IMPORTANT:
This resource was developed to support a previous version of the syllabus.
It may contain content that differs from the current syllabus.

2019 HSC Mathematics Extension 1 Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	A
2	D
3	C
4	B
5	A
6	B
7	C
8	A
9	D
10	A

Section II

Question 11 (a)

Criteria	Marks
• Provides correct solution	2
• Finds the slope of $x-2y+1=0$, or equivalent merit	1

Sample answer:

$$x-2y+1=0 \qquad y=3x-4$$

$$m_1=\frac{1}{2} \qquad m_2=3$$

$$\tan\theta=\left|\frac{3-\frac{1}{2}}{1+3\left(\frac{1}{2}\right)}\right|$$

$$=\frac{5}{2}\div\frac{5}{2}$$

$$\tan\theta=1$$

$$\therefore \quad \theta=\frac{\pi}{4}$$

Question 11 (b)

Criteria	Marks
• Provides correct solution	3
• Factorises a relevant quadratic, or equivalent merit	2
• Observes that $x \neq -1$, or equivalent merit	1

Sample answer:

$$\frac{x}{x+1} < 2$$

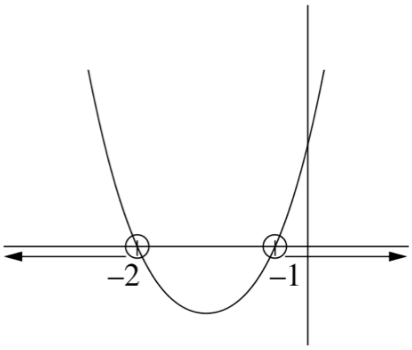
$$(x+1)^2\left(\frac{x}{x+1}\right) < 2(x+1)^2$$

$$x(x+1) < 2(x+1)^2$$

$$0 < 2(x+1)^2 - x(x+1)$$

$$0 < (x+1)[2(x+1) - x]$$

$$0 < (x+1)(x+2)$$

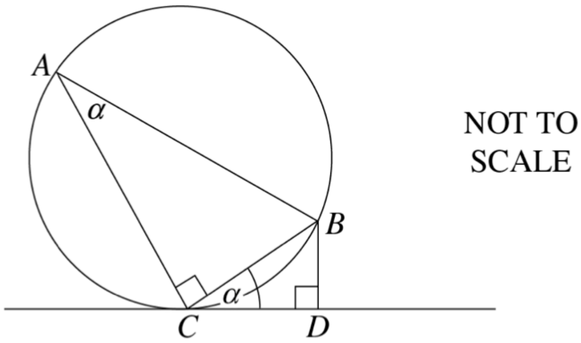


$$x < -2,\ x > -1$$

Question 11 (c)

Criteria	Marks
• Provides correct solution	3
• Shows two pairs of angles are equal with correct reasons, or equivalent merit	2
• Observes that $\angle ACB$ is a right angle, or equivalent merit	1

Sample answer:



Let $\angle BCD = \alpha$

$\therefore \angle CAB = \alpha$ (angle in alt segment)

$\angle ACB = 90°$ (angle in a semi-circle)

$\therefore \angle ABC = 90 - \alpha$ (angle sum $\triangle$)

$\angle CBD = 90 - \alpha$ (angle sum $\triangle$)

$\therefore \angle ABC = \angle CBD$

Question 11 (d)

Criteria	Marks
• Provides correct solution	2
• Attempts long division of relevant polynomials, or equivalent merit	1

Sample answer:

$$\begin{array}{r} x^2+4x+5 \\ x-2\overline{\smash{\big)}\,x^3+2x^2-3x-7} \\ \underline{x^3-2x^2} \\ 4x^2-3x \\ \underline{4x^2-8x} \\ 5x-7 \\ \underline{5x-10} \\ 3 \end{array}$$

$$x^3+2x^2-3x-7=(x-2)\left(x^2+4x+5\right)+3$$

$$\therefore\ Q(x)=x^2+4x+5$$

Question 11 (e)

Criteria	Marks
• Provides correct solution	2
• Correctly uses double angle result, or equivalent merit	1

Sample answer:

$$\int 2\sin^2 4x\,dx$$

$$=\int 2\left(\frac{1-\cos 2(4x)}{2}\right)dx$$

$$=\int 1-\cos 8x\,dx$$

$$=x-\frac{\sin 8x}{8}+C$$

Question 11 (f) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$p = 0.05 \qquad n = 8$

