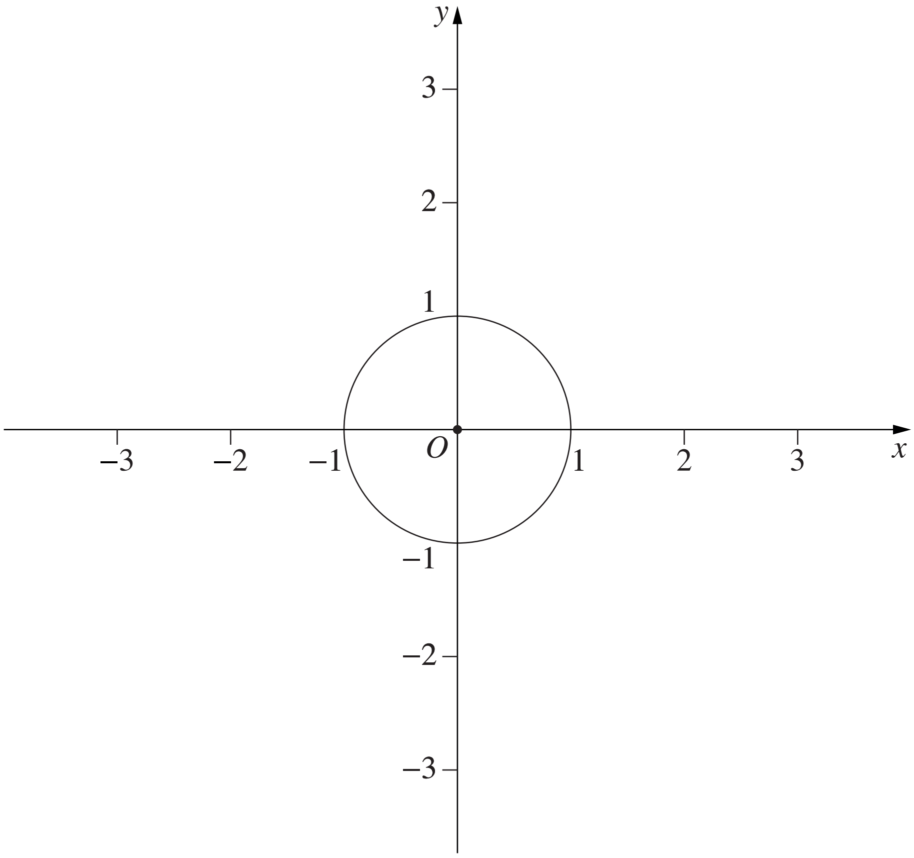
A. $\frac{1}{8}$

B. $\frac{\sqrt{7}}{2}$

C. $\frac{3}{2}$

D. $\frac{7}{4}$

7 The complex number z lies on the unit circle.



What is the range of $\text{Arg}(z - 2i)$?

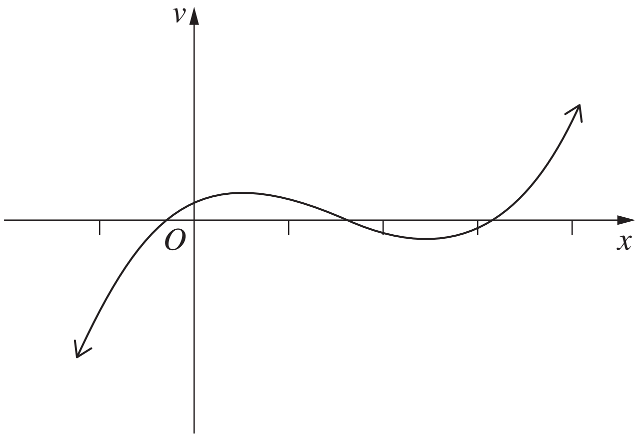
A. $\frac{\pi}{6} \leq \text{Arg}(z - 2i) \leq \frac{5\pi}{6}$

B. $\frac{\pi}{3} \leq \text{Arg}(z - 2i) \leq \frac{2\pi}{3}$

C. $-\frac{5\pi}{6} \leq \text{Arg}(z - 2i) \leq -\frac{\pi}{6}$

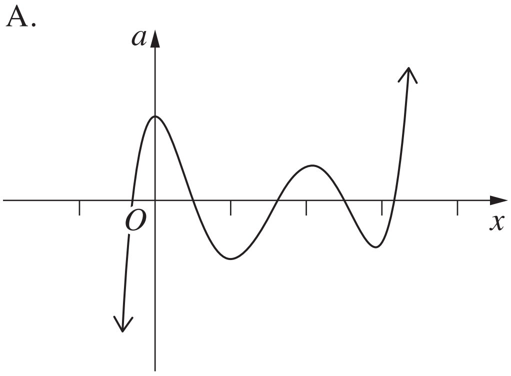
D. $-\frac{2\pi}{3} \leq \text{Arg}(z - 2i) \leq -\frac{\pi}{3}$

