2025 HSC Mathematics Extension 2
Marking Guidelines

Section I

Multiple-choice Answer Key

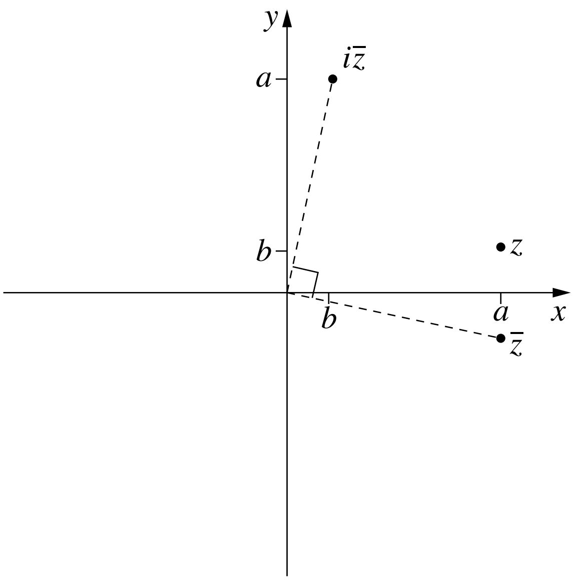
Question	Answer
1	D
2	B
3	C
4	B
5	A
6	B
7	D
8	A
9	C
10	D

Section II

Question 11 (a)

Criteria	Marks
• Indicates the locations of $\overline{z}$ and $i\overline{z}$	2
• Indicates the location of $\overline{z}$, or equivalent merit	1

Sample answer:



Let $z = a + ib$

$\overline{z} = a - ib$

$$i\overline{z} = i(a - ib)$$
$$= -i^2b + ia$$
$$= b + ia$$

Question 11 (b)

Criteria	Marks
• Provides correct solution	2
• Provides the modulus or argument	1

Sample answer:

$w = 2e^{\frac{i\pi}{6}}, \ z = 3e^{\frac{i\pi}{6}}$

$\therefore \quad wz = (2 \times 3)e^{i\pi\left(\frac{1}{6}+\frac{1}{6}\right)}$

$wz = 6e^{\frac{i\pi}{3}}$

$\therefore \quad \text{Modulus} = 6$

$\text{Argument} = \frac{\pi}{3}$

Question 11 (c) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\begin{aligned} z^2 &= (x+iy)(x+iy) \\ &= x^2 + 2ixy - y^2 \\ &= \left(x^2 - y^2\right) + 2xyi \end{aligned}$$

Question 11 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Attempts to realise the denominator of $\frac{1}{x+iy}$	1

Sample answer:

$$\begin{aligned} \frac{1}{z} &= \frac{1}{x+iy} \\ &= \frac{x-iy}{(x+iy)(x-iy)} \\ &= \frac{x-iy}{x^2+y^2} \\ &= \frac{x}{x^2+y^2} - \frac{y}{x^2+y^2}i \end{aligned}$$

Question 11 (d) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$\left|2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}\right| = \sqrt{2^2 + (-2)^2 + 1^2} = 3$$

$$\left|\underset{\sim}{F}_1\right| = 12$$

$$\therefore \quad \underset{\sim}{F}_1 = 4\left(2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}\right) = 8\underset{\sim}{i} - 8\underset{\sim}{j} + 4\underset{\sim}{k}$$

Question 11 (d) (ii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$\underset{\sim}{F}_3 = \underset{\sim}{F}_1 + \underset{\sim}{F}_2$$

$$= (8-6)\underset{\sim}{i} + (-8+12)\underset{\sim}{j} + (4+4)\underset{\sim}{k}$$

$$= 2\underset{\sim}{i} + 4\underset{\sim}{j} + 8\underset{\sim}{k}$$

Question 11 (d) (iii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\underset{\sim}{F}_3 \cdot \underset{\sim}{d}$$

$$= \left(2\underset{\sim}{i} + 4\underset{\sim}{j} + 8\underset{\sim}{k}\right) \cdot \left(\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}\right)$$

$$= (2 \times 1) + (4 \times 1) + (8 \times 2)$$

$$= 22$$

Question 11 (e)

Criteria	Marks
• Provides correct proof	2
• Assumes negation is true	1

Sample answer:

Assume

$$\sqrt{3} + \sqrt{5} \le \sqrt{11}$$

Squaring,

$$\left(\sqrt{3} + \sqrt{5}\right)^2 \le \left(\sqrt{11}\right)^2$$

$$8 + 2\sqrt{15} \le 11$$

$$2\sqrt{15} \le 3$$

Squaring,

$$60 \le 9$$

which is not true, contradicting the assumption.

$$\therefore \quad \sqrt{3} + \sqrt{5} > \sqrt{11}$$

Question 11 (f)

Criteria	Marks
• Provides correct solution	2
• Completes the square	1

Sample answer:

$$\begin{aligned} \int \frac{5}{\sqrt{7 - x^2 - 6x}}\,dx &= \int \frac{5}{\sqrt{-\left[x^2 + 6x - 7\right]}}\,dx \\ &= \int \frac{5}{\sqrt{-\left[(x+3)^2 - 16\right]}}\,dx \\ &= \int \frac{5}{\sqrt{16 - (x+3)^2}}\,dx \\ &= 5\sin^{-1}\left(\frac{x+3}{4}\right) + C \end{aligned}$$

Question 12 (a)

Criteria	Marks
• Provides correct solution	3
• Uses integration by parts	2
• Attempts to use integration by parts	1

Sample answer:

$$\int_0^{\frac{\pi}{2}} x \sin x \, dx$$

Using integration by parts

Let $u = x, \quad u' = 1$

Let $v' = \sin x, \quad v = -\cos x$

$$\therefore \int_0^{\frac{\pi}{2}} x \sin x \, dx = \left[-x \cos x\right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x \, dx$$

$$= \left(-\frac{\pi}{2} \cdot \cos\left(\frac{\pi}{2}\right) + 0 \cdot \cos(0)\right) + \left[\sin x\right]_0^{\frac{\pi}{2}}$$

$$= (0 + 0) + \left(\sin\left(\frac{\pi}{2}\right) - \sin(0)\right)$$

$$= 1$$

Question 12 (b)

Criteria	Marks
• Provides correct proof	3
• Establishes that $\frac{d^k y}{dx^k} = P_k(x) \Rightarrow \frac{d^{k+1}y}{dx^{k+1}} = P_{k+1}(x)$, or equivalent merit	2
• Establishes initial statement, or equivalent merit	1

Sample answer:

Let $P_n(x) = \left(2^n x + n2^{n-1}\right)e^{2x}$.

