NSW Education Standards Authority

IMPORTANT:
This resource was developed to support a previous version of the syllabus.
It may contain content that differs from the current syllabus.

2019 HSC Mathematics Extension 2 Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	A
2	A
3	C
4	D
5	B
6	B
7	D
8	C
9	D
10	A

Section II

Question 11 (a) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$z + \overline{w}$

$= 1 + 3i + 2 + i$

$= 3 + 4i$

Question 11 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Multiplies numerator and denominator by the conjugate of w, or equivalent merit	1

Sample answer:

$$\frac{z}{w}$$

$$= \frac{1+3i}{2-i} \times \frac{2+i}{2+i}$$

$$= \frac{(1+3i)(2+i)}{(2-i)(2+i)}$$

$$= \frac{2+6i+i-3}{4-(-1)}$$

$$= \frac{7i-1}{5}$$

$$= \frac{-1}{5} + \frac{7i}{5}$$

Question 11 (b)

Criteria	Marks
• Provides correct solution	3
• Determines the location of the foci and the equations of the directrices, or equivalent merit	2
• Sketches an ellipse, showing correct intercepts, or equivalent merit	1

Sample answer:

$$\frac{x^2}{4} + 4^2 = 1$$

$a = 2$ $\qquad$ $b = 1$

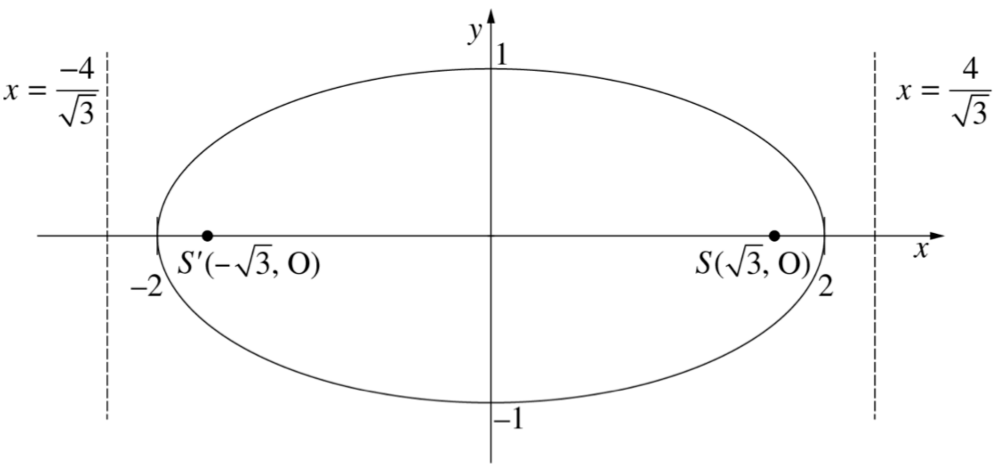
$b^2 = a^2\left(1 - e^2\right)$ $\qquad$ Foci at $x = \pm ae = \pm\sqrt{3}$

$1 = 4\left(1 - e^2\right)$ $\qquad$ directrices $x = \pm\frac{a}{e} = \pm\frac{4}{\sqrt{3}}$

$$1 - e^2 = \frac{1}{4}$$

$$e^2 = \frac{3}{4}$$

$$e = \frac{\sqrt{3}}{2}$$



Question 11 (c)

Criteria	Marks
• Provides correct solution	2
• Completes the square of the denominator, or equivalent merit	1

Sample answer:

$$\int \frac{dx}{x^2 + 10x + 29}$$

$$= \int \frac{dx}{(x+5)^2 + 4}$$

$$= \int \frac{dx}{(x+5)^2 + 2^2}$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{x+5}{2}\right) + c$$

Question 11 (d)

Criteria	Marks
• Provides correct solution	3
• Correctly writes the integrand as the sum of partial fractions, or equivalent merit	2
• Attempts to write the integrand as the sum of partial fractions, or equivalent merit	1

Sample answer:

$$\int \frac{6}{x^2 - 9}\,dx$$

$$\frac{6}{x^2 - 9} = \frac{a}{x+3} + \frac{b}{x-3}$$

$$a(x-3) + b(x+3) = 6$$

Let $x = 3$ and $x = -3$

$b(6) = 6$ $\quad$ $a(-6) = 6$

$\therefore b = 1$ $\quad$ $a = -1$

$$\therefore \int \frac{1}{x-3} - \frac{1}{x+3}\,dx$$

$$= \ln|x-3| - \ln|x+3| + c$$

Question 11 (e) (i)

Criteria	Marks
• Provides correct solution	2
• Obtains the modulus, or equivalent merit	1

Sample answer:

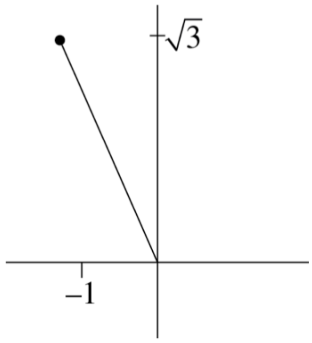
$z = -1 + i\sqrt{3}$

$r = \sqrt{1+3}$ $\qquad \tan\theta = \dfrac{\sqrt{3}}{1}$

$r = \sqrt{4}$ $\qquad \theta = \dfrac{\pi}{3}$

$= 2$ $\qquad \arg(z) = \dfrac{2\pi}{3}$

$z = 2\left(\cos\left(\dfrac{2\pi}{3}\right) + i\sin\left(\dfrac{2\pi}{3}\right)\right)$



Question 11 (e) (ii)

Criteria	Marks
• Provides correct solution	2
• Uses de Moivre’s theorem, or equivalent merit	1