8 The graph shows the velocity of a particle as a function of its displacement.

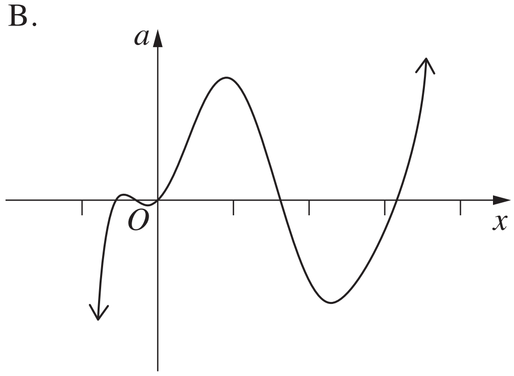


Which of the following graphs best shows the acceleration of the particle as a function of its displacement?

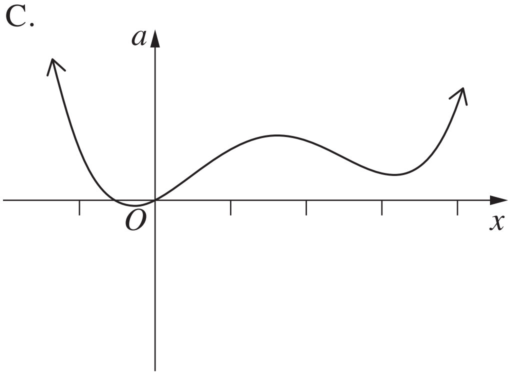
A.



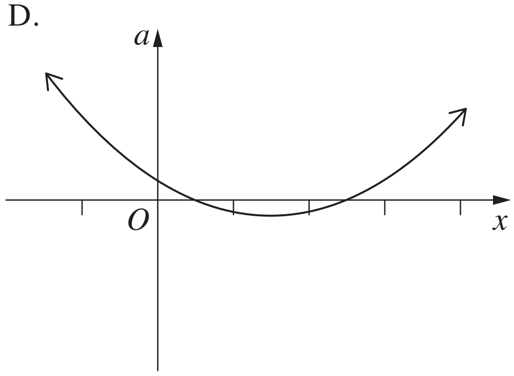
B.



C.



D.



9 The points U, V, W and Z represent the complex numbers u, v, w and z respectively. It is given that $v + z = u + w$ and $u + kiz = w + kiv$ where $k \in \mathbb{R}$, $k > 1$.

Which quadrilateral best describes $UVWZ$?

A. Parallelogram

B. Rectangle

C. Rhombus

D. Square

10 Which of the following gives the same curve as $\begin{pmatrix} \cos(t) \\ -t \\ \sin(t) \end{pmatrix}$ for $t \in \mathbb{R}$?

A. $\begin{pmatrix} \cos(2t) \\ 2t \\ \sin(2t) \end{pmatrix}$

B. $\begin{pmatrix} \cos\left(t^2 + \frac{\pi}{2}\right) \\ t^2 + \frac{\pi}{2} \\ \sin\left(t^2 + \frac{\pi}{2}\right) \end{pmatrix}$

C. $\begin{pmatrix} \cos\left(t^2\right) \\ -t^2 \\ \sin\left(t^2\right) \end{pmatrix}$

D. $\begin{pmatrix} \cos\left(2t + \frac{\pi}{2}\right) \\ 2t + \frac{\pi}{2} \\ -\sin\left(2t + \frac{\pi}{2}\right) \end{pmatrix}$

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (14 marks) Use the Question 11 Writing Booklet

(a) The location of the complex number z is shown on the diagram on page 1 of the Question 11 Writing Booklet. **2**

On the diagram provided in the writing booklet, indicate the locations of $\overline{z}$ and $i\overline{z}$.

(b) The complex numbers w and z are given by $w = 2e^{\frac{i\pi}{6}}$ and $z = 3e^{\frac{i\pi}{6}}$. **2**

Find the modulus and argument of wz.

(c) The complex number z is given by $x + iy$.

Find, in Cartesian form:

(i) z^2 **1**

(ii) $\frac{1}{z}$. **2**

Question 11 continues on page 8

(d) (i) Force $\underset{\sim}{F}_1$ has magnitude 12 newtons in the direction of vector $2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$. **1**

Show that $\underset{\sim}{F}_1 = 8\underset{\sim}{i} - 8\underset{\sim}{j} + 4\underset{\sim}{k}$.