The first derivative is $\frac{dy}{dx} = 2xe^{2x} + e^{2x} = \left(2x + 1 \times 2^0\right)e^{2x} = P_1(x)$. So the statement is true for $n = 1$.

If the statement is true for $n = k$, that is $\frac{d^k y}{dx^k} = P_k(x)$,

then $\frac{d^{k+1}y}{dx^{k+1}} = \frac{d}{dx}\left(\frac{d^k y}{dx^k}\right) = \frac{d}{dx}\left(P_k(x)\right) = \frac{d}{dx}\left(\left(2^k x + k2^{k-1}\right)e^{2x}\right)$

$= 2\left(2^k x + k2^{k-1}\right)e^{2x} + 2^k e^{2x}$

$= \left(2^{(k+1)}x + k2^k + 2^k\right)e^{2x}$

$= \left(2^{(k+1)}x + (k+1)\,2^k\right)e^{2x} = P_{k+1}(x)$.

So the statement is also true for $n = k + 1$.

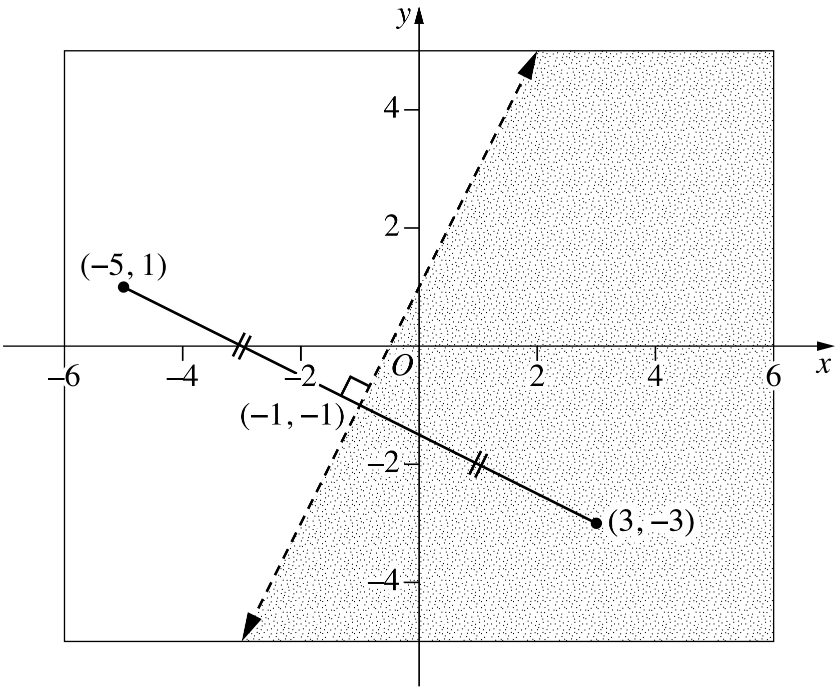
Hence the statement is true for all positive integers by mathematical induction.

Question 12 (c)

Criteria	Marks
• Provides correct sketch	3
• Sketches the perpendicular bisector, or equivalent merit	2
• Plots the point $(-5, 1)$ or $(3, -3)$, or equivalent merit	1

Sample answer:

The region is to the right of the perpendicular bisector of the line joining $(-5, 1)$ and $(3, -3)$, as shown.



Answers could include:

$$|z + 5 - i| > |z - 3 + 3i|$$

$$|(x + 5) + i(y - 1)| > |(x - 3) + i(y + 3)|$$

$$\sqrt{(x + 5)^2 + (y - 1)^2} > \sqrt{(x - 3)^2 + (y + 3)^2}$$

$$y < 2x + 1$$

Question 12 (d)

Criteria	Marks
• Provides correct solution	4
• Obtains one anti-derivative	3
• Shows $\dfrac{x^2-2x+9}{(4-x)(x^2+1)} = \dfrac{1}{4-x} + \dfrac{2}{x^2+1}$, or equivalent merit	2
• Attempts to use partial fractions, or equivalent merit	1

Sample answer:

$$\int \frac{x^2-2x+9}{(4-x)(x^2+1)}\,dx$$

For partial fractions, let

$$\frac{A}{4-x} + \frac{Bx+C}{x^2+1} = \frac{x^2-2x+9}{(4-x)(x^2+1)}$$

$$A(x^2+1) + (Bx+C)(4-x) = x^2-2x+9$$

Put $x = 4$

$$17A = 16 - 8 + 9$$

$$\boxed{A = 1}$$

Put $x = 0$

$$A + 4C = 9$$

$$4C = 8$$

$$\boxed{C = 2}$$

Compare coefficients of x^2

$$A - B = 1$$

$$\boxed{B = 0}$$

$$\int \frac{dx}{4-x} + \int \frac{2}{x^2+1}\,dx$$

$$= -\ln|4-x| + 2\tan^{-1}x + c$$

Question 12 (e)

Criteria	Marks
• Provides correct solution	3
• Integrates to find an expression in x	2
• Obtains force equation, or equivalent merit	1

Sample answer:

The total force on the particle is $-mk - mv^2$

$$m\ddot{x} = -mk - mv^2$$

$$\ddot{x} = -\left(k + v^2\right)$$

$$v\frac{dv}{dx} = -\left(k + v^2\right)$$

$$\int \frac{v\,dv}{k + v^2} = -\int dx$$

$$\frac{1}{2}\ln\left(k + v^2\right) = -x + C$$

When $x = 0,\ v = V_0$

$$\therefore \quad \frac{1}{2}\ln\left(k + {V_0}^2\right) = C$$

$$\therefore \quad \frac{1}{2}\ln\left(k + v^2\right) = -x + \frac{1}{2}\ln\left(k + {V_0}^2\right)$$

$$x = \frac{1}{2}\ln\left(\frac{k + {V_0}^2}{k + v^2}\right)$$

When $v = 0$

$$x = \frac{1}{2}\ln\left(\frac{k + {V_0}^2}{k}\right)$$

Question 12 (e) (continued)

Alternative solution

The total force on the particle is $-mk - mv^2$ newtons.

$$\therefore \quad m\ddot{x} = -mk - mv^2$$
$$\ddot{x} = -\left(k + v^2\right)$$
$$\frac{1}{2}\frac{d}{dx}\left(v^2\right) = -\left(k + v^2\right)$$
$$-\frac{1}{2}\int \frac{d\left(v^2\right)}{k + v^2} = \int 1\, dx$$

