KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 10

MATHEMATICS

COMMON TEST

JUNE 2022

MARKS:

TIME: **2 hours**

This question paper consists of 10 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 9 questions.

2. Read the questions carefully.

3. Answer ALL the questions.

4. Number your answers exactly as the questions are numbered.

5. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.

6. Answers only will NOT necessarily be awarded full marks.

7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

8. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.

9. Diagrams are NOT necessarily drawn to scale.

10. Write neatly and legibly.

QUESTION 1

1.1 Simplify the following expression fully:

(2)

1.2 Factorise the following expression fully:

$(a+1)^2 - 4b^2$ (2)

1.3 Solve for x :

1.3.1 $(x+m)(x-n)=0$ (2)

1.3.2 $4^x - 4^0 - 255 = 0$ (3)

1.4 Simplify the following:

$$\frac{2^{x+2}.7}{2^{x+4} - 6.2^{x+1}}$$ (4)

[13]

QUESTION 2

2.1 Consider the general term: $T_n = \frac{1}{-2n+5}$

2.1.1 Write down the first TWO terms of the sequence. (2)

2.1.2 Determine which term in the sequence has a value of $-\frac{1}{395}$. (2)

2.2 Consider the following sequence: $5 \; ; \; \frac{7}{4} \; ; \; a \; ; \; b \; ; \; \frac{13}{9} \; ; \; c$

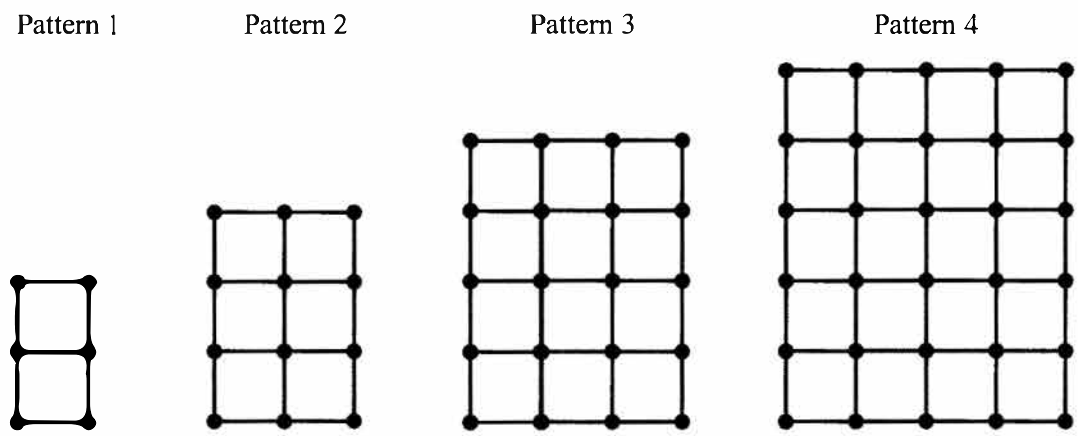
The general term for the sequence is $T_n = \frac{2n+3}{n^2}$.

Determine the values of a , b and c. (3)

2.3 The first four patterns in a sequence are shown below.

Each pattern is made from dots and one-centimetre lines.

The area of each small square is 1 cm^2.



2.3.1 Use the table below and pattern above to determine the values of a, b, c, and d.

Pattern	1	2	3	4	5
Area (cm^2)	2	6	12	20	***a***
Number of dots	6	***b***	20	30	***c***
Number of one centimetre lines	7	17	31	49	***d***

(4)