(ii) Force $\underset{\sim}{F}_1$ from part (i) and a second force, $\underset{\sim}{F}_2 = -6\underset{\sim}{i} + 12\underset{\sim}{j} + 4\underset{\sim}{k}$, both act upon a particle. **1**

Show that the resultant force acting on the particle is given by:

$$\underset{\sim}{F}_3 = 2\underset{\sim}{i} + 4\underset{\sim}{j} + 8\underset{\sim}{k}.$$

(iii) Calculate $\underset{\sim}{F}_3 \cdot \underset{\sim}{d}$, where $\underset{\sim}{F}_3$ is the resultant force from part (ii) and $\underset{\sim}{d} = \underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}$. **1**

(e) Prove by contradiction that $\sqrt{3} + \sqrt{5} > \sqrt{11}$. **2**

(f) Find $\displaystyle\int \frac{5}{\sqrt{7 - x^2 - 6x}}\,dx$. **2**

End of Question 11

Question 12 (16 marks) Use the Question 12 Writing Booklet

(a) Using integration by parts, evaluate $\displaystyle\int_0^{\frac{\pi}{2}} x \sin x \, dx$. **3**

(b) Given the function $y = xe^{2x}$, use mathematical induction to prove that $\dfrac{d^n y}{dx^n} = \left(2^n x + n2^{n-1}\right)e^{2x}$ for all positive integers n, where $\dfrac{d^n y}{dx^n}$ is the nth derivative of y and $\dfrac{d}{dx}\left(\dfrac{d^n y}{dx^n}\right) = \dfrac{d^{n+1} y}{dx^{n+1}}$. **3**

(c) Sketch the region of the complex plane defined by $|z + 5 - i| > |z - 3 + 3i|$. **3**

(d) Find $\displaystyle\int \frac{x^2 - 2x + 9}{(4 - x)\left(x^2 + 1\right)} \, dx$. **4**

(e) A particle of mass m kg moves along a horizontal line with an initial velocity of V_0 m s^{-1}. **3**

The motion of the particle is resisted by a constant force of mk newtons and a variable force of mv^2 newtons, where k is a positive constant and v m s^{-1} is the velocity of the particle at t seconds.

Show that the distance travelled when the particle is brought to rest is $\dfrac{1}{2}\ln\left(\dfrac{k + V_0^{\,2}}{k}\right)$ metres.

Question 13 (15 marks) Use the Question 13 Writing Booklet

(a) It is given that $A = \int_2^4 \frac{e^x}{x-1}\,dx$. **3**

Show that $\int_{m-4}^{m-2} \frac{e^{-x}}{x-m+1}\,dx = kA$, where k and m are constants.

(b) Let $\underset{\sim}{c} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$ be a unit vector that is perpendicular to both $\underset{\sim}{a} = 2\underset{\sim}{i} + 4\underset{\sim}{j} - 3\underset{\sim}{k}$ and $\underset{\sim}{b} = -4\underset{\sim}{i} - 5\underset{\sim}{j} + 3\underset{\sim}{k}$. **4**

Find all possible vectors $\underset{\sim}{c}$.

(c) (i) For positive real numbers a and b, prove that $\frac{a+b}{2} \geq \sqrt{ab}$. **2**

(ii) Hence, or otherwise, show that $\frac{2n+1}{2n+2} < \frac{\sqrt{2n+1}}{\sqrt{2n+3}}$ for any integer $n \geq 0$. **2**

(d) Evaluate $\int_0^{\frac{\pi}{2}} \frac{u}{1+\sin u+\cos u}\,du$, by first using the substitution $u = \frac{\pi}{2} - x$. **4**

Question 14 (15 marks) Use the Question 14 Writing Booklet

(a) Let $I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^{2n}\theta \, d\theta$ for integers $n \geq 0$.

(i) Show that $I_n = \dfrac{1}{2n-1} - I_{n-1}$ for $n > 0$, given that $\dfrac{d}{d\theta}\cot\theta = -\operatorname{cosec}^2\theta$. **3**

(ii) Hence, or otherwise, calculate I_2. **1**

(b) The acceleration of a particle is given by $\ddot{x} = 32x\left(x^2 + 3\right)$, where x is the displacement of the particle from a fixed-point O after t seconds, in metres. Initially the particle is at O and has a velocity of 12 m s^{-1} in the negative direction.