Since v^2 goes from ${V_0}^2$ to 0 while x goes from 0 to X:

$$-\frac{1}{2}\int_{{V_0}^2}^{0} \frac{d\left(v^2\right)}{k + v^2} = \int_0^X 1\, dx$$

$$-\frac{1}{2}\left[\ln\left(k + v^2\right)\right]_{{V_0}^2}^{0} = [x]_0^X \qquad \text{Using } v^2 \text{ as the variable of integration}$$

$$-\frac{1}{2}\left[\ln(k) - \ln\left(k + {V_0}^2\right)\right] = X - 0$$

$$\therefore \quad X = \frac{1}{2}\left[\ln\left(k + {V_0}^2\right) - \ln(k)\right]$$
$$= \frac{1}{2}\ln\left(\frac{k + {V_0}^2}{k}\right)$$

Question 13 (a)

Criteria	Marks
• Provides correct solution	3
• Obtains correct integral after substitution	2
• Attempts to integrate by substitution	1

Sample answer:

$$I = \int_{m-4}^{m-2} \frac{e^{-x}}{x - m + 1} dx$$

Let $u = m - x$

When $x = m - 2$ then $u = 2$,
when $x = m - 4$ then $u = 4$,

$$\frac{du}{dx} = -1.$$

So

$$I = \int_4^2 \frac{e^{u-m}}{1-u}(-1)du$$

$$= -e^{-m} \int_4^2 \frac{e^u du}{1-u}$$

$$= -e^{-m} \int_2^4 \frac{e^u}{u-1} du$$

$$= -Ae^{-m}$$

$$= kA \text{ where } k = -e^{-m}.$$

Question 13 (b)

Criteria	Marks
• Provides correct solution	4
• Finds one value of $\underset{\sim}{c}$	3
• Obtains a relationship between x and y or z, or equivalent merit	2
• Evaluates $\underset{\sim}{a} \cdot \underset{\sim}{c} = 0$ or $\underset{\sim}{b} \cdot \underset{\sim}{c} = 0$, or equivalent merit	1

Sample answer:

Since $\underset{\sim}{c}$ is perpendicular to both $\underset{\sim}{a}$ and $\underset{\sim}{b}$,

$\underset{\sim}{a} \cdot \underset{\sim}{c} = 0$: $\quad 2x + 4y - 3z = 0 \quad ①$

$\underset{\sim}{b} \cdot \underset{\sim}{c} = 0$: $\quad -4x - 5y + 3z = 0 \quad ②$

① + ②: $\quad -2x - y = 0 \Rightarrow y = -2x$

Substitute this into equation ①.

$2x - 8x - 3z = 0 \Rightarrow z = -2x$

Since $\underset{\sim}{c}$ is a unit vector, $x^2 + y^2 + z^2 = 1 \quad ③$

Substituting into equation ③:

$$
\begin{aligned}
x^2 + (-2x)^2 + (-2x)^2 &= 1 \\
9x^2 &= 1 \\
x^2 &= \frac{1}{9} \\
x &= \pm\frac{1}{3} \\
y = z = -2x &= \mp\frac{2}{3}
\end{aligned}
$$

$$\underset{\sim}{c} = \pm\left(\frac{1}{3}\underset{\sim}{i} - \frac{2}{3}\underset{\sim}{j} - \frac{2}{3}\underset{\sim}{k}\right)$$

Question 13 (c) (i)

Criteria	Marks
• Provides correct solution	2
• Begins proof with $(a-b)^2 \geq 0$, or equivalent merit	1

Sample answer:

$$(a-b)^2 \geq 0$$

$$a^2 - 2ab + b^2 \geq 0$$

$$a^2 + 2ab + b^2 \geq 4ab$$

$$(a+b)^2 \geq 4ab$$

$$a + b \geq 2\sqrt{ab} \qquad \begin{pmatrix} \text{ignore negative root} \\ \text{since } a, b \text{ positive} \end{pmatrix}$$

$$\frac{a+b}{2} \geq \sqrt{ab}$$

Question 13 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Attempts to use AM−GM inequality, or equivalent merit	1

Sample answer:

Apply AM−GM inequality

$$\sqrt{ab} \leq \frac{a+b}{2} \qquad \text{with } a = 2n+1,\ b = 2n+3.$$

Since $a \neq b$, this becomes $\sqrt{ab} < \dfrac{a+b}{2}$

$$\sqrt{(2n+1)(2n+3)} < \frac{(2n+1)+(2n+3)}{2}$$

$$\frac{\sqrt{(2n+1)(2n+3)}}{2n+1} < \frac{2n+2}{2n+1}$$

$$\frac{\sqrt{2n+3}}{\sqrt{2n+1}} < \frac{2n+2}{2n+1}$$

$$\therefore \qquad \frac{2n+1}{2n+2} < \frac{\sqrt{2n+1}}{\sqrt{2n+3}} \qquad \text{as required.}$$

Question 13 (d)

Criteria	Marks
• Provides correct solution	4
• Attempts to use t-substitution, or equivalent merit	3
• Obtains two times original integral, or equivalent merit	2
• Uses given substitution	1

Sample answer:

Let $u = \frac{\pi}{2} - x$. When $u = \frac{\pi}{2}$ then $x = 0$, when $u = 0$ then $x = \frac{\pi}{2}$ and $\frac{du}{dx} = (-1)$.
Hence,

$$\int_0^{\frac{\pi}{2}} \frac{u}{1 + \sin u + \cos u}\, du = \int_{\frac{\pi}{2}}^{0} \frac{\frac{\pi}{2} - x}{1 + \sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} (-1)\, dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2} - x}{1 + \cos x + \sin x}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2}}{1 + \cos x + \sin x}\, dx - \int_0^{\frac{\pi}{2}} \frac{x}{1 + \cos x + \sin x}\, dx$$

$$= \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{dx}{1 + \cos x + \sin x} - \int_0^{\frac{\pi}{2}} \frac{u}{1 + \cos u + \sin u}\, du$$

$$\therefore \quad 2 \int_0^{\frac{\pi}{2}} \frac{u}{1 + \sin u + \cos u}\, du = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{dx}{1 + \cos x + \sin x}$$

$$\therefore \quad \int_0^{\frac{\pi}{2}} \frac{u}{1 + \sin u + \cos u}\, du = \frac{\pi}{4} \int_0^{\frac{\pi}{2}} \frac{dx}{1 + \cos x + \sin x}$$

Question 13 (d) sample answer (continued)

Putting $t = \tan\dfrac{x}{2}$, $\cos(x) = \dfrac{1 - t^2}{1 + t^2}$, $\sin(x) = \dfrac{2t}{1 + t^2}$, $\dfrac{dt}{dx} = \dfrac{1 + t^2}{2}$.