$q = 0.95$

$$P(0 \text{ wins}) = (0.95)^8$$
$$\doteqdot 0.663$$

Question 11 (f) (ii)

Criteria	Marks
• Provides correct solution	2
• Correctly finds one non-trivial probability, or equivalent merit	1

Sample answer:

$$P(\text{at least } 2) = 1 - P(0 \text{ or } 1)$$
$$= 1 - \left[0.95^8 + {}^8C_1(0.05)^1(0.95)^7\right]$$
$$\doteqdot 0.057$$

Question 12 (a)

Criteria	Marks
• Provides correct solution	3
• Finds $\frac{dA}{dt}$ in terms of B, or equivalent merit	2
• Finds $\frac{dA}{dB}$, or equivalent merit	1

Sample answer:

$$A = 9B^{-1}$$

$$\frac{dA}{dB} = -9B^{-2}$$

$$= \frac{-9}{B^2}$$

$$A = 12$$

$$12 = \frac{9}{B}$$

$$B = 0.75$$

$$\frac{dB}{dt} = 0.2$$

$$\therefore \frac{dA}{dt} = \frac{dA}{dB} \times \frac{dB}{dt}$$

$$= \frac{-9}{(0.75)^2} \times 0.2$$

$$\therefore \frac{dA}{dt} = -3.2$$

Question 12 (b) (i)

Criteria	Marks
• Provides correct solution	2
• Finds R, or equivalent merit	1

Sample answer:

$$R\cos(3t-\alpha) = R\cos 3t\cos\alpha + R\sin 3t\sin\alpha$$

Let $R\cos 3t\cos\alpha + R\sin 3t\sin\alpha = \sqrt{2}\cos 3t + \sqrt{6}\sin 3t$

$\therefore R\cos\alpha = \sqrt{2}$ — ① $\quad R\sin\alpha = \sqrt{6}$ — ②

①^2 + ②^2 $\quad R^2\cos^2\alpha + R^2\sin^2\alpha = (\sqrt{2})^2 + (\sqrt{6})^2$

$$R^2 = 2+6$$

$$R = 2\sqrt{2}$$

$\frac{②}{①} \quad \frac{R\sin\alpha}{R\cos\alpha} = \frac{\sqrt{6}}{\sqrt{2}}$

$$\tan\alpha = \sqrt{3}$$

$$\alpha = \frac{\pi}{3}$$

$$\therefore\ x = 2\sqrt{2}\cos\left(3t - \frac{\pi}{3}\right)$$

Question 12 (b) (ii)

Criteria	Marks
• Provides correct answers	1

Sample answer:

At rest when $x = \pm 2\sqrt{2}$.

Question 12 (b) (iii)

Criteria	Marks
• Provides correct solution	2
• Finds half the maximum speed, or equivalent merit	1

Sample answer:

$$x = 2\sqrt{2}\cos\left(3t - \frac{\pi}{3}\right)$$

$$v = -6\sqrt{2}\sin\left(3t - \frac{\pi}{3}\right)$$

Max speed $= +6\sqrt{2}$

Half max speed $= 3\sqrt{2}$

$$\therefore \quad -\frac{1}{2} = \sin\left(3t - \frac{\pi}{3}\right)$$

$$3t - \frac{\pi}{3} = -\frac{\pi}{6}$$

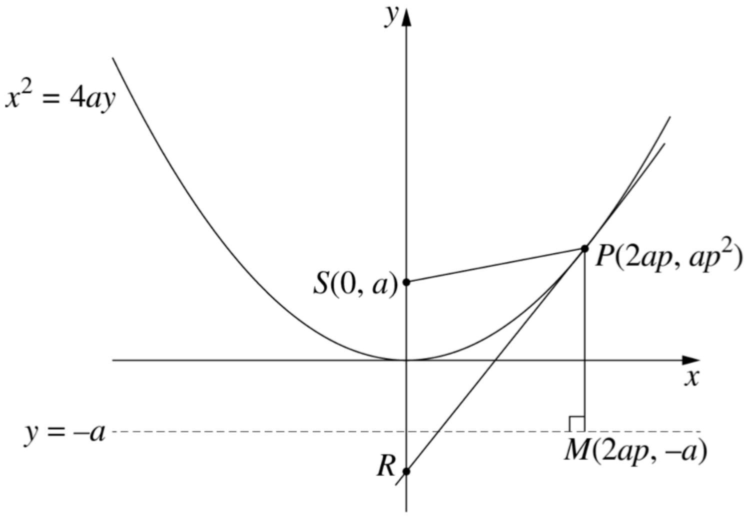
$$3t = \frac{\pi}{6}$$

$$t = \frac{\pi}{18}$$

Question 12 (c)

Criteria	Marks
• Provides correct solution	3
• Finds the length of SP, or equivalent merit	2
• Finds coordinates of R, or equivalent merit	1

Sample answer:



$SP = PM$ $\qquad\qquad$ $PM = ap^2 + a$

Point P is equidistant to $S(0, a)$ and directrix.