(i) Show that the velocity of the particle is given by $v = -4\left(x^2 + 3\right)$. **2**

(ii) Find the time taken for the particle to travel 3 metres from the origin. **2**

(c) Let w be a complex number such that $1 + w + w^2 + \cdots + w^6 = 0$.

(i) Show that w is a 7th root of unity. **1**

The complex number $\alpha = w + w^2 + w^4$ is a root of the equation $x^2 + bx + c = 0$, where b and c are real and α is not real.

(ii) Find the other root of $x^2 + bx + c = 0$ in terms of positive powers of w. **2**

(iii) Find the numerical value of c. **1**

(d) Positive real numbers a, b, c and d are chosen such that $\dfrac{1}{a}, \dfrac{1}{b}, \dfrac{1}{c}$ and $\dfrac{1}{d}$ are consecutive terms in an arithmetic sequence with common difference k, where $k \in \mathbb{R}$, $k > 0$. **3**

Show that $b + c < a + d$.

Question 15 (15 marks) Use the Question 15 Writing Booklet

(a) The adjacent sides of a parallelogram are represented by the vectors $\underset{\sim}{a} = 4\underset{\sim}{i} + 3\underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}$. **4**

Show that the area of the parallelogram is $6\sqrt{10}$ square units.

(b) A particle moves in simple harmonic motion about the origin with amplitude A, and it completes two cycles per second. When it is $\frac{1}{4}$ metres from the origin, its speed is half its maximum speed. **4**

Find the maximum positive acceleration of the particle during its motion.

(c) (i) Show that **3**

$$\frac{1 + \cos\theta + i\sin\theta}{1 - \cos\theta - i\sin\theta} = i\cot\frac{\theta}{2}.$$

(ii) Use De Moivre's theorem to show that the sixth roots of -1 are given by $\cos\left(\frac{(2k+1)\pi}{6}\right) + i\sin\left(\frac{(2k+1)\pi}{6}\right)$ for $k = 0, 1, 2, 3, 4, 5$. **2**

(iii) Hence, or otherwise, show the solutions to $\left(\frac{z-1}{z+1}\right)^6 = -1$ are $z = i\cot\left(\frac{\pi}{12}\right)$, $i\cot\left(\frac{3\pi}{12}\right)$, $i\cot\left(\frac{5\pi}{12}\right)$, $i\cot\left(\frac{7\pi}{12}\right)$, $i\cot\left(\frac{9\pi}{12}\right)$, and $i\cot\left(\frac{11\pi}{12}\right)$. **2**

Question 16 (15 marks) Use the Question 16 Writing Booklet

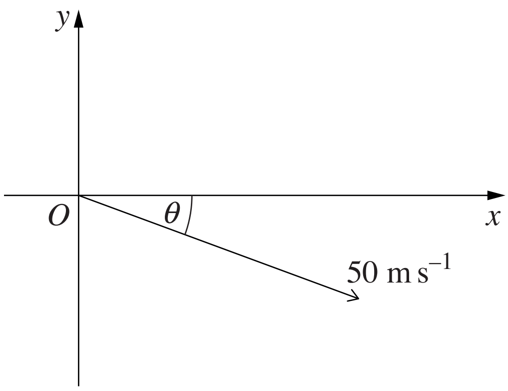
(a) Consider the equation **4**

$$z^n \cos[n\theta] + z^{n-1}\cos[(n-1)\theta] + z^{n-2}\cos[(n-2)\theta] + \cdots + z\cos[\theta] = 1$$

where $z \in \mathbb{C}$, $\theta \in \mathbb{R}$, and n is a positive integer.

Using a proof by contradiction and the triangle inequality, or otherwise, prove that all the solutions to the equation lie outside the circle $|z| = \frac{1}{2}$ on the complex plane.

(b) A particle of mass 1 kg is projected from the origin with a speed of 50 m s^{-1}, at an angle of θ below the horizontal into a resistive medium. **5**



The position of the particle t seconds after projection is (x, y), and the velocity of the particle at that time is $\underset{\sim}{v} = \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$.

The resistive force, $\underset{\sim}{R}$, is proportional to the velocity of the particle, so that $\underset{\sim}{R} = -k\underset{\sim}{v}$, where k is a positive constant.

Taking the acceleration due to gravity to be 10 m s^{-2}, and the upwards vertical direction to be positive, the acceleration of the particle at time t is given by:

$$\underset{\sim}{a} = \begin{pmatrix} -k\dot{x} \\ -k\dot{y} - 10 \end{pmatrix}. \qquad \text{(Do NOT prove this.)}$$

Derive the Cartesian equation of the motion of the particle, given $\sin\theta = \frac{3}{5}$.