When $x = \dfrac{\pi}{2}$, $t = 1$, and when $x = 0$, $t = 0$.

$$\int_0^{\frac{\pi}{2}} \frac{u}{1 + \sin u + \cos u}\, du = \frac{\pi}{4} \int_0^1 \frac{\dfrac{2dt}{\left(1 + t^2\right)}}{\dfrac{\left(1 + t^2\right) + \left(1 - t^2\right) + 2t}{\left(1 + t^2\right)}}$$

$$= \frac{\pi}{4} \int_0^1 \frac{2dt}{2 + 2t} = \frac{\pi}{4} \int_0^1 \frac{dt}{1 + t} = \frac{\pi}{4} \Big[\ln|1 + t| \Big]_0^1$$

$$= \frac{\pi}{4} \ln 2$$

Question 14 (a) (i)

Criteria	Marks
• Provides correct solution	3
• Replaces integral with I_{n-1}	2
• Uses the Pythagorean identity, or equivalent merit	1

Sample answer:

$$I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^{2n}\theta \, d\theta$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^{2(n-1)}\theta \cot^2\theta \, d\theta$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^{2(n-1)}\theta \left(\operatorname{cosec}^2\theta - 1\right) d\theta$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^{2(n-1)}\theta \operatorname{cosec}^2\theta \, d\theta - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^{2(n-1)}\theta \, d\theta$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^{2(n-1)}\theta \operatorname{cosec}^2\theta \, d\theta - I_{n-1}$$

$$= -\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(-\operatorname{cosec}^2\theta\right) \cot^{2(n-1)}\theta \, d\theta - I_{n-1}$$

$$= -\left[\frac{\cot^{2(n-1)+1}\theta}{2n-1}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} - I_{n-1}$$

$$= -\left(\frac{0-1}{2n-1}\right) - I_{n-1}$$

$$= \frac{1}{2n-1} - I_{n-1}$$

Question 14 (a) (ii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

Since $I_0 = \frac{\pi}{4}$, using the recurrence relation we have

$I_1 = 1 - I_0 = 1 - \frac{\pi}{4}$

$I_2 = \frac{1}{3} - I_1 = -\frac{2}{3} + \frac{\pi}{4}$.

Question 14 (b) (i)

Criteria	Marks
• Provides correct solution	2
• Integrates to get an expression for v^2 in terms of x	1

Sample answer:

Since $v\frac{dv}{dx} = 32x\left(x^2+3\right)$ we have $\int v\,dv = \int 32x\left(x^2+3\right)dx$.

Hence,

$$\frac{v^2}{2} = 8\left(x^2+3\right)^2 + C.$$

Since the initial velocity is -12, we have

$$\frac{144}{2} = 72 + C \text{ and so } C = 0.$$

Hence $v^2 = 16\left(x^2+3\right)^2$ and $v = \pm 4\left(x^2+3\right)$.

Since the initial velocity is negative, $v = -4\left(x^2+3\right)$.

Question 14 (b) (ii)

Criteria	Marks
• Provides correct solution	2
• Integrates to find a relation between time and displacement, or equivalent merit	1

Sample answer:

Given $\frac{dx}{dt} = -4\left(x^2 + 3\right)$ we have

$$\int \frac{dx}{x^2 + 3} = \int -4\, dt\ .$$

Hence $\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) = -4t + C$

Using initial conditions, $C = 0$.

Since $x < 0$ at all times we let $x = -3$. Then

$$\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{-3}{\sqrt{3}}\right) = -4t$$

$$\frac{1}{\sqrt{3}} \cdot \left(-\frac{\pi}{3}\right) = -4t$$

and $t = \frac{\pi}{12\sqrt{3}}$ seconds.

Question 14 (c) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$1 + w + w^2 + \cdots + w^6 = 0$$

$$\therefore \quad (1 - w)\left(1 + w + w^2 + \cdots + w^6\right) = 0$$

$$1 + w + w^2 + \cdots + w^6 - \left(w + w^2 + w^3 + \cdots + w^7\right) = 0$$

$$1 - w^7 = 0$$

$$\therefore \quad w^7 = 1$$

Question 14 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Recognises that the complex conjugate of $w + w^2 + w^4$ is the other root	1

Sample answer:

w is a 7th root of unity, $w = e^{\frac{2i\pi k}{7}}$ $(k \in \mathbb{Z})$

$$\begin{aligned}\therefore \quad \bar{w} &= e^{\frac{-2i\pi k}{7}} \\ &= \frac{1}{w} \\ &= \frac{w^7}{w} \\ &= w^6\end{aligned}$$

$$\begin{aligned}\therefore \quad \bar{\alpha} &= \overline{w + w^2 + w^4} \\ &= \bar{w} + \left(\bar{w}\right)^2 + \left(\bar{w}\right)^4 \\ &= w^6 + \left(w^6\right)^2 + \left(w^6\right)^4 \\ &= w^6 + w^{12} + w^{24} \\ &= w^6 + w^{7+5} + w^{(3\times 7)+3} \\ &= w^6 + w^5 + w^3\end{aligned}$$

Question 14 (c) (iii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$c = \alpha\bar{\alpha}$

$$\begin{aligned}\left(w + w^2 + w^4\right)\left(w^3 + w^5 + w^6\right) &= w^4 + w^6 + w^7 + w^5 + w^7 + w^8 + w^7 + w^9 + w^{10} \\ &= w^4 + w^6 + 1 + w^5 + 1 + w + 1 + w^2 + w^3 \\ &= 3 + (-1)\end{aligned}$$

$$\therefore \quad c = 2$$

Question 14 (d)

Criteria	Marks
• Provides correct solution	3
• Writes $a+d$ and $b+c$ in terms of a and k, or equivalent merit	2
• Writes $b = \dfrac{a}{1+ak}$, or equivalent merit	1

Sample answer:

Since $\dfrac{1}{b} = \dfrac{1}{a} + k = \dfrac{1+ak}{a}$ then $b = \dfrac{a}{1+ak}$. Similarly $c = \dfrac{a}{1+2ak}$, and $d = \dfrac{a}{1+3ak}$

$$a + d = a + \frac{a}{1+3ak}$$
$$= \frac{2a + 3a^2k}{1+3ak}$$

$$b + c = \frac{a}{1+ak} + \frac{a}{1+2ak}$$
$$= \frac{2a+3a^2k}{(1+ak)(1+2ak)}$$
$$= \frac{2a+3a^2k}{1+3ak+2a^2k^2}$$