$\therefore SP = ap^2 + a$ — ①

tangent at $P \rightarrow y = px - ap^2$

at R $\quad x = 0$

$\therefore y = -ap^2$ ie $R\left(0, -ap^2\right)$

$\therefore SR = a + ap^2$

$\therefore SR = SP$ $\quad$ from ①

$\therefore$ In $\triangle SPR$

$SP = SR$

$\therefore \angle SPR = \angle SRP$ (base $\angle$s of isosceles)

Question 12 (d) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$T = 3 + Ae^{kt} \qquad Ae^{kt} = T - 3$$

$$\frac{dT}{dt} = k.Ae^{kt}$$

$$\therefore \frac{dT}{dt} = k(T-3)$$

Question 12 (d) (ii)

Criteria	Marks
• Provides correct solution	3
• Finds the value of k, or equivalent merit	2
• Finds the value of A, or equivalent merit	1

Sample answer:

$t = 60$

$$\frac{dT}{dt} = ?$$

$t = 0 \quad T = 30$

$$\begin{aligned} T &= 3 + Ae^{kt} \\ 30 &= 3 + Ae^{0} \\ A &= 27 \end{aligned}$$

$t = 15 \quad T = 28$

$$\begin{aligned} 28 &= 3 + 27e^{15k} \\ \frac{25}{27} &= e^{15k} \\ k &= \frac{1}{15}\ln\left(\frac{25}{27}\right) \end{aligned}$$

$t = 60$

$$\begin{aligned} T &= 3 + 27e^{k.60} \\ &= 22.8458\ldots \end{aligned}$$

$\therefore t = 60$

$$\begin{aligned} \frac{dT}{dt} &= \frac{1}{15}\ln\left(\frac{25}{27}\right)(22.8458 - 3) \\ &= -0.1018\ldots \end{aligned}$$

$\therefore$ Temperature is decreasing at 0.102° per min.

Question 13 (a)

Criteria	Marks
• Provides correct solution	3
• Finds correct primitive, or equivalent merit	2
• Finds $\frac{du}{dx}$, or equivalent merit	1

Sample answer:

$$\int_0^{\frac{\pi}{4}} \frac{\sin 2x}{4+\cos^2 x}\,dx$$

let $u = \cos^2 x$

$$\frac{du}{dx} = -2\cos x \sin x$$

$$\therefore \; du = -\sin 2x \, dx$$

when $x = 0$ $u = 1$

when $x = \frac{\pi}{4}$ $u = \frac{1}{2}$

$$\therefore \text{The integral} = \int_1^{\frac{1}{2}} \frac{-du}{4+u}$$

$$= \int_{\frac{1}{2}}^{1} \frac{du}{4+u}$$

$$= \Big[\ln(4+u) \Big]_{\frac{1}{2}}^{1}$$

$$= \ln 5 - \ln\frac{9}{2}$$

$$= \ln\left(\frac{10}{9}\right)$$

Question 13 (b)

Criteria	Marks
• Provides correct solution	3
• Finds a correct equation in k, or equivalent merit	2
• Uses the binomial theorem to find a coefficient, or equivalent merit	1

Sample answer:

$$\text{Coefficient of } x^k = \binom{20}{k} 5^k\, 2^{20-k}$$

$$\text{Coefficient of } x^{k+1} = \binom{20}{k+1} 5^{k+1}\, 2^{19-k}$$

Equating these coefficients,

$$\frac{20!}{k!(20-k)!} 5^k\, 2^{20-k} = \frac{20!}{(k+1)!(19-k)!} 5^{k+1}\, 2^{19-k}$$

$$(k+1)2 = (20-k)5$$

$$2k+2 = 100-5k$$

$$7k = 98$$

$$k = 14$$

Question 13 (c) (i)

Criteria	Marks
• Provides correct solution	2
• Integrates to find $\frac{1}{2}v^2$, or equivalent merit	1

Sample answer:

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -2e^{-x}$$

$$\frac{1}{2}v^2 = \int -2e^{-x}\,dx$$

$$= 2e^{-x} + c$$

when $x = 0 \quad v = 2$

$$\frac{1}{2} \times 2^2 = 2e^0 + c$$

$$\therefore \; c = 0$$

$$\therefore \; v^2 = 4e^{-x}$$

$$\therefore \; v = \pm 2e^{-\frac{x}{2}}$$

We disregard the negative case since it is given that v is always positive

$$\therefore v = 2e^{-\frac{x}{2}}$$

Question 13 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Integrates to find t, or equivalent merit	1

Sample answer:

$$\frac{dx}{dt} = 2e^{-\frac{x}{2}}$$

$$\frac{dt}{dx} = \frac{1}{2}e^{\frac{x}{2}}$$

$$t = \int \frac{1}{2}e^{\frac{x}{2}}\,dx$$

$$= e^{\frac{x}{2}} + C_1$$

when $t = 0 \quad x = 0$

$$0 = e^0 + C_1$$

$$\therefore C_1 = -1$$

$$t = e^{\frac{x}{2}} - 1$$

$$e^{\frac{x}{2}} = t + 1$$

$$x = 2\ln(t+1)$$

Question 13 (d) (i)

Criteria	Marks
• Provides correct solution	2
• Finds the time when the particle lands on the sloping plane, or equivalent merit	1

Sample answer:

We want $y = -x$

$$18\sqrt{3}t - 5t^2 = -18t$$

$$\therefore\ 5t^2 = \left(18\sqrt{3} + 18\right)t$$

$$\therefore t = 0 \quad \text{or} \quad t = \frac{18\left(\sqrt{3}+1\right)}{5}$$

At this time $x = 18.\dfrac{18\left(\sqrt{3}+1\right)}{5}$

$$\therefore OA = \sqrt{x^2 + x^2} = \sqrt{2}\,x$$

$$= \frac{18^2\sqrt{2}\left(\sqrt{3}+1\right)}{5}$$

$$\doteqdot 250.37 \text{ m}$$

Question 13 (d) (ii)

Criteria	Marks
• Provides correct solution	3
• Finds the slope of the path at A, or equivalent merit	2
• Finds a relevant derivative, or equivalent merit	1

Sample answer:

Find the components of the velocity

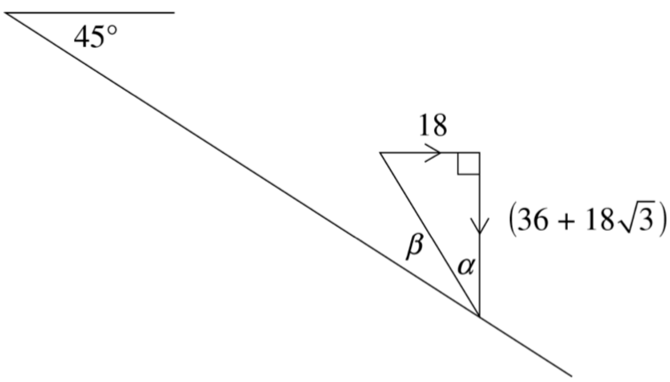
$$\dot{x} = 18$$

$$\dot{y} = 18\sqrt{3} - 10t$$

when $t = \dfrac{18\left(\sqrt{3}+1\right)}{5}$

$$\dot{y} = 18\sqrt{3} - \left(36\sqrt{3} + 36\right)$$

$$= -36 - 18\sqrt{3}$$



$$\tan\alpha = \frac{18}{36 + 18\sqrt{3}}$$

$$= \frac{1}{2+\sqrt{3}}$$

$$\doteqdot 0.2679$$

$$\therefore\ \alpha = 15°$$

$\therefore$ angle with sloping plane is $\beta = 45° - \alpha = 30°$.

Question 14 (a)

Criteria	Marks
• Provides correct solution	3
• Establishes the inductive step, or equivalent merit	2
• Establishes the $n = 1$ case, or equivalent merit	1

Sample answer:

$n = 1$: LHS $= 1(1!) = 1$
RHS $= (1 + 1)! - 1 = 1 =$ LHS

True when $n = 1$

Suppose true for $n = k$

So $1(1!) + 2(2!) + \cdots + k(k!) = (k+1)! - 1$

Want to show it is then true for $n = k + 1$

that is $1(1!) + \cdots + k(k!) + (k+1)\big((k+1)!\big) = \big((k+1)+1\big)! - 1$

$$\begin{aligned}
\text{LHS} &= 1(1!) + \cdots + k(k!) + (k+1)\big((k+1)!\big) \\
&= \big((k+1)! - 1\big) + (k+1)\big((k+1)!\big) \\
&= (k+1)! + (k+1)(k+1)! - 1 \\
&= (k+1)!(1 + k + 1) - 1 \\
&= (k+1)!(k+2) - 1 \\
&= (k+2)! - 1 \\
&= \text{RHS}
\end{aligned}$$

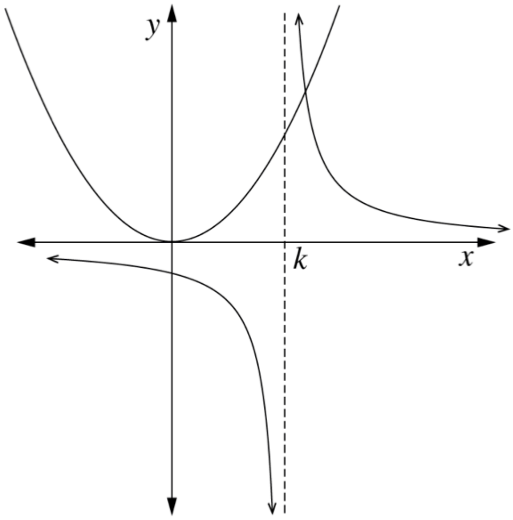
If true for $n = k$, then true for $n = k + 1$

Hence, by mathematical induction, true for $n \geq 1$.

Question 14 (b) (i)

Criteria	Marks
• Provides correct solution	2
• States that the two curves intersect at exactly one point, or equivalent merit	1

Sample answer:



Adding $y = x^2$ to the sketch we can see that there is exactly one point of intersection, so $x^2 = \dfrac{1}{x-k}$ has exactly one solution.

If $f(x) = 0$

then $x^3 - kx^2 - 1 = 0$

$$x^2(x-k) = 1$$

$$x^2 = \frac{1}{x-k}$$

which has exactly one solution.

Question 14 (b) (ii)

Criteria	Marks
• Provides correct solution	2
• Uses Newton's method, or equivalent merit	1

Sample answer:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$f'(x) = 3x^2 - 2kx$$

$$\text{so} \quad x_2 = k - \frac{(\cancel{k^3} - \cancel{k^3} - 1)}{3k^2 - 2k^2}$$

$$= k + \frac{1}{k^2}$$

Question 14 (b) (iii)

Criteria	Marks
• Provides correct solution	3
• Shows $f(x_2) > 0$, or equivalent merit	2
• Shows $f(x_1) < 0$, or equivalent merit	1

Sample answer:

$$
\begin{aligned}
f(x_1) &= f(k) \\
&= k^3 - k^3 - 1 \\
&= -1 \\
&< 0
\end{aligned}
$$

$$
\begin{aligned}
f(x_2) &= \left(k + \frac{1}{k^2}\right)^3 - k\left(k + \frac{1}{k^2}\right)^2 - 1 \\
&= \cancel{k^3} + \cancel{3} + \frac{3}{k^3} + \frac{1}{k^6} - \cancel{k^3} - \cancel{2} - \frac{1}{k^3} - \cancel{1} \\
&= \frac{2}{k^3} + \frac{1}{k^6} \\
&> 0
\end{aligned}
$$

as k is positive.

As $f(x_1) < 0$ and $f(x_2) > 0$ there must be a zero of $f(x)$ between x_1 and x_2.

As α is the only zero of $f(x)$ we have $x_1 < \alpha < x_2$.

Question 14 (c) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

The curves have a common tangent at x_0 and so have the same slope there.

$$y = \sin x$$
$$\frac{dy}{dx} = \cos x$$
$$y = \sin(x - \alpha) + k$$
$$\frac{dy}{dx} = \cos(x - \alpha)$$

and so $\cos x_0 = \cos(x_0 - \alpha)$.