Question 16 continues on page 14

(c) Consider the point B with three-dimensional position vector $\underset{\sim}{b}$ and the line ℓ: $\underset{\sim}{a} + \lambda \underset{\sim}{d}$, where $\underset{\sim}{a}$ and $\underset{\sim}{d}$ are three-dimensional vectors, $\left| \underset{\sim}{d} \right| = 1$ and λ is a parameter.

Let $f(\lambda)$ be the distance between a point on the line ℓ and the point B.

(i) Find λ_0, the value of λ that minimises f, in terms of $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{d}$. **2**

(ii) Let P be the point with position vector $\underset{\sim}{a} + \lambda_0 \underset{\sim}{d}$. **1**

Show that PB is perpendicular to the direction of the line ℓ.

(iii) Hence, or otherwise, find the shortest distance between the line ℓ and the sphere of radius 1 unit, centred at the origin O, in terms of $\underset{\sim}{d}$ and $\underset{\sim}{a}$. **3**

You may assume that if B is the point on the sphere closest to ℓ, then OBP is a straight line.

End of paper

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2025 **HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Extension 2

Writing Booklet

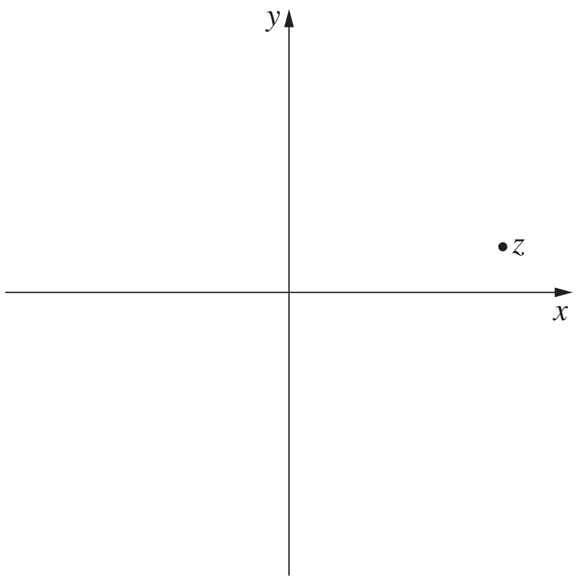
Question 11

Instructions

- Use this Writing Booklet to answer Question 11 → **11**
- Write the number of this booklet and the total number of booklets that you have used for this question (eg: 1 of 3) → [ ] of [ ] (this booklet / number of booklets for this question)
- Write your Centre Number and Student Number at the top of this page
- Write using black pen
- You may ask for an extra writing booklet if you need more space
- If you have not attempted the question(s), you must still hand in the writing booklet, with 'NOT ATTEMPTED' written clearly on the front cover
- You may NOT take any writing booklets, used or unused, from the examination room

Start here for
Question Number: **11**

(a) The location of the complex number z is shown on the diagram. Indicate the locations of $\overline{z}$ and $i\overline{z}$.



Additional writing space on back page.

Tick this box if you have continued this answer in another writing booklet.

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2025 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}, r \neq 1$$

$$S = \frac{a}{1-r}, |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

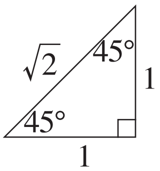
$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2} ab \sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

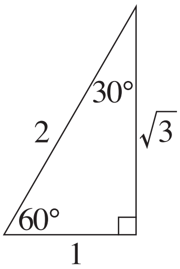


$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2} r^2 \theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

If $t = \tan \frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A \sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

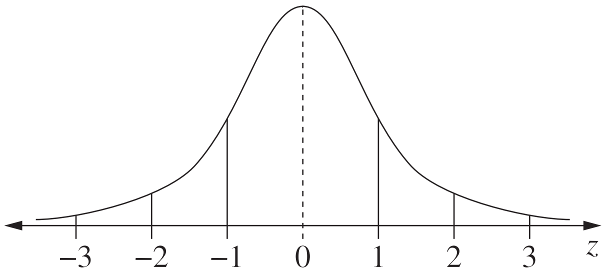
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n - r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \quad P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n - x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = n f'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)\, f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)\, f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1 - \left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1 - \left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1 + \left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \left[f(x)\right]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + \left[f(x)\right]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left| \underset{\sim}{u} \right| = \left| x\underset{\sim}{i} + y\underset{\sim}{j} \right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left| \underset{\sim}{u} \right| \left| \underset{\sim}{v} \right| \cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^{n} &= r^{n}(\cos n\theta + i\sin n\theta) \\ &= r^{n}e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^{2}(x-c)$$