Sample answer:

$$z^3 = 2^3\left(\cos 3\left(\frac{2\pi}{3}\right) + i\sin 3\left(\frac{2\pi}{3}\right)\right)$$

$$= 8(\cos 2\pi + i\sin 2\pi)$$

$$= 8(1 + 0i)$$

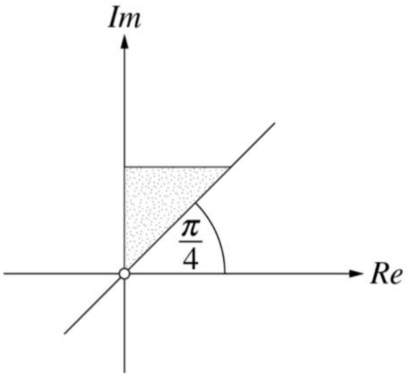
$$= 8 + 0i$$

$$\therefore x = 8,\ y = 0$$

Question 12 (a)

Criteria	Marks
• Provides correct solution	2
• Correctly interprets one condition, or equivalent merit	1

Sample answer:



Question 12 (b) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$x^2 + y^2 - 2xy\cos\frac{2\pi}{3} = 70^2$$

$$x^2 + y^2 - 2xy\left(\frac{-1}{2}\right) = 70^2$$

$$x^2 + y^2 + xy = 70^2$$

Question 12 (b) (ii)

Criteria	Marks
• Provides correct solution	3
• Finds the value of $\frac{dy}{dx}$, or equivalent merit	2
• Implicitly differentiates $x^2 + xy + y^2 = 70^2$ with respect to x, or equivalent merit	1

Sample answer:

$$x^2 + xy + y^2 = 70^2$$

$$\frac{d}{dx}\left(x^2\right) + \frac{d}{dx}(xy) + \frac{d}{dx}\left(y^2\right) = \frac{d}{dx}\left(70^2\right)$$

$$2x + x.\frac{dy}{dx} + y.1 + 2y.\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-(y + 2x)}{x + 2y}$$

$$\text{at} \quad x = 30 \;\; y = 50$$

$$\frac{dy}{dx} = \frac{-(50 + 2(30))}{30 + 2(50)}$$

$$= \frac{-110}{130}$$

$$= \frac{-11}{13}$$

$$\frac{dx}{dt} = \frac{dx}{dy}.\frac{dy}{dt}$$

$$\therefore \frac{dx}{dt} = \frac{-13}{11} \times 4$$

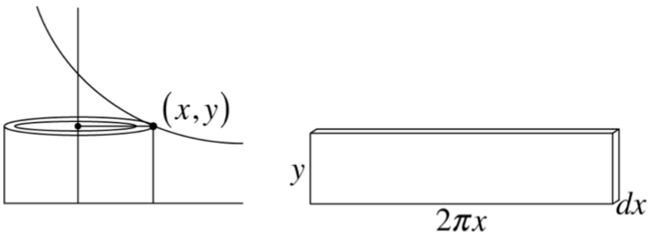
$$= \frac{-52}{11}$$

$$= -4.7$$

Question 12 (c)

Criteria	Marks
• Provides correct solution	4
• Correctly applies integration by parts to an integral for the volume, or equivalent merit	3
• Obtains an integral from which the volume can be evaluated, or equivalent merit	2
• Obtains the volume of a shell, or equivalent merit	1

Sample answer:



$$\text{Volume} = 2A\pi \int_{10}^{40} xe^{-kx}\,dx$$

$$= 2A\pi \left[\left[-\frac{1}{k} xe^{-kx} \right]_{10}^{40} + \frac{1}{k} \int_{10}^{40} e^{-kx}\,dx \right]$$

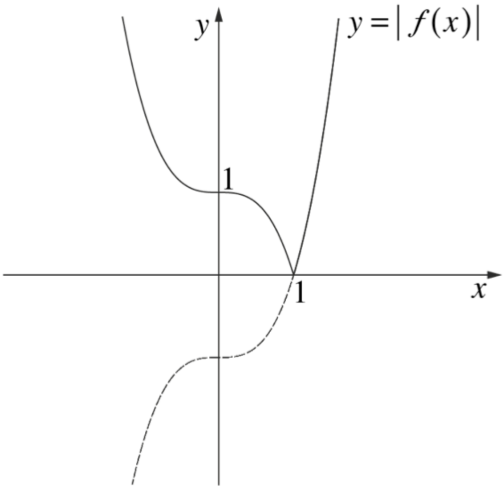
$$= 2A\pi \left[\left[-\frac{1}{k} xe^{-kx} \right]_{10}^{40} + \frac{1}{k} \left[\frac{e^{-kx}}{-k} \right]_{10}^{40} \right]$$

$$= 2A\pi \left[\left(\frac{-40}{k} e^{-40k} + \frac{10}{k} e^{-10k} \right) - \frac{1}{k^2} \left(e^{-40k} - e^{-10k} \right) \right]$$

Question 12 (d) (i)

Criteria	Marks
• Provides correct sketch	1

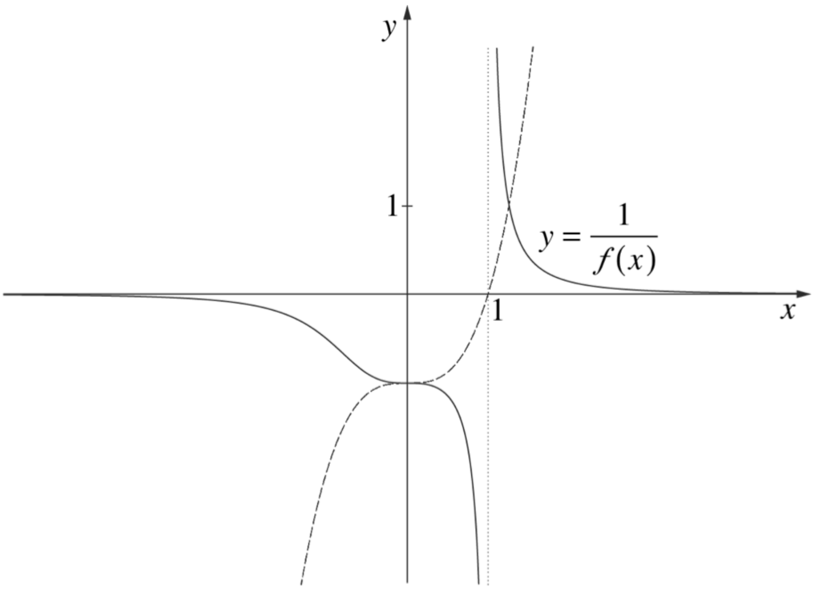
Sample answer:



Question 12 (d) (ii)

Criteria	Marks
• Provides correct sketch	2
• Provides sketch with correct asymptote, or equivalent merit	1

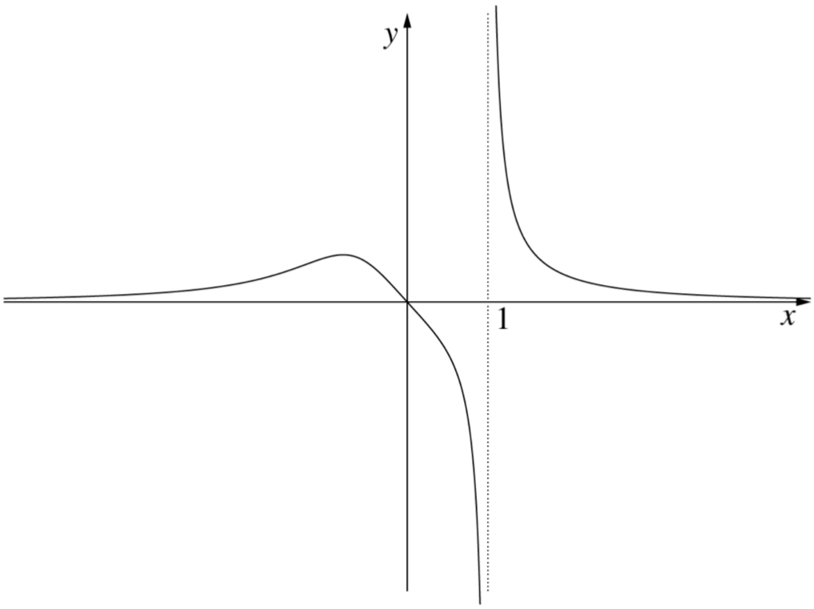
Sample answer:



Question 12 (d) (iii)

Criteria	Marks
• Provides correct sketch	2
• Provides sketch of curve passing through the origin which is not defined at $x = 1$, or equivalent merit	1

Sample answer:



Question 13 (a) (i)

Criteria	Marks
• Provides correct solution	2
• Finds the slope of the hyperbola at P, or equivalent merit	1

Sample answer:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$$\frac{d}{dx}\left(\frac{x^2}{a^2}\right)-\frac{d}{dx}\left(\frac{y^2}{b^2}\right)=\frac{d}{dx}(1)$$

$$\frac{2x}{a^2}-\frac{2y}{b^2}\left(\frac{dy}{dx}\right)=0$$

$$\frac{2x}{a^2}=\frac{2y}{b^2}\left(\frac{dy}{dx}\right)$$

$$\frac{\not{2}xb^2}{\not{2}ya^2}=\frac{dy}{dx}$$

$$\therefore \frac{dy}{dx}=\frac{xb^2}{ya^2}$$

at $P(a\sec\theta, b\tan\theta)$

$$m=\frac{b^2a\sec\theta}{a^2b\tan\theta}$$

$$m=\frac{b\sec\theta}{a\tan\theta}$$

Equation of tangent at P.

$$y-b\tan\theta=\frac{b\sec\theta}{a\tan\theta}(x-a\sec\theta)$$

$$a\tan\theta(y-b\tan\theta)=b\sec\theta(x-a\sec\theta)$$

$$ay\tan\theta-ab\tan^2\theta=bx\sec\theta-ab\sec^2\theta$$

$$ab\sec^2\theta-ab\tan^2\theta=bx\sec\theta-ay\tan\theta$$

$$ab\left(\sec^2\theta-\tan^2\theta\right)=bx\sec\theta-ay\tan\theta$$

$$\therefore ab=bx\sec\theta-ay\tan\theta$$

Question 13 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Attempts to find the point of intersection of the two tangents, or equivalent merit	1

Sample answer:

$bx\sec\theta = ay\tan\theta + ab$ (tangent at P)

$$\therefore \frac{bx\sec\theta - ab}{a\tan\theta} = y$$

$bx\sec\phi = ay\tan\phi + ab$ (tangent at Q)

$$\therefore \frac{bx\sec\phi - ab}{a\tan\phi} = y$$

$$\therefore \frac{bx\sec\theta - ab}{a\tan\theta} = \frac{bx\sec\phi - ab}{a\tan\phi}$$

$$a\tan\phi(bx\sec\theta - ab) = a\tan\theta(bx\sec\phi - ab)$$

$$abx\sec\theta\tan\phi - a^2b\tan\phi = abx\tan\theta\sec\phi - a^2b\tan\theta$$

$$a^2b\tan\theta - a^2b\tan\phi = abx\tan\theta\sec\phi - abx\sec\theta\tan\phi$$

$$a^2b(\tan\theta - \tan\phi) = abx(\tan\theta\sec\phi - \sec\theta\tan\phi)$$

At the point of intersection

$$x_0 = \frac{a(\tan\theta - \tan\phi)}{\tan\theta\sec\phi - \sec\theta\tan\phi}$$

Question 13 (a) (iii)

Criteria	Marks
• Provides correct solution	3
• Finds the slopes of *OM* and *OT*, or equivalent merit	2
• Finds the slope of *OM*, or equivalent merit	1

Sample answer:

Find M.

$$M = \left(\frac{a(\sec\theta + \sec\phi)}{2}, \frac{b(\tan\theta + \tan\phi)}{2} \right)$$

Find gradient of OM.

$$\frac{\dfrac{b(\tan\theta + \tan\phi)}{2}}{\dfrac{a(\sec\theta + \sec\phi)}{2}} = \frac{b(\tan\theta + \tan\phi)}{a(\sec\theta + \sec\phi)}$$

Find gradient of OT.