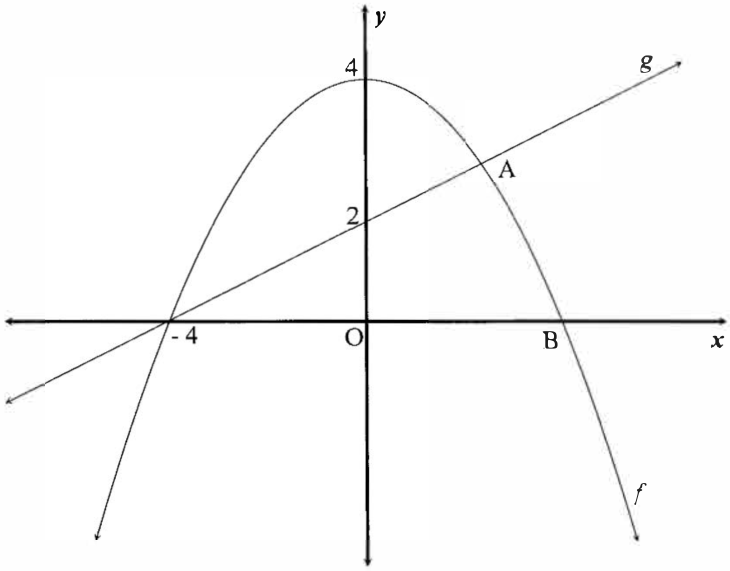
2.3.2 The area of pattern n can be written as $T_n = n(n+1)\,cm^2$.
Find the area of pattern 50. (1)

[12]

QUESTION 3

3.1 The graphs of $f(x) = ax^2 + q$ and $g(x) = mx + c$ are sketched below.

The graph of f intersects the x–axis at (– 4; 0) and B, and the y–axis at (0 ; 4) which is also the turning point of f. One of the points of intersection of f and g is A.



Use the graphs and the information given above to determine:

3.1.1 the equation of g. (2)

3.1.2 the values of a and q. (3)

3.1.3 the coordinates of B. (1)

3.1.4 the domain and range of f. (3)

3.1.5 the equation of $k(x)$, if k is the graph of f reflected about the x–axis. (2)

3.1.6 one value of x for which $f(x) = 0$. (1)

3.1.7 the value(s) of x for which $f(x) > g(x)$, given that A (2 ; 3). (2)

3.2 The function $h(x) = k^x + q$, is described with the following properties:

$k > 0 ; k \neq 1$
$h(0) = -7$
$h(3) = 0$
horizontal asymptote : $y = -8$

Using the information provided, draw a neat sketch graph of $h(x) = k^x + q$ (3)

[17]

QUESTION 4

Given: $f(x)=\frac{12}{x}+3$

4.1 Write down the equations of the asymptotes. (2)

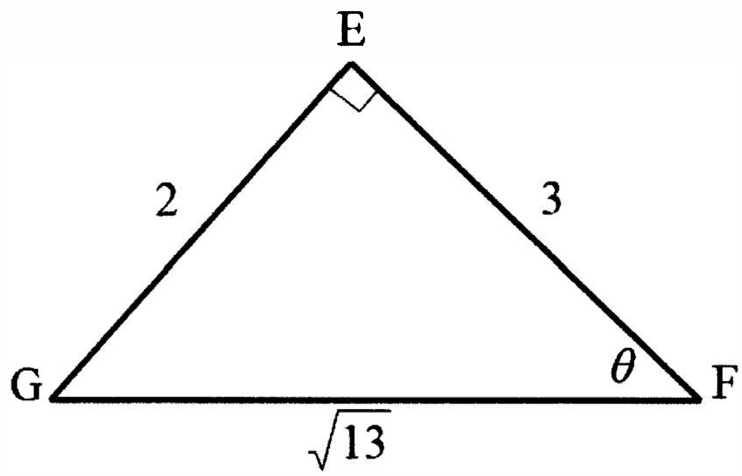
4.2 Determine the equation of g , the axes of symmetry of f, with a positive gradient. (2)

4.3 Sketch the graph of f and g on the same system of axes, indicating all intercepts with the axes and asymptotes. (4)

[8]

QUESTION 5

5.1 In ΔEFG, $\hat{E}=90°$ and $\hat{F}=\theta$. $EG=2$ units , $EF=3$ units and FG=$\sqrt{13}$ units.



Write down the values of:

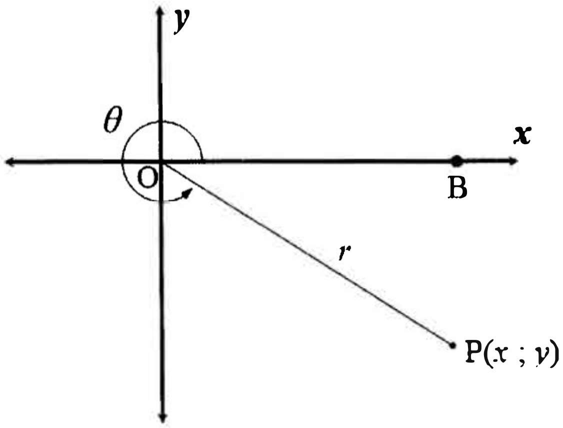
5.1.1 $\tan\theta$ (1)

5.1.2 $\text{cosec}\,\theta$ (1)

5.1.3 $\cos(90°-\theta)$ (2)

5.2 In the diagram below, P(x ; y) is a point in the fourth quadrant.

$B\hat{O}P = \theta$ and $17\cos\theta - 15 = 0$.



Make use of the information provided and the diagram to:

5.2.1 determine the length of OP. (1)

5.2.2 calculate the value of $\tan\theta$. (2)

5.2.3 prove that $\cos^2\theta + \sin^2\theta = 1$ (2)

[9]

QUESTION 6

6.1 If $x = 15°$ and $y = 22,5°$, use your calculator and determine (correct to TWO decimal places) the following:

6.1.1 $\cos^3 x - \sin^2 x$ (2)

6.1.2 $\dfrac{5\sec 2y}{\cot x}$ (2)

6.1.3 $\sqrt{\text{cosec}(y - x)}$ (2)

6.2 Solve for x, correct to **ONE** decimal place, where $0° \le x \le 90°$:

6.2.1 $\tan 2x = 1,01$ (2)

6.2.2 $\dfrac{\sin(2x - 20°)}{3} = 0,099$ (3)

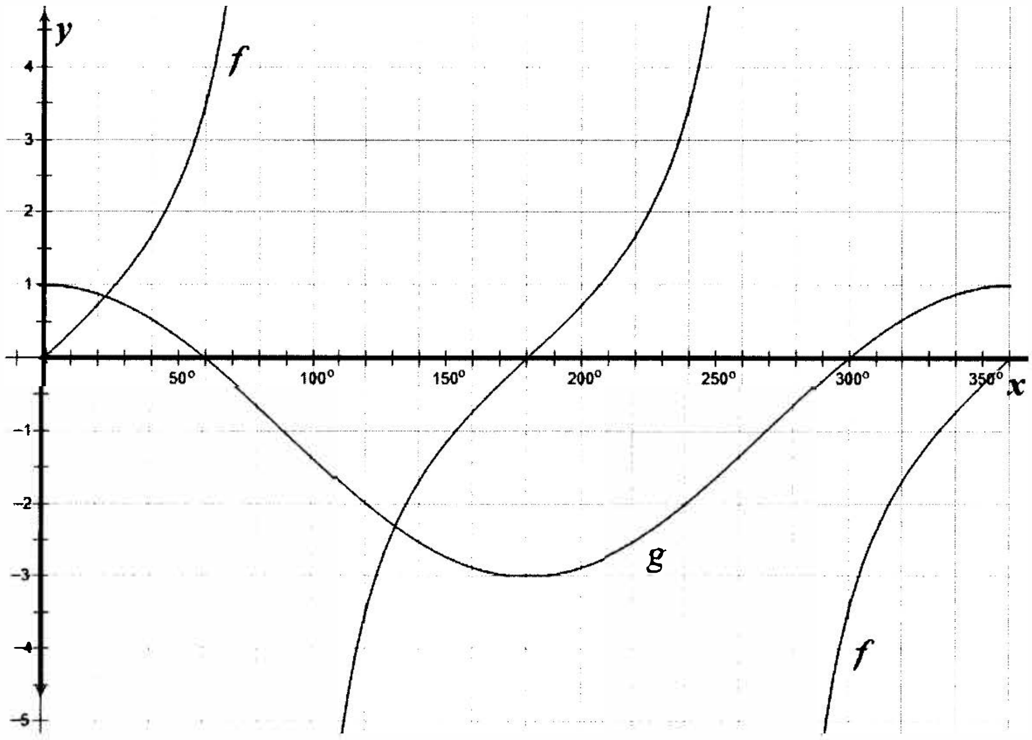
6.3 Without the use of a calculator, showing all working, determine the value of:

$$\frac{\sin 60°.\sec 30° + \tan^2 60°}{2\tan 45°}$$ (5)

[16]

QUESTION 7

The graphs of $f(x) = a\tan x$ *and* $g(x) = 2\cos x + p$ for $x \in [0°;360°]$ are sketched below.



Use the graphs and the information provided to answer the following questions.

7.1 Write down the values of a and p. (2)

7.2 Write down the following:

7.2.1 the amplitude of g. (1)

7.2.2 the range of f. (1)

7.2.3 the period of f. (1)

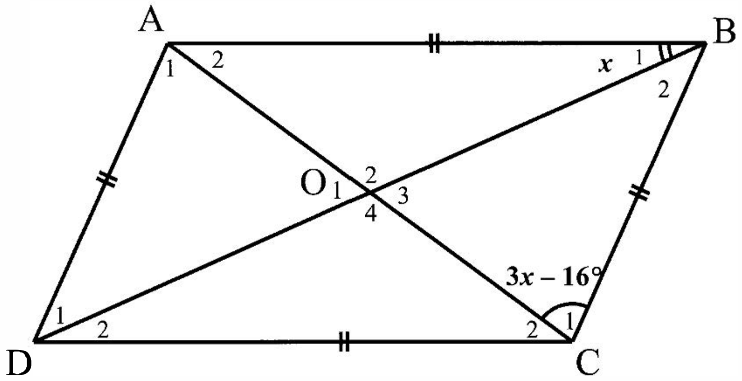
7.3 Use the graphs to determine the number of solutions to $f(x) = g(x)$ in the interval $x \in [0°;180°]$ (1)

[6]

Give reasons for ALL geometry statements in QUESTIONS 8 and 9.

QUESTION 8

8.1 In the diagram below, the diagonals of a rhombus ABCD intersect at O.
$\hat{B}_1 = x$ and $\hat{C}_1 = 3x - 16°$.

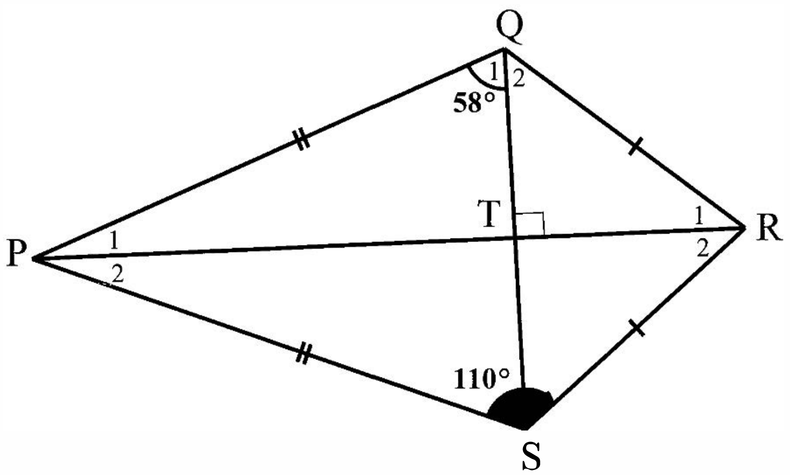


8.1.1 State the value of $\hat{O}_3$. (1)

8.1.2 Calculate the value of x. (3)

8.1.3 Hence, find the value of $\hat{A}_1$. (2)

8.2 In the quadrilateral PQRS below, $\hat{Q}_1 = 58°$ and $P\hat{S}R = 110°$.
$QS \perp PR$, $QR = RS$ and $PQ = PS$.



Use the diagram to calculate, with reasons, the size of:

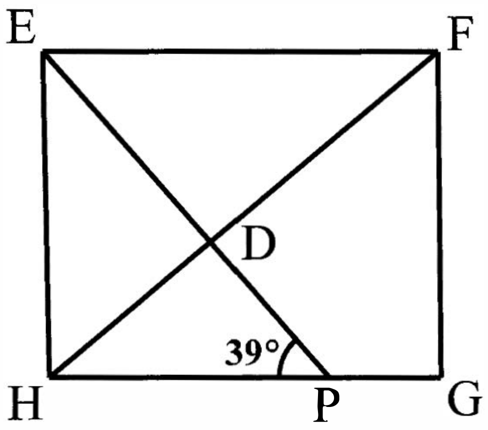
8.2.1 $\hat{P}_1$ (2)

8.2.2 $\hat{R}_2$ (3)

[11]

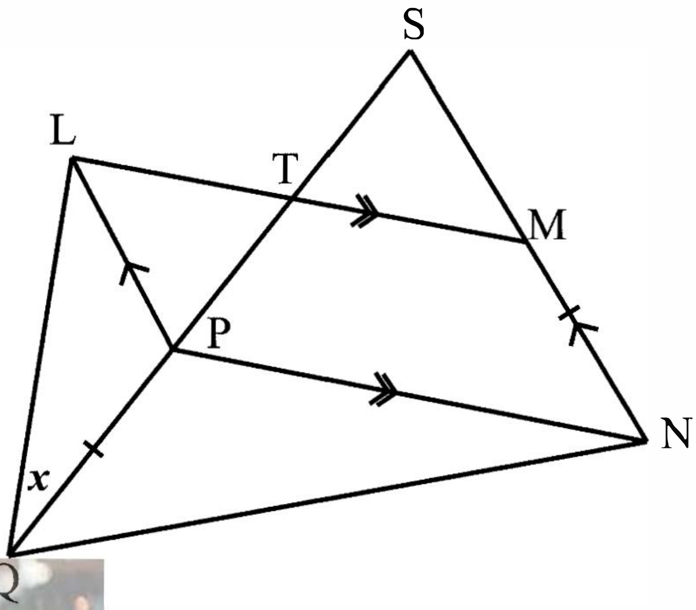
QUESTION 9

9.1 In the diagram below, $EFGH$ is a square and P is a point on HG such that $E\hat{P}H = 39°$. EP and FH intersect at D.

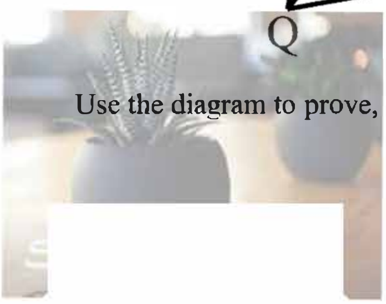


Calculate the size of $E\hat{D}F$. (4)

9.2 In the diagram below $LP \parallel SN, LM \parallel PN$ and $PQ = MN$. Let $L\hat{Q}S = x$.



Use the diagram to prove, with reasons, that $Q\hat{S}N = 2L\hat{Q}S$ (5)



[9]

TOTAL: 100

EDUCATION
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MARKING GUIDELINE

MARKS: 100

TIME: 2 hours

NOTE:

- If a candidate answered a QUESTION TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guidelines.
- Assuming values/answer in order to solve a problem is unacceptable.

This marking guideline consists of 9 pages.