Since $k \in \mathbb{R}$, $k > 0$

$$2a^2k^2 > 0$$

$$\therefore \quad 1 + 3ak + 2a^2k^2 > 1 + 3ak$$

$$\therefore \quad \frac{1}{1+3ak+2a^2k^2} < \frac{1}{1+3ak}$$

Multiply by $2a + 3a^2k$

$$\frac{2a+3a^2k}{1+3ak+2a^2k^2} < \frac{2a+3a^2k}{1+3ak}$$

$$\therefore \quad b + c < a + d$$

<u>Alternate solution</u>:

$$\frac{1}{a} < \frac{1}{b} < \frac{1}{c} < \frac{1}{d}$$

$$\therefore \quad a > b > c > d$$

$$\therefore \quad ab > cd$$

$$\frac{ab}{cd} > 1$$

Now, $\dfrac{1}{b} - \dfrac{1}{a} = k$

and $\dfrac{1}{d} - \dfrac{1}{c} = k$

$$\therefore \quad \frac{1}{b} - \frac{1}{a} = \frac{1}{d} - \frac{1}{c}$$

$$\frac{a-b}{ab} = \frac{c-d}{cd}$$

$$(a-b) = (c-d)\,.\,\frac{ab}{cd} > (c-d)$$

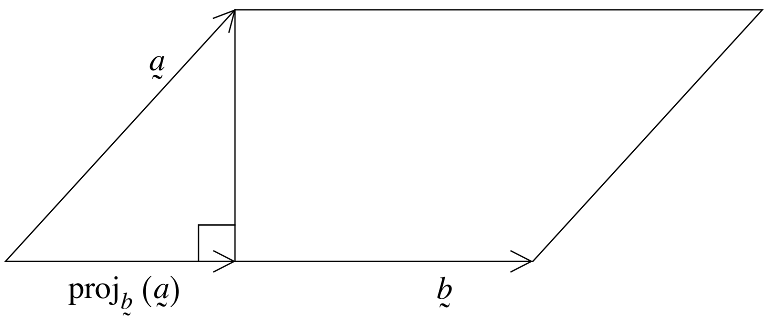
$$\therefore \quad a - b > c - d$$

$$\therefore \quad a + d > b + c$$

Question 15 (a)

Criteria	Marks
• Provides correct solution	4
• Calculates the perpendicular height, or equivalent merit	3
• Calculates a projection, or equivalent merit	2
• Finds $\lvert \underset{\sim}{a} \rvert$ or $\lvert \underset{\sim}{b} \rvert$, or equivalent merit	1

Sample answer:



Area of parallelogram is base × perpendicular height

$$\text{base} = \lvert \underset{\sim}{b} \rvert = \sqrt{2^2 + (-1)^2 + 3^2} = \sqrt{14}$$

$$\left\lvert \text{proj}_{\underset{\sim}{b}} (\underset{\sim}{a}) \right\rvert = \left\lvert \frac{\underset{\sim}{b} \cdot \underset{\sim}{a}}{\lvert \underset{\sim}{b} \rvert^2} \underset{\sim}{b} \right\rvert = \frac{\lvert 8 - 3 - 3 \rvert}{14} \cdot \sqrt{14} = \frac{1}{7} \cdot \sqrt{14}$$

Hence perpendicular height $= \sqrt{\lvert \underset{\sim}{a} \rvert^2 - \left\lvert \text{proj}_{\underset{\sim}{b}} (\underset{\sim}{a}) \right\rvert^2}$

$$= \sqrt{(16 + 9 + 1) - \frac{14}{49}}$$

$$= \sqrt{26 - \frac{2}{7}} = 3\sqrt{\frac{20}{7}}$$

$$\text{Area} = \sqrt{14} \times 3\sqrt{\frac{20}{7}} = 3 \times \sqrt{2} \times \sqrt{20} = 6\sqrt{10} \text{ square units.}$$

Question 15 (b)

Criteria	Marks
• Provides correct solution	4
• Finds $A = \frac{1}{2\sqrt{3}}$	3
• Finds maximum speed in terms of A, or equivalent merit	2
• Finds period is $\frac{1}{2}$, or equivalent merit	1

Sample answer:

Two cycles per second $\Rightarrow$ period $T = \frac{1}{2}$

$$T = \frac{2\pi}{n} \Rightarrow n = 4\pi$$

Let A be the amplitude

$$\ddot{x} = -n^2 x \quad \therefore \text{ maximum positive acceleration } = \left| -(4\pi)^2 \cdot A \right|$$
$$= 16\pi^2 A$$

Let $x(t) = A\cos(4\pi t + \alpha)$

$\therefore \quad v(t) = -4\pi A\sin(4\pi t + \alpha)$

$\therefore \quad$ Maximum speed $= \left| -4\pi A \right| = 4\pi A$

Let $t = 0$ be the time when $x = \frac{1}{4}$ and $v = \frac{1}{2} \times 4\pi A = 2\pi A$

$\therefore \qquad A\cos\alpha = \frac{1}{4}$

and $\qquad \left| -4\pi A\sin\alpha \right| = 2\pi A$

$$4\pi A\sin\alpha = 2\pi A$$

$$\therefore \quad \sin\alpha = \frac{1}{2}$$

$$\therefore \quad \alpha = \frac{\pi}{6}$$

$\therefore \quad A\cos\frac{\pi}{6} = \frac{1}{4}$

$$A = \frac{1}{2\sqrt{3}}$$

$\therefore \quad$ Maximum positive acceleration $= 16\pi^2 \times \frac{1}{2\sqrt{3}}$

$$= \frac{8\pi^2}{\sqrt{3}} \text{ m s}^{-2}$$

Question 15 (b) sample answer (continued)

Alternative solution

The particle is in simple harmonic motion about the origin with period $\frac{1}{2}$ and amplitude A, so the displacement at time t is

$$x(t) = A\cos(4\pi t + \alpha).$$

The velocity is

$$\dot{x}(t) = -4\pi A\sin(4\pi t + \alpha). \quad \text{The maximum speed is } 4\pi A.$$

Let t_0 be the time when the particle is $\frac{1}{4}$ m from the origin

$$x(t_0) = A\cos(4\pi t_0 + \alpha) = \frac{1}{4}$$

and the speed is half its maximum

$$\dot{x}(t_0) = \frac{4\pi A}{2} = 2\pi A = -4\pi A\sin(4\pi t_0 + \alpha).$$

This means $\sin(4\pi t_0 + \alpha) = -\frac{1}{2}$ and so $\cos(4\pi t_0 + \alpha) = \pm\frac{\sqrt{3}}{2}$.