$$\frac{\dfrac{b(\sec\theta - \sec\phi)}{\sec\phi\tan\theta - \sec\theta\tan\phi}}{\dfrac{a(\tan\theta - \tan\phi)}{\sec\phi\tan\theta - \sec\theta\tan\phi}} = \frac{b(\sec\theta - \sec\phi)}{a(\tan\theta - \tan\phi)}$$

Points will be collinear if the 2 gradients are equal.

$$\text{Gradient of } OM = \frac{b(\tan\theta + \tan\phi)}{a(\sec\theta + \sec\phi)} \times \frac{(\sec\theta - \sec\phi)}{(\sec\theta - \sec\phi)}$$

$$= \frac{b(\tan\theta + \tan\phi)(\sec\theta - \sec\phi)}{a\left(\sec^2\theta - \sec^2\phi\right)}$$

$$= \frac{b}{a}\frac{(\tan\theta + \tan\phi)(\sec\theta - \sec\phi)}{\left(1 + \tan^2\theta - 1 - \tan^2\phi\right)}$$

$$= \frac{b}{a}\frac{(\tan\theta + \tan\phi)(\sec\theta - \sec\phi)}{\left(\tan^2\theta - \tan^2\phi\right)}$$

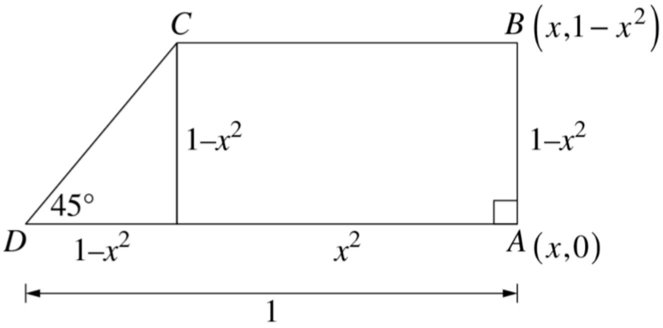
$$= \frac{b}{a}\frac{\cancel{(\tan\theta + \tan\phi)}(\sec\theta - \sec\phi)}{(\tan\theta - \tan\phi)\cancel{(\tan\theta + \tan\phi)}}$$

$$= \frac{b}{a}\frac{(\sec\theta - \sec\phi)}{(\tan\theta - \tan\phi)} = \text{gradient of } OT.$$

Question 13 (b) (i)

Criteria	Marks
• Provides correct solution	2
• Finds the length of CB, or equivalent merit	1

Sample answer:



B lies on the parabola $y = 1 - x^2$ so $|AB| = 1 - x^2$.

D lies on line parallel to x-axis and 1 unit away, known from isosceles triangle of height 1 in the centre of the diagram. So $|AD| = 1$.

$$\begin{aligned}\text{Area of trapezium } ABCD &= \frac{1}{2}(1-x^2)\left[1+1-(1-x^2)\right] \\ &= \frac{1}{2}(1-x^2)(1+x^2) \\ &= \frac{1}{2}(1-x^4)\end{aligned}$$

Question 13 (b) (ii)

Criteria	Marks
• Provides correct volume	1

Sample answer:

$$V = \int_{-1}^{1} \frac{1}{2}\left(1 - x^4\right) dx$$

$$= 2\int_{0}^{1} \frac{1}{2}\left(1 - x^4\right) dx$$

$$= \int_{0}^{1} \left(1 - x^4\right) dx$$

$$= \left[x - \frac{x^5}{5}\right]_0^1 = 1 - \frac{1}{5} = \frac{4}{5}$$

Question 13 (c) (i)

Criteria	Marks
• Provides correct solution	2
• Finds the time at which Object 1 lands, or equivalent merit	1

Sample answer:

For object 1

$$y = -4.9t^2 + 20\sin\frac{\pi}{3}t$$

$$= -4.9t^2 + 10\sqrt{3}t$$

When $y = 0$ $\quad t\left(-4.9t + 10\sqrt{3}\right) = 0$

$$\therefore t = 0 \qquad or \qquad t = \frac{10\sqrt{3}}{4.9}$$

When $t = \dfrac{10\sqrt{3}}{4.9}$

$$x = 20\left(\frac{10\sqrt{3}}{4.9}\right)\cos\frac{\pi}{3}$$

$$= 200\left(\frac{\sqrt{3}}{4.9}\right)\frac{1}{2}$$

$$= \frac{100\sqrt{3}}{4.9}$$

Question 13 (c) (ii)

Criteria	Marks
• Provides correct solution	3
• Obtains two equations for the speed and angle of projection of Object 2, or equivalent merit	2
• Finds the time of flight of Object 2, or equivalent merit	1

Sample answer:

Let V be the initial speed and θ the angle of projection of object 2. Let T be time (in seconds) after projection.