QUESTION 1

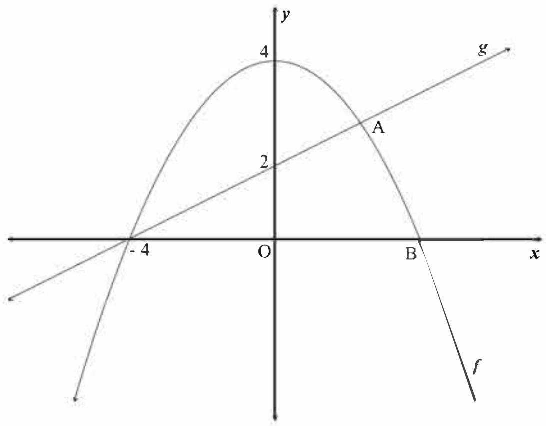
1.1	$\frac{1}{3}x^2y\left(6x-\frac{9}{2}x^{-2}y^2\right)$ $=2x^3y-\frac{3}{2}y^3$	✓ $2x^3y$ ✓ $-\frac{3}{2}y^3$	(2)
1.2	$(a+1)^2-4b^2$ $=[(a+1)-2b][(a+1)+2b]$ $=(a+1-2b)(a+1+2b)$	✓ $(a+1-2b)$ ✓ $(a+1+2b)$	(2)
1.3.1	$(x+m)(x-n)=0$ $x=-m$ or $x=n$	✓ $x=-m$ ✓ $x=n$	(2)
1.3.2	$4^x-4^0-255=0$ $4^x-1-255=0$ $4^x=256$ $2^{2x}=2^8$ $\therefore 2x=8$ $\therefore x=4$	✓ simplification ✓ prime base ✓ answer	(3)
1.4	$\frac{2^{x+2}.7}{2^{x+4}-6.2^{x+1}}$ $=\frac{2^x.2^2.7}{2^x.2^4-6.2^x.2^1}$ $=\frac{2^x.2^2.7}{2^x(2^4-6.2)}$ $=\frac{28}{4}$ $=7$	✓ $2^x.2^2.7$ ✓ $2^x.2^4-6.2^x.2^1$ ✓ common factor: 2^x ✓ answer	(4)
			[13]

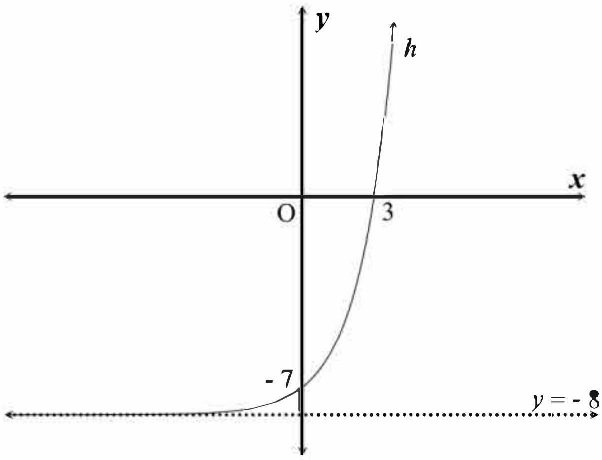
QUESTION 2

2.1.1	$\frac{1}{3};1$	✓ $\frac{1}{3}$ ✓ 1	(2)
2.1.2	$\frac{1}{-2n+5}=-\frac{1}{395}$ $-2n+5=-395$ $-2n=-400$ $n=200$	✓ substitution ✓ answer	(2)