Question 14 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Observes that $\sin x_0 = \sin(x_0 - \alpha) + k$, or equivalent merit	1

Sample answer:

As $\cos x_0 = \cos(x_0 - \alpha)$

We have

$$\begin{aligned}\sin^2 x_0 &= 1 - \cos^2 x_0 \\ &= 1 - \cos^2(x_0 - \alpha) \qquad \text{(from (i))} \\ &= \sin^2(x_0 - \alpha) \\ \sin x_0 &= \pm \sin(x_0 - \alpha)\end{aligned}$$

But at x_0

$$\sin x_0 = \sin(x_0 - \alpha) + k$$

with $\quad k > 0$

so $\quad \sin x_0 \neq \sin(x_0 - \alpha)$

Hence $\quad \sin x_0 = -\sin(x_0 - \alpha)$

Question 14 (c) (iii)

Criteria	Marks
• Provides correct solution	2
• Shows $\sin x_0 = \frac{k}{2}$, or equivalent merit	1

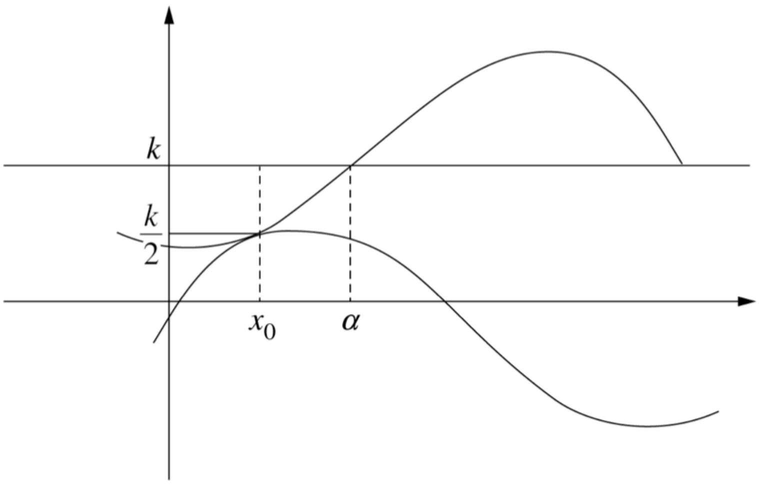
Sample answer:

At x_0 $\quad \sin x_0 = \sin(x_0 - \alpha) + k$

$\sin x_0 = -\sin x_0 + k \qquad$ from (ii)

$2\sin x_0 = k$

$\sin x_0 = \frac{k}{2}$



Now $0 < \alpha < \pi$ and $0 < x_0 < \frac{\pi}{2}$

So $-\pi < x_0 - \alpha < \frac{\pi}{2}$

$\cos(x_0 - \alpha) = \cos x_0$

$\sin(x_0 - \alpha) = -\sin x_0$

Hence $\quad x_0 - \alpha = 2n\pi - x_0$

and so $\quad x_0 - \alpha = -x_0 \qquad as \quad -\pi < x_0 < \frac{\pi}{2}$

$2x_0 = \alpha$

$x_0 = \frac{\alpha}{2}$

Thus $\quad k = 2\sin\frac{\alpha}{2}$

2019 HSC Mathematics Extension 1 Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	4.1, 12.2	HE7
2	1	2.9	PE3
3	1	15.5	HE4
4	1	10.5	HE4
5	1	14.4	HE3
6	1	5.7	PE6
7	1	16.3	PE3
8	1	18.1	PE3
9	1	15.3	HE4
10	1	15.1	HE4

Section II

Question	Marks	Content	Syllabus outcomes
11 (a)	2	6.6	PE6
11 (b)	3	1.4	PE3
11 (c)	3	2.9	PE3
11 (d)	2	16.2	PE3
11 (e)	2	13.6	HE6
11 (f) (i)	1	18.2	HE3
11 (f) (ii)	2	18.2	HE3
12 (a)	3	14.1	HE5
12 (b) (i)	2	5.9	HE3
12 (b) (ii)	1	14.4	HE3
12 (b) (iii)	2	14.4	HE3
12 (c)	3	9.6	PE3, PE4
12 (d) (i)	1	14.2	HE3
12 (d) (ii)	3	14.2	HE3
13 (a)	3	5.9, 11.5	HE6
13 (b)	3	17.3	HE2
13 (c) (i)	2	14.3	HE5
13 (c) (ii)	2	14.3	HE5
13 (d) (i)	2	14.3	HE3
13 (d) (ii)	3	14.3	HE3
14 (a)	3	7.4	HE2
14 (b) (i)	2	4.3	PE3, PE6
14 (b) (ii)	2	16.4	PE3, HE4

Question	Marks	Content	Syllabus outcomes
14 (b) (iii)	3	16.1, 16.4	HE7
14 (c) (i)	1	13.5, 13.7	HE7
14 (c) (ii)	2	13.5, 13.7	HE7
14 (c) (iii)	2	13.5, 13.7	HE7