$$y = -4.9T^2 + VT\sin\theta$$

When $y = 0$ $\qquad T(-4.9T + V\sin\theta) = 0$

$$\therefore T = 0 \text{ or } T = \frac{V\sin\theta}{4.9}$$

$$\text{Distance travelled} = V\left(\frac{V\sin\theta}{4.9}\right)\cos\theta$$

$$= \frac{V^2\sin\theta\cos\theta}{4.9}$$

Equating these distances

$$\frac{V^2\sin\theta\cos\theta}{4.9} = \frac{100\sqrt{3}}{4.9}$$

$$V^2\sin\theta\cos\theta = 100\sqrt{3} \qquad ①$$

Equating the times

$$\frac{V\sin\theta}{4.9} = \frac{10\sqrt{3}}{4.9} - 2$$

$$V\sin\theta = 10\sqrt{3} - 9.8 \qquad ②$$

$$V^2\sin\theta\cos\theta = 100\sqrt{3} \qquad ①$$

$$V\sin\theta = 10\sqrt{3} - 9.8 \approx 7.52$$

$$V^2\sin^2\theta = 56.55 \qquad ③$$

$$\tan\theta = \frac{56.55}{100\sqrt{3}} \qquad ③ \div ①$$

$$= 0.326$$

$$\theta = 18.1°$$

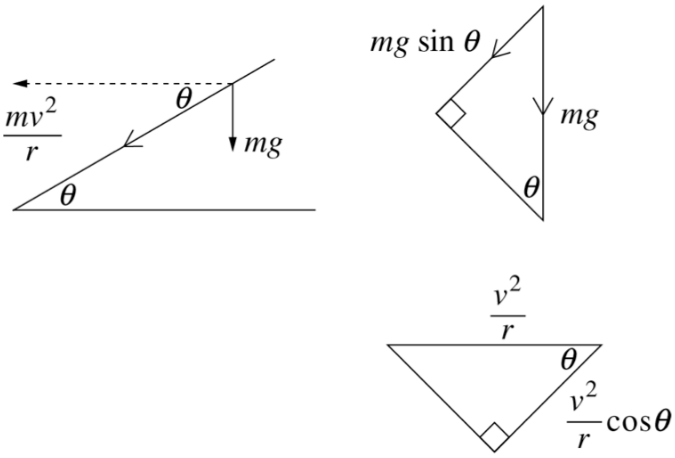
$$V = \frac{7.52}{\sin 18.08°}$$

$$= 24.2$$

Question 14 (a)

Criteria	Marks
• Provides correct solution	3
• Uses the fact that there is no net force parallel to the track to equate forces, or equivalent merit	2
• Correctly resolves forces horizontally and vertically, or equivalent merit	1

Sample answer:



Friction force parallel to slope = mass x acceleration along slope.

$$F + mg\sin\theta = \frac{mv^2}{r}\cos\theta$$

If there are no net forces parallel to track, $F = 0$.

That is,

$$mg\sin\theta = \frac{mv^2}{r}\cos\theta$$

$$\frac{v^2}{r}\cos\theta = g\sin\theta$$

$$v^2 = \frac{gr\sin\theta}{\cos\theta}$$

$$= gr\tan\theta$$

Question 14 (b) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

Terminal velocity is a solution of $\ddot{x} = 0$.

So $0 = g - kv$

$g = kv$

$v = \frac{g}{k}$

So w = terminal velocity = $\frac{g}{k}$.

Question 14 (b) (ii)

Criteria	Marks
• Provides correct solution	4
• Finds the integral and evaluates the constant of integration, or equivalent merit	3
• Finds the integral, or equivalent merit	2
• Obtains $\int \frac{dv}{g-kv} = \int dt$, or equivalent merit	1

Sample answer:

$v = 1.6\,w$ when parachute opens. Let that time be $t = 0$.

$v = 1.6 \times \frac{g}{k}$ from previous part.

$$\ddot{x} = \frac{dv}{dt} = g - kv$$

$$\int \frac{dv}{g-kv} = \int dt$$

$$\frac{-1}{k}\ln|g-kv| = t + c$$

when $t = 0$ $\quad v = \frac{1.6g}{k}$

so $\frac{-1}{k}\ln\left|g - \cancel{k}.\frac{1.6g}{\cancel{k}}\right| = c$

$$\frac{-1}{k}\ln|0.6g| = c$$

so $\frac{-1}{k}\ln|g-kv| = t - \frac{1}{k}\ln|-0.6g|$

Find t when $v = 1.1\,w$.

$$\frac{-1}{k}\ln\left|g - \cancel{k} \times 1.1 \times \frac{g}{\cancel{k}}\right| = t - \frac{1}{k}\ln|-0.6g|$$

$$t = \frac{1}{k}\ln|-0.6g| - \frac{1}{k}\ln|g - 1.1g|$$

$$= \frac{1}{k}\ln\left|\frac{-0.6\cancel{g}}{-0.1\cancel{g}}\right| = \frac{1}{k}\ln 6$$

Question 14 (b) (iii)

Criteria	Marks
• Provides correct solution	3
• Finds an expression for x in terms of t, or equivalent merit	2
• Obtains an expression for v as a function of t, or equivalent merit	1

Sample answer:

$$\frac{-1}{k}\ln\left|g-kv\right| = t - \frac{1}{k}\ln\left|-0.6g\right|$$

$$\ln\left|g-kv\right| = -kt + \ln\left|-0.6g\right|$$

$$\left|g-kv\right| = e^{-kt+\ln\left|-0.6g\right|}$$

$$= \left|-0.6g\right|e^{-kt}$$

$$g-kv = -0.6ge^{-kt}, \text{ since } g-kv<0$$

$$-kv = -0.6ge^{-kt} - g$$

$$v = \frac{0.6}{k}ge^{-kt} + \frac{g}{k}$$

$$v = \frac{g}{k}\left[0.6e^{-kt}+1\right]$$

$$v = \frac{dx}{dt}, \text{ so } x = \int \frac{g}{k}\left[0.6e^{-kt}+1\right]dt$$

$$x = \frac{g}{k}\left[\frac{-1}{k}0.6e^{-kt}+t\right]+c$$

when $x=0$ $t=0$

$$0 = \frac{g}{k}\left[\frac{-1}{k}0.6\right]+c$$

$$c = \frac{g}{k^2}\times 0.6$$

$$\therefore \quad x = \frac{g}{k}\left[-\frac{1}{k}0.6e^{-kt}+t\right]+\frac{g}{k^2}(0.6)$$

when $x = D$, $v = 1.1w$ and $\text{t}=\frac{1}{k}\ln 6$

$$D = \frac{g}{k}\left[-\frac{1}{k}0.6e^{-\ln 6} + \frac{1}{k}\ln 6\right] + \frac{g}{k^2}(0.6)$$

$$= \frac{-g}{k^2}0.6 \times \frac{1}{6} + \frac{g}{k^2}\ln 6 + \frac{g}{k^2}(0.6)$$

$$= \frac{g}{k^2}(0.6 - 0.1 + \ln 6)$$

$$= \frac{g}{k^2}\left(\frac{1}{2} + \ln 6\right)$$

Question 14 (c) (i)