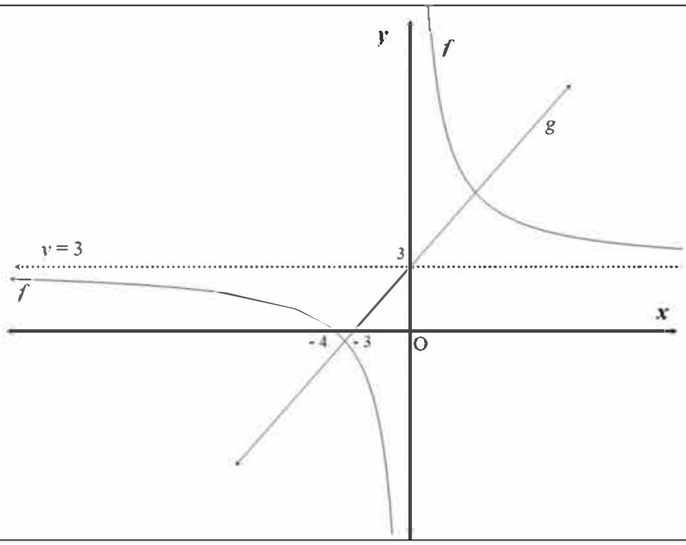
2.2	$T_3 = \frac{2(3)+3}{(3)^2} =$ $\quad a = 1$ $T_4 = \frac{2(4)+3}{(4)^2} =$ $\quad b = \frac{11}{16}$ $T_6 = \frac{2(5)+3}{(5)^2} =$ $\quad c = \frac{13}{25}$	✓ $a = 1$ ✓ $b = \frac{11}{16}$ ✓ $c = \frac{13}{25}$	(3)
2.3.1	2 ; 6 ; 12 ; 20 ; **30** 6 ; **12** ; 20 ; 30 ; **42** 7 ; 17 ; 31 ; 49 ; **71**	✓ $a = 30$ ✓ $b = 12$ ✓ $c = 42$ ✓ $d = 71$	(4)
2.3.2	$T_n = n(n+1)$ $T_{50} = 50(50+1)$ $T_{50} = 2550\,cm^2$	✓ answer	(1)
			[12]

QUESTION 3

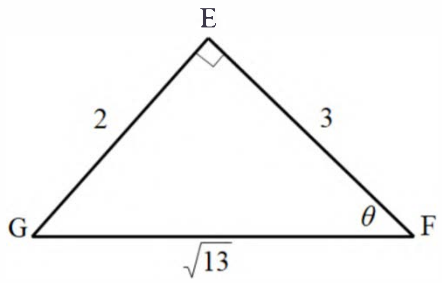
3.1			
3.1.1	$g(x) = \frac{1}{2}x + 2$	✓ gradient ✓ y intercept	(2)
3.1.2	$q = 4$ $y = ax^2 + 4$ $sub\ (-4;0) \rightarrow \quad 0 = a(-4)^2 + 4$ $a = \frac{-4}{16} = -\frac{1}{4}$	✓ $q = 4$ ✓ substitution ✓ $a = -\frac{1}{4}$	(3)
3.1.3	$B(4;0)$	✓ answer	(1)
3.1.4	$y \leq 4$ or $y \in (-\infty; 4]$	✓ values ✓ notation	(2)
3.1.5	$k(x) = \frac{1}{4}x^2 - 4$	✓ $\frac{1}{4}$ ✓ -4	(2)
3.1.6	$x = 4$ or $x = -4$	✓ either value	(1)

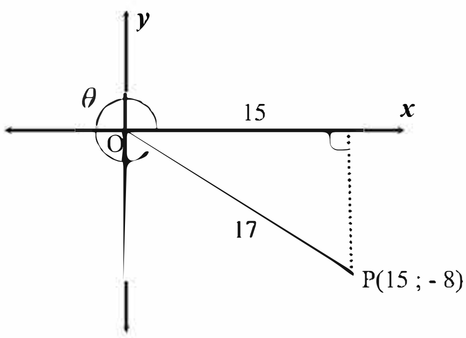
3.1.7	$-4 < x < 2$ *or* $x \in (-4;2)$	✓ values ✓ notation	(2)
3.2		✓ shape: increasing ✓ intercepts ✓ asymptote	(3)
			[16]

QUESTION 4

4.1	$x = 0$ $y = 3$	✓ answer ✓ answer	(2)
4.2	$y = x + c$ *sub* $(0;3) \rightarrow \quad y = x + 3$	✓ gradient = +1 ✓ *y*-intercept: (0;3)	(2)
4.3		*f*: ✓ shape: increasing ✓ *x*-intercept ✓ asymptote *y* = 3 ✓ *g*: *x*- and *y*- intercepts	(4)
			[8]