Hence $A\cos(4\pi t_0 + \alpha) = \pm\frac{A\sqrt{3}}{2} = \pm\frac{1}{4}$ and so $A = \frac{1}{2\sqrt{3}}$. (Since amplitude is not negative.)

The acceleration is $\ddot{x}(t) = -16\pi^2 A\cos(4\pi t + \alpha)$. The maximum positive acceleration is $16\pi^2 A = \frac{8\pi^2}{\sqrt{3}}$ m s^{-2}.

Question 15 (c) (i)

Criteria	Marks
• Provides correct solution	3
• Uses double angle formulas	2
• Realises denominator, or equivalent merit	1

Sample answer:

$$\text{LHS} = \frac{1+\cos\theta + i\sin\theta}{1-\cos\theta - i\sin\theta} \times \frac{1-\cos\theta + i\sin\theta}{1-\cos\theta + i\sin\theta}$$

$$= \frac{2i\sin\theta}{2-2\cos\theta}$$

$$= \frac{4i\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{4\sin^2\frac{\theta}{2}}$$

$$= i\cot\frac{\theta}{2}$$

Question 15 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Uses De Moivre's theorem	1

Sample answer:

$w^6 = -1 \qquad \therefore \ |w| = 1$

Let $w = \cos\theta + i\sin\theta$

$$(\cos\theta + i\sin\theta)^6 = 1$$

$$\cos 6\theta + i\sin 6\theta = \cos\pi + i\sin\pi$$

Equating real parts,

$$6\theta = \pi + 2k\pi, \qquad k = 0, 1, 2, 3, 4, 5$$

$$\theta = \frac{\pi}{6} + \frac{2k\pi}{6}$$

$$= \frac{(2k+1)\pi}{6}$$

$$\therefore \ w = \cos\left(\frac{(2k+1)\pi}{6}\right) + i\sin\left(\frac{(2k+1)\pi}{6}\right), \qquad k = 0, 1, 2, 3, 4, 5$$

Question 15 (c) (iii)

Criteria	Marks
• Provides correct solution	2
• Finds an expression for z in terms of $\cos$ and $\sin$, or equivalent merit	1

Sample answer:

Let $\alpha_k = \cos\left(\frac{(2k+1)\pi}{6}\right) + i\sin\left(\frac{(2k+1)\pi}{6}\right)$ for $k = 0, 1, 2, 3, 4, 5$

From part (ii) $\quad \frac{z-1}{z+1} = \alpha_k$

$$\therefore \quad z - 1 = z\alpha_k + \alpha_k$$

$$z(1 - \alpha_k) = 1 + \alpha_k$$

$$z = \frac{1+\alpha_k}{(1-\alpha_k)}$$

$$= \frac{1 + \cos\left(\frac{(2k+1)\pi}{6}\right) + i\sin\left(\frac{(2k+1)\pi}{6}\right)}{1 - \cos\left(\frac{(2k+1)\pi}{6}\right) - i\sin\left(\frac{(2k+1)\pi}{6}\right)}$$

$$= i\cot\left(\frac{(2k+1)\pi}{2\times 6}\right) \qquad \text{from part (i)}$$

$$= i\cot\left(\frac{(2k+1)\pi}{12}\right) \qquad \text{for } k = 0, 1, 2, 3, 4, 5$$

$$= i\cot\frac{\pi}{12},\ i\cot\frac{3\pi}{12},\ i\cot\frac{5\pi}{12},\ i\cot\frac{7\pi}{12},$$

$$i\cot\frac{9\pi}{12} \text{ and } i\cot\frac{11\pi}{12}$$

Question 16 (a)

Criteria	Marks
• Provides correct solution	4
• States and uses the assumption that at least one solution lies inside the circle	3
• Obtains a geometric series, or equivalent merit	2
• Uses triangle inequality, or equivalent merit	1

Sample answer:

Assume for contradiction that w is a solution to the equation and $|w| \le \frac{1}{2}$.

$$\therefore \quad w^n \cos n\theta + w^{n-1}\cos(n-1)\theta + w^{n-2}\cos(n-2)\theta + \cdots + w\cos\theta = 1$$

$$\therefore \left| w^n \cos n\theta + w^{n-1}\cos(n-1)\theta + w^{n-2}\cos(n-2)\theta + \cdots + w\cos\theta \right| = |1| = 1$$

$$\therefore 1 \le \left| w^n \cos n\theta \right| + \left| w^{n-1}\cos(n-1)\theta \right| + \left| w^{n-2}\cos(n-2)\theta \right| + \cdots + |w\cos\theta| \quad \left(\text{by triangle inequality}\right)$$

$$= |w^n||\cos n\theta| + |w^{n-1}||\cos(n-1)\theta| + |w^{n-2}||\cos(n-2)\theta| + \cdots + |w||\cos\theta| \quad \left(\text{property of modulus}\right)$$

$$\le |w^n| + |w^{n-1}| + |w^{n-2}| + \cdots + |w| \quad (\text{since } |\cos\alpha| \le 1 \ \forall\ \alpha)$$

$$= |w|^n + |w|^{n-1} + |w|^{n-2} + \cdots + |w| \quad (\text{property of modulus})$$

$$\le \left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^{n-1} + \left(\frac{1}{2}\right)^{n-2} + \cdots + \frac{1}{2} \quad (\text{by assumption})$$

$$= \frac{1}{2}\left(\frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}\right) \quad (\text{sum of GP})$$

$$= 1 - \frac{1}{2^n}$$

$$< 1$$

This is a contradiction, so original assumption must be false, and all the solutions lie outside the circle $|z| = \frac{1}{2}$.

Question 16 (b)

Criteria	Marks
• Provides correct solution	5
• Obtains x and y, or equivalent merit	4
• Obtains $(x$ and $\dot{y})$ or $(y$ and $\dot{x})$, or equivalent merit	3
• Obtains x or y or both $\dot{x}$ and $\dot{y}$, or equivalent merit	2
• Obtains $\dot{x}$ or $\dot{y}$, or equivalent merit	1

Sample answer:

Horizontal motion:

$$\ddot{x} = -k\dot{x}$$

$$\frac{d(\dot{x})}{dt} = -k\dot{x}$$

$$\therefore \quad \dot{x} = Ae^{-kt}$$

When $t = 0,\ \dot{x} = 50\cos\theta = 50 \times \frac{4}{5} = 40 \quad \therefore\ A = 40$

$$\therefore \quad \dot{x} = 40e^{-kt}$$

Integrating,

$$\therefore \quad x = -\frac{40}{k}e^{-kt} + C$$

When $t = 0,\ x = 0 \quad \therefore\ C = \frac{40}{k}$

$$\therefore \quad x = -\frac{40}{k}e^{-kt} + \frac{40}{k}$$

$$x = \frac{40}{k}\left(1 - e^{-kt}\right)$$

(∗) Rearranging,

$$1 - \frac{kx}{40} = e^{-kt}$$

$$\therefore \quad e^{kt} = \frac{40}{40 - kx}$$

$$\therefore \quad t = \frac{1}{k}\ln\left(\frac{40}{40 - kx}\right)$$

Question 16 (b) sample answer (continued)