Criteria	Marks
• Provides correct solution	2
• Starting with one side, uses a double-angle result, or equivalent merit	1

Sample answer:

$\cot x - \cot 2x$

$= \dfrac{\cos x}{\sin x} - \dfrac{\cos 2x}{\sin 2x}$

$= \dfrac{\cos x}{\sin x} - \dfrac{\cos^2 - \sin^2 x}{2\sin x \cos x}$

$= \dfrac{2\cos^2 x - \left(\cos^2 x - \sin^2 x\right)}{2\sin x \cos x}$

$= \dfrac{\cos^2 x + \sin^2 x}{2\sin x \cos x} = \dfrac{1}{\sin 2x}$

$= \operatorname{cosec} 2x$

Question 14 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Attempts to use the fact that $2^{k+1} = 2 \times 2^k$ when establishing the inductive step, or equivalent merit	1

Sample answer:

Statement is true when $n = 1$ (from part (i)).

Assume statement is true for $n = k$.

That is $\operatorname{cosec}(2x) + \operatorname{cosec}(4x) + \cdots + \operatorname{cosec}(2^k x)$

$= \cot x - \cot(2^k x)$

Then

$\operatorname{cosec}(2x) + \operatorname{cosec}(4x) + \cdots + \operatorname{cosec}(2^k x) + \operatorname{cosec}(2^{k+1} x)$

$= \cot x - \cot(2^k x) + \operatorname{cosec}(2^{k+1} x)$ (by assumption)

$= \cot x - \cot(2^k x) + \cot(2^k x) - \cot(2^{k+1} x)$

Using part (i)

since $2^{\mathrm{k+1}} x = 2 \times 2^k x$

$= \cot x - \cot(2^{k+1} x)$ as required.

Therefore statement is true for all $n \geq 1$ by induction.

Question 15 (a) (i)

Criteria	Marks
• Provides correct solution	2
• Uses the substitution $u = -x$, or equivalent merit	1

Let $I = \int_{-a}^{a} \frac{f(x)}{f(x) + f(-x)}\, dx$

Let $u = x$ so that $du = -dx$.

$$I = \int_{a}^{-a} \frac{f(-u)}{f(-u) + f(u)}(-du)$$

$$= \int_{-a}^{a} \frac{f(-u)}{f(u) + f(-u)}\, du = \int_{-a}^{a} \frac{f(-x)}{f(x) + f(-x)}\, dx$$

Question 15 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Uses part (i), or equivalent merit	1

Sample answer:

From part (i).

$$\int_{-1}^{1} \frac{e^x}{e^x + e^{-x}}\, dx = \int_{-1}^{1} \frac{e^{-x}}{e^{-x} + e^x}\, dx$$

$$\therefore 2\int_{-1}^{1} \frac{e^x}{e^x + e^{-x}}\, dx = \int_{-1}^{1} \frac{e^x}{e^x + e^{-x}}\, dx + \int_{-1}^{1} \frac{e^{-x}}{e^x + e^{-x}}\, dx$$

$$= \int_{-1}^{1} \frac{e^x + e^{-x}}{e^x + e^{-x}}\, dx$$

$$= \int_{-1}^{1} dx$$

$$= [x]_{-1}^{1} = 2$$

$$\therefore \int_{-1}^{1} \frac{e^x}{e^x + e^{-x}}\, dx = 1$$

Question 15 (b) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

The probability that A wins on the first draw is $\frac{1}{2}$. If A doesn't win on the first draw then A still has a further chance of winning.

$$\therefore P(A \text{ wins}) > \frac{1}{2}$$

$$\therefore P(B \text{ wins}) < \frac{1}{2}$$

Question 15 (b) (ii)

Criteria	Marks
• Provides correct solution	3
• Finds the limiting sum for $P(A)$ or $P(B)$ and forms an inequality, or equivalent merit	2
• Finds an expression for the probability that one player wins, or equivalent merit	1

Sample answer:

$$P(B) = \frac{y}{w+y} \times \frac{y}{w+y} + \frac{y}{w+y} \times \frac{w}{w+y} \times \frac{y}{w+y} \times \frac{y}{w+y} + \cdots$$

$$\text{Limiting sum} = \frac{\dfrac{y^2}{(w+y)^2}}{1 - \dfrac{wy}{(w+y)^2}}$$

$$= \frac{y^2}{(w+y)^2 - wy}$$

For B to have a greater chance,

$$P(B) > \frac{1}{2}$$

$$\therefore \frac{y^2}{(w+y)^2 - wy} > \frac{1}{2}$$

$$2y^2 > w^2 + 2wy + y^2 - wy$$

$$y^2 - wy - w^2 > 0$$

$$\left(\frac{y}{w}\right)^2 - \frac{y}{w} - 1 > 0$$

$$\text{Roots of quadratic are } \frac{1 \pm \sqrt{5}}{2}$$

$$\frac{y}{w} > \frac{1+\sqrt{5}}{2} \quad \text{(Positive root of quadratic).}$$

Question 15 (c) (i)

Criteria	Marks
• Provides correct solution	2
• Finds correct primitive, or equivalent merit	1

Sample answer:

Let $u = x + 1$

$du = dx$

$$\int_0^1 \frac{x}{(x+1)^2}\,dx$$

$$= \int_1^2 \frac{u-1}{u^2}\,du$$

$$= \int_1^2 u^{-1} - u^{-2}\,du$$

$$= \left[\ln u + u^{-1}\right]_1^2$$

$$= \left(\ln 2 + 2^{-1}\right) - (\ln 1 + 1)$$

$$= \ln 2 + \frac{1}{2} - 1$$

$$= \ln 2 - \frac{1}{2}$$

Question 15 (c) (ii)

Criteria	Marks
• Provides correct solution	3
• Completes integration by parts and writes the result in terms of I_n, I_{n-1} and constants, or equivalent merit	2
• Attempts to use integration by parts, or equivalent merit	1