QUESTION 5

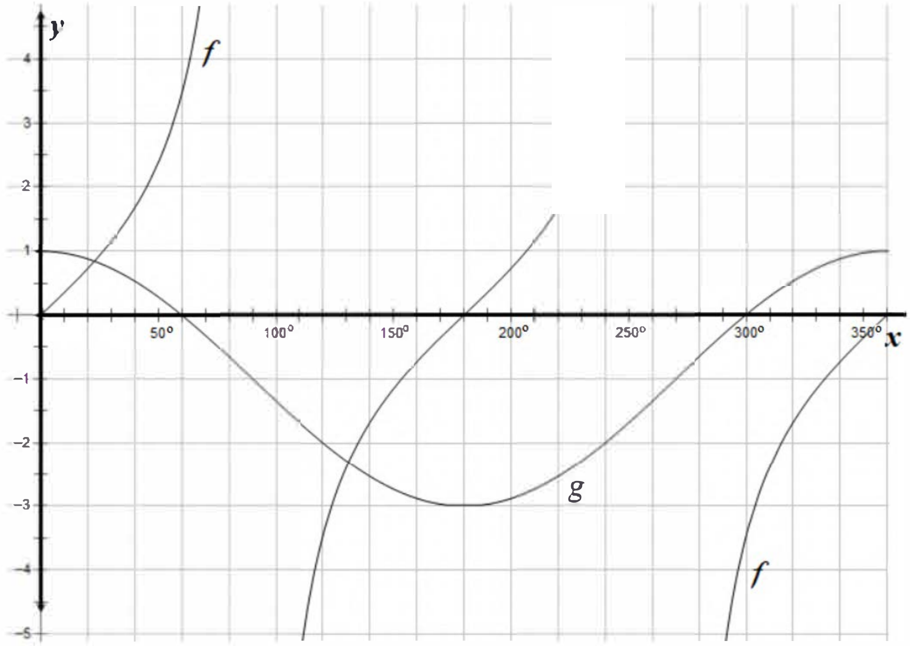


5.1.1	$\tan\theta = \frac{EG}{EF} = \frac{2}{3}$	✓ answer	(1)
5.1.2	$\text{cosec}\,\theta = \frac{GF}{GE} = \frac{\sqrt{13}}{2}$	✓ answer	(1)
5.1.3	$\cos(90° - \theta) = \cos G = \frac{2}{\sqrt{13}}$	✓ answer	(1)
5.2			
5.2.1	$\cos\theta = \frac{15}{17}$ $\therefore OP = 17\ units$	✓ answer	(1)
5.2.2	$y^2 = 17^2 - 15^2$ $y^2 = 64$ $\therefore y = -8$ (4*th* quadrant). $\therefore \tan\theta = \frac{y}{x} = \frac{-8}{15}$	✓ $y = -8$ ✓ answer	(2)
5.2.3	LHS: $\cos^2\theta + \sin^2\theta$ $= \left(\frac{15}{17}\right)^2 + \left(\frac{-8}{17}\right)^2$ $= \frac{289}{289}$ $= 1$ $= RHS$ $\therefore \cos^2\theta + \sin^2\theta = 1$	✓ substitution ✓ $\frac{289}{289} = 1$	(2)
			[8]

QUESTION 6

6.1.1	$\cos^3 x - \sin^2 x$ $= \cos^3 15° - \sin^2 15°$ $= 0,83$ Answer only: full marks	✓ substitution ✓ answer	(2)
6.1.2	$\dfrac{5\sec 2y}{\cot x}$ $= \dfrac{5\sec 2(22,5°)}{\cot 15°}$ $= \dfrac{5\tan 15°}{\cos 2(22,5°)}$ $= 1,89$ Answer only: full marks	✓ substitution ✓ answer	(2)
6.1.3	$\sqrt{\text{cosec}(y-x)}$ $= \sqrt{\