<u>Vertical motion</u>:

$$\ddot{y} = -k\dot{y} - 10$$

$$\frac{d(\dot{y})}{dt} = -k\dot{y} - 10$$

$$\int \frac{d(\dot{y})}{k\dot{y} + 10} = -\int dt$$

$$\therefore \quad t = -\frac{1}{k}\ln\left|(k\dot{y} + 10)\right| + C$$

$$Ae^{-kt} = k\dot{y} + 10$$

When $t = 0$, $\dot{y} = -50 \times \sin\theta$

$$= -50 \times \frac{3}{5}$$

$$= -30$$

$$\therefore \quad A = -30k + 10$$

$$\therefore \quad \dot{y} = \frac{1}{k}\left\{(10 - 30k)e^{-kt} - 10\right\}$$

$$= \frac{-10}{k}\left\{(3k - 1)e^{-kt} + 1\right\}$$

Integrating,

$$y = \frac{-10}{k}\left\{(3k - 1)\frac{e^{-kt}}{-k} + t\right\} + C$$

$$= \frac{10}{k}\left\{\frac{(3k - 1)e^{-kt}}{k} - t\right\} + C$$

When $t = 0$, $y = 0$

$$\therefore \quad C = \frac{-10}{k^2}(3k - 1)$$

$$\therefore \quad y = \frac{10}{k}\left\{\frac{(3k - 1)}{k}e^{-kt} - t - \frac{(3k - 1)}{k}\right\}$$

$$= \frac{10}{k}\left\{\frac{(3k - 1)}{k}\left[e^{-kt} - 1\right] - t\right\}$$

Question 16 (b) sample answer (continued)

Using ⊛, $e^{-kt} - 1 = \dfrac{-kx}{40}$

and $t = \dfrac{1}{k}\ln\left(\dfrac{40}{40 - kx}\right)$

$$\therefore \quad y = \frac{10}{k}\left\{\frac{(3k-1)}{k}\left(\frac{-kx}{40}\right) - \frac{1}{k}\ln\left(\frac{40}{40-kx}\right)\right\}$$

$$= \frac{10}{k^2}\left\{\frac{(1-3k)kx}{40} + \ln\left(\frac{40-kx}{40}\right)\right\}$$

Question 16 (c) (i)

Criteria	Marks
• Provides correct solution	2
• Finds an expression for distance, or equivalent merit	1

Sample answer:

The distance is minimised when the squared distance is minimised. The squared distance is

$$\begin{aligned} g(\lambda) &= \left| \underset{\sim}{b} - \left(\underset{\sim}{a} + \lambda \underset{\sim}{d}\right) \right|^2 \\ &= \left| \left(\underset{\sim}{b} - \underset{\sim}{a}\right) - \lambda \underset{\sim}{d} \right|^2 \\ &= \left(\left(\underset{\sim}{b} - \underset{\sim}{a}\right) - \lambda \underset{\sim}{d}\right) \cdot \left(\left(\underset{\sim}{b} - \underset{\sim}{a}\right) - \lambda \underset{\sim}{d}\right) \\ &= \left| \underset{\sim}{b} - \underset{\sim}{a} \right|^2 - 2\left(\underset{\sim}{b} - \underset{\sim}{a}\right) \cdot \underset{\sim}{d}\lambda + \lambda^2 \qquad \text{since, } \underset{\sim}{d} \cdot \underset{\sim}{d} = \left|\underset{\sim}{d}\right|^2 = 1 \end{aligned}$$

This is a concave-up parabola. The smallest value occurs on the axis of symmetry.

$$\begin{aligned} \lambda_0 &= \frac{2\left(\underset{\sim}{b} - \underset{\sim}{a}\right) \cdot \underset{\sim}{d}}{2} \\ &= \left(\underset{\sim}{b} - \underset{\sim}{a}\right) \cdot \underset{\sim}{d} \end{aligned}$$

Question 16 (c) (ii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

The direction of line ℓ is $\underset{\sim}{d}$.

$$\begin{aligned} \overrightarrow{PB} &= \underset{\sim}{b} - \left(\underset{\sim}{a} + \lambda_0 \underset{\sim}{d}\right) \\ &= \left(\underset{\sim}{b} - \underset{\sim}{a}\right) - \lambda_0 \underset{\sim}{d} \end{aligned}$$

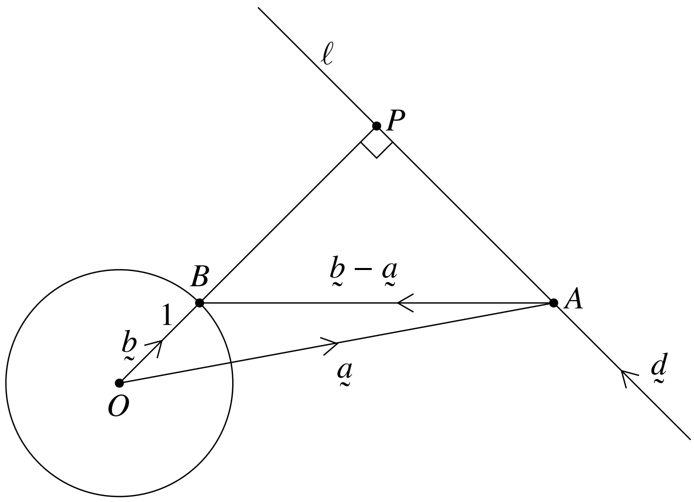
Hence
$$\begin{aligned} \overrightarrow{PB} \cdot \underset{\sim}{d} &= \left(\left(\underset{\sim}{b} - \underset{\sim}{a}\right) - \lambda_0 \underset{\sim}{d}\right) \cdot \underset{\sim}{d} \\ &= \left(\underset{\sim}{b} - \underset{\sim}{a}\right) \cdot \underset{\sim}{d} - \lambda_0 \underset{\sim}{d} \cdot \underset{\sim}{d} \\ &= \lambda_0 - \lambda_0 \left|\underset{\sim}{d}\right|^2 \\ &= \lambda_0 - \lambda_0 \\ &= 0 \end{aligned}$$

So the vectors are perpendicular.

Question 16 (c) (iii)

Criteria	Marks
• Provides correct solution	3
• Uses Pythagoras' theorem, or equivalent merit	2
• Uses the projection formula, or equivalent merit	1

Sample answer:



Let A be the point on line ℓ with position vector $\underset{\sim}{a}$ and let B be the point on the sphere closest to the line.

$\overrightarrow{BP} \perp \overrightarrow{AP}$ from part (ii)

$\overrightarrow{AP}$ is the projection of $\overrightarrow{AB}$ onto $\underset{\sim}{d}$. So $\overrightarrow{AP} = \dfrac{(\underset{\sim}{b} - \underset{\sim}{a}) \cdot \underset{\sim}{d}}{|\underset{\sim}{d}|^2} \underset{\sim}{d} = \lambda_0 \underset{\sim}{d}$

By Pythagoras,

$$|\overrightarrow{OA}|^2 = |\overrightarrow{OP}|^2 + |\overrightarrow{AP}|^2$$

$$|\underset{\sim}{a}|^2 = \left(1 + |\overrightarrow{BP}|\right)^2 + |\overrightarrow{AP}|^2$$

$$\therefore \left(1 + |\overrightarrow{BP}|\right)^2 = |\underset{\sim}{a}|^2 - |\overrightarrow{AP}|^2$$

$$= |\underset{\sim}{a}|^2 - |\lambda_0 \underset{\sim}{d}|^2$$

$$= |\underset{\sim}{a}|^2 - \lambda_0^2 |\underset{\sim}{d}|^2$$

$$= |\underset{\sim}{a}|^2 - \lambda_0^2$$

Question 16 (c) (iii) sample answer (continued)

Now $\underset{\sim}{b} \cdot \underset{\sim}{d} = 0$ from part (ii) since *OBP* is a straight line. So $\lambda_0 = \underset{\sim}{b} \cdot \underset{\sim}{d} - \underset{\sim}{a} \cdot \underset{\sim}{d} = -\underset{\sim}{a} \cdot \underset{\sim}{d}$.

$$\therefore \quad \left(1 + \left|\overrightarrow{BP}\right|\right)^2 = \left|\underset{\sim}{a}\right|^2 - \left[0 - \left(\underset{\sim}{a} \cdot \underset{\sim}{d}\right)\right]^2$$

$$= \left|\underset{\sim}{a}\right|^2 - \left(\underset{\sim}{a} \cdot \underset{\sim}{d}\right)^2$$

$$1 + \left|\overrightarrow{BP}\right| = \sqrt{\left|\underset{\sim}{a}\right|^2 - \left(\underset{\sim}{a} \cdot \underset{\sim}{d}\right)^2}$$

$$\therefore \quad \left|\overrightarrow{BP}\right| = \sqrt{\left|\underset{\sim}{a}\right|^2 - \left(\underset{\sim}{a} \cdot \underset{\sim}{d}\right)^2} - 1$$

unless the line intersects the sphere, in which case the distance is zero.

2025 HSC Mathematics Extension 2 Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	MEX-V1 Further Work with Vectors	MEX12-3
2	1	MEX-P1 The Nature of Proof	MEX12-8
3	1	MEX-N1 Introduction to Complex Numbers	MEX12-4
4	1	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
5	1	MEX-P1 The Nature of Proof	MEX12-8
6	1	MEX-N2 Using Complex Numbers	MEX12-4
7	1	MEX-N2 Using Complex Numbers	MEX12-4
8	1	MEX-M1 Applications of Calculus to Mechanics	MEX12-7
9	1	MEX-N2 Using Complex Numbers	MEX12-4
10	1	MEX-V1 Further Work with Vectors	MEX12-3

Section II

Question	Marks	Content	Syllabus outcomes
11 (a)	2	MEX-N2 Using Complex Numbers	MEX12-4
11 (b)	2	MEX-N1 Introduction to Complex Numbers	MEX12-4
11 (c) (i)	1	MEX-N1 Introduction to Complex Numbers	MEX12-1
11 (c) (ii)	2	MEX-N1 Introduction to Complex Numbers	MEX12-1
11 (d) (i)	1	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
11 (d) (ii)	1	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
11 (d) (iii)	1	MEX-V1 Further Work with Vectors	MEX12-3
11 (e)	2	MEX-P1 The Nature of Proof	MEX12-2
11 (f)	2	MEX-C1 Further Integration	MEX12-5
12 (a)	3	MEX-C1 Further Integration	MEX12-5
12 (b)	3	MEX-P2 Further Proof by Mathematical Induction	MEX12-2
12 (c)	3	MEX-N2 Using Complex Numbers	MEX12-1
12 (d)	4	MEX-C1 Further Integration	MEX12-5
12 (e)	3	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
13 (a)	3	MEX-C1 Further Integration	MEX12-5
13 (b)	4	MEX-V1 Further Work with Vectors	MEX12-3
13 (c) (i)	2	MEX-P1 The Nature of Proof	MEX12-2
13 (c) (ii)	2	MEX-P1 The Nature of Proof	MEX12-2
13 (d)	4	MEX-C1 Further Integration	MEX12-5
14 (a) (i)	3	MEX-C1 Further Integration	MEX12-5
14 (a) (ii)	1	MEX-C1 Further Integration	MEX12-5
14 (b) (i)	2	MEX-M1 Applications of Calculus to Mechanics	MEX12-6

Question	Marks	Content	Syllabus outcomes
14 (b) (ii)	2	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
14 (c) (i)	1	MEX-N2 Using Complex Numbers	MEX12-4
14 (c) (ii)	2	MEX-N2 Using Complex Numbers	MEX12-4
14 (c) (iii)	1	MEX-N2 Using Complex Numbers	MEX12-4
14 (d)	3	MEX-P1 The Nature of Proof	MEX12-1
15 (a)	4	MEX-V1 Further Work with Vectors	MEX12-3
15 (b)	4	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
15 (c) (i)	3	MEX-N1 Introduction to Complex Numbers	MEX12-1
15 (c) (ii)	2	MEX-N2 Using Complex Numbers	MEX12-1
15 (c) (iii)	2	MEX-N2 Using Complex Numbers	MEX12-4
16 (a)	4	MEX-P1 The Nature of Proof MEX-N1 Introduction to Complex Numbers	MEX12-2, MEX12-4
16 (b)	5	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
16 (c) (i)	2	MEX-V1 Further Work with Vectors	MEX12-3, MEX12-7
16 (c) (ii)	1	MEX-V1 Further Work with Vectors	MEX12-3, MEX12-7
16 (c) (iii)	3	MEX-V1 Further Work with Vectors	MEX12-3, MEX12-7