Sample answer:

$$I_n = \int_0^1 \frac{x^n}{(x-1)^2}\,dx$$

$$= \int_0^1 x^n.\frac{d}{dx}\left(\frac{-1}{x+1}\right)dx \qquad \text{(Integrating by parts)}$$

$$= \left[\frac{-x^n}{x+1}\right]_0^1 + n\int_0^1 \frac{x^{n-1}}{x+1}\,dx$$

$$= \frac{-1}{2} + n\int_0^1 \frac{x^{n-1}}{(x+1)^2}.(x+1)\,dx$$

$$= \frac{-1}{2} + n\int_0^1 \frac{x^n}{(x+1)^2} + \frac{x^{n-1}}{(x+1)^2}\,dx$$

$$\therefore I_n = -\frac{1}{2} + nI_n + nI_{n-1}$$

$$\therefore I_n = \frac{1}{2(n-1)} - \frac{n}{n-1}I_{n-1}$$

Question 15 (c) (iii)

Criteria	Marks
• Provides correct solution	2
• Finds I_2, or equivalent merit	1

Sample answer:

$$I_1 = \ln 2 - \frac{1}{2}$$

$$I_2 = \frac{1}{2} - 2\left(\ln 2 - \frac{1}{2}\right)$$

$$= \frac{3}{2} - 2\ln 2$$

$$I_3 = \frac{1}{4} - \frac{3}{2}\left(\frac{3}{2} - 2\ln 2\right)$$

$$= 3\ln 2 - 2$$

Question 16 (a) (i)

Criteria	Marks
• Provides correct solution	2
• Obtains a cubic in $\cos\theta$, p, q, or equivalent merit	1

Sample answer:

Let $x = r\cos\theta$

$r = \sqrt{\dfrac{4p}{3}}$ and so $r^3 = \dfrac{4p}{3}r$

$x^3 - px + q = r^3\cos^3\theta - pr\cos\theta + q$

$= \dfrac{4p}{3}r\cos^3\theta - pr\cos\theta + q$

$= \dfrac{pr}{3}\left(4\cos^3\theta - 3\cos\theta\right) + q$

$= \dfrac{pr}{3}\cos 3\theta + q$

$= \dfrac{p\cancel{r}}{3}\left(\dfrac{-4q}{r^2\cancel{r^3}}\right) + q$

$= \dfrac{-\cancel{4}\cancel{p}q}{\cancel{3}}\,\dfrac{\cancel{3}}{\cancel{4}\cancel{p}} + q$

$= 0$

So $r\cos\theta$ is a root of $x^3 - px + q = 0$.

Question 16 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Writes down a cubic with roots $\alpha + 3$, $\beta + 3$, $\gamma + 3$, or equivalent merit	1

Sample answer:

$\alpha + 3,\ \beta + 3,\ \gamma + 3$ will be roots of

$0 = (x-3)^3 + 9(x-3)^2 + 15(x-3) - 17$

$= x^3 - \cancel{9x^2} + 27x - 27 + \cancel{9x^2} - 54x + 81 + 15x - 45 - 17$

$= x^3 - 12x - 8$

Question 16 (a) (iii)

Criteria	Marks
• Provides correct solution	3
• Finds three values for θ, or equivalent merit	2
• Calculates r (from part (i)), or equivalent merit	1

Sample answer:

Use $p = 12, \quad q = -8 \quad$ in (i)

$$r = \sqrt{\frac{4p}{3}} = \sqrt{16} = 4$$

$$\cos 3\theta = \frac{-4(-8)}{4^3} = \frac{1}{2}$$

$$3\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$$

$$\theta = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$$

So $4\cos\frac{\pi}{9}, 4\cos\frac{5\pi}{9}$ and $4\cos\frac{7\pi}{9}$

are roots of $x^3 - 12x - 8 = 0$,

and so the roots of

$x^3 + 9x^2 + 15x - 17 = 0$

are $4\cos\frac{\pi}{9} - 3,\ 4\cos\frac{5\pi}{9} - 3,\ 4\cos\frac{7\pi}{9} - 3$

Question 16 (b) (i)

Criteria	Marks
• Provides correct solution	3
• Simplifies the sum of the products of pairs of roots, or equivalent merit	2
• Finds the sum of the roots, or equivalent merit	1

Sample answer:

$$\alpha + \overline{\alpha} + \beta + \overline{\beta} = 2k$$

$$2\mathrm{Re}(\alpha) + 2\mathrm{Re}(\beta) = 2k \qquad ①$$

$$\alpha\overline{\alpha} + \alpha\beta + \alpha\overline{\beta} + \overline{\alpha}\beta + \overline{\alpha}\overline{\beta} + \beta\overline{\beta} = 2k^2$$

$$\alpha\left(\beta + \overline{\beta}\right) + \overline{\alpha}\left(\beta + \overline{\beta}\right) = 2k^2 - 2$$

$$2\alpha\,\mathrm{Re}(\beta) + 2\overline{\alpha}\,\mathrm{Re}(\beta) = 2k^2 - 2$$

$$2\mathrm{Re}(\beta)(\alpha + \overline{\alpha}) = 2k^2 - 2$$

$$4\mathrm{Re}(\alpha)\mathrm{Re}(\beta) = 2k^2 - 2$$

$$2\mathrm{Re}(\alpha)\mathrm{Re}(\beta) = k^2 - 1$$

From ①

$$2\mathrm{Re}(\alpha) + 2\mathrm{Re}(\beta) = 2k$$

$$\mathrm{Re}(\alpha) + \mathrm{Re}(\beta) = k$$

$$\mathrm{Re}(\alpha)^2 + 2\mathrm{Re}(\alpha)\mathrm{Re}(\beta) + \mathrm{Re}(\beta)^2 = k^2$$

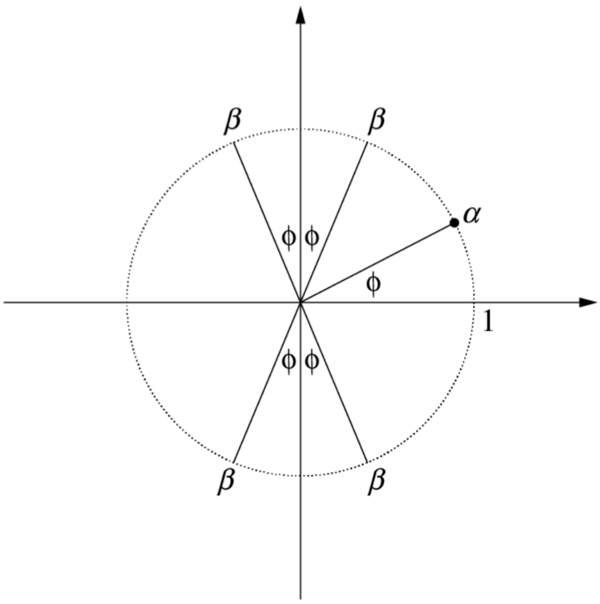
$$\mathrm{Re}(\alpha)^2 + k^2 - 1 + \mathrm{Re}(\beta)^2 = k^2$$

So $\mathrm{Re}(\alpha)^2 + \mathrm{Re}(\beta)^2 = 1$

Question 16 (b) (ii)

Criteria	Marks
• Provides correct sketch	2
• Finds one correct possibility for β, or equivalent merit	1

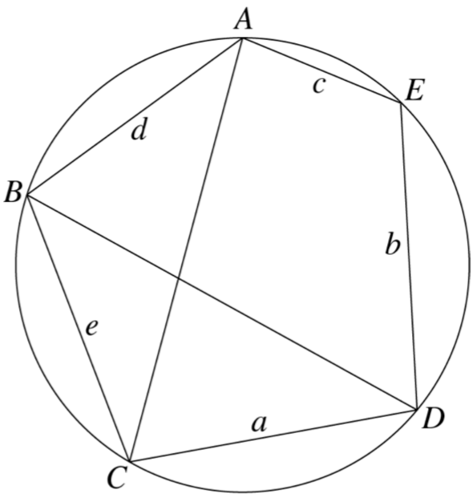
Sample answer:



Question 16 (c)

Criteria	Marks
• Provides correct solution	3
• Shows that $\dfrac{a}{\sin(B+E-\pi)} = \dfrac{d}{\sin(C+E-\pi)}$, or equivalent merit	2
• Shows that $\dfrac{a}{\sin DBC} = \dfrac{d}{\sin ACB}$, or equivalent merit	1

Sample answer:



Sine rule in $\triangle ABC$

$$\frac{d}{\sin ACB} = \frac{e}{\sin BAC}$$

in $\triangle DBC$

$$\frac{a}{\sin DBC} = \frac{e}{\sin BDC}$$

But $\angle BDC$ and $\angle BAC$ stand on same chord and so are equal.

$$\frac{a}{\sin DBC} = \frac{e}{\sin BDC} = \frac{e}{\sin BAC} = \frac{d}{\sin ACB}$$

$ABDE$ is cyclic so

$$E + ABD = \pi$$

$$\begin{aligned} \text{but } B &= ABD + DBC \\ &= \pi - E + DBC \\ DBC &= B + E - \pi \\ \sin DBC &= \sin(B + E - \pi) \\ &= -\sin(B + E) \end{aligned}$$

similarly $ACB = (C + E - \pi)$ $\quad (C - ACD$, and $ACD = ABD = \pi - E)$

$$\sin ACB = -\sin(C+E)$$

$$\frac{a}{\sin DBC} = \frac{d}{\sin ACB}$$

$$-\frac{a}{\sin(B+E)} = -\frac{d}{\sin(C+E)}$$

Hence $\dfrac{a}{\sin(B+E)} = \dfrac{d}{\sin(C+E)}$

2019 HSC Mathematics Extension 2 Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	2.1	E3
2	1	4.1	E8
3	1	4.1	E8
4	1	7.2	E4
5	1	2.5, 3.1	E3
6	1	1.7	E6
7	1	1.2, 1.6, 4.1	E6
8	1	2.2	E3
9	1	1.5, 8	E6
10	1	8	E2

Section II

Question	Marks	Content	Syllabus outcomes
11 (a) (i)	1	2.1	E3
11 (a) (ii)	2	2.1	E3
11 (b)	3	3.1	E3
11 (c)	2	4.1	E8
11 (d)	3	4.1	E8
11 (e) (i)	2	2.2	E3
11 (e) (ii)	2	2.2, 2.4	E3
12 (a)	2	2.5	E3
12 (b) (i)	1	8	E2
12 (b) (ii)	3	1.8, 8	E6
12 (c)	4	4.1, 5.1	E7, E8
12 (d) (i)	1	1.3	E6
12 (d) (ii)	2	1.5	E6
12 (d) (iii)	2	1.5	E6
13 (a) (i)	2	3.2	E3, E4
13 (a) (ii)	2	3.2	E4
13 (a) (iii)	3	3.2	E4
13 (b) (i)	2	5.1	E8
13 (b) (ii)	1	5.1	E8
13 (c) (i)	2	6.1	E5
13 (c) (ii)	3	6.1	E5
14 (a)	3	6.3.4	E5
14 (b) (i)	1	6.2.3	E5

Question	Marks	Content	Syllabus outcomes
14 (b) (ii)	4	6.2.3	E5
14 (b) (iii)	3	6.2.3	E5
14 (c) (i)	2	8	E2, E9
14 (c) (ii)	2	8.2	E2, E9
15 (a) (i)	2	4.1	E8
15 (a) (ii)	2	4.1	E8
15 (b) (i)	1	8	E9
15 (b) (ii)	3	8	E9
15 (c) (i)	2	4.1	E8
15 (c) (ii)	3	4.1	E8
15 (c) (iii)	2	4.1	E8
16 (a) (i)	2	7.5	E4
16 (a) (ii)	2	7.5	E4
16 (a) (iii)	3	7.5	E4
16 (b) (i)	3	2	E3, E4
16 (b) (ii)	2	2	E3
16 (c)	3	8.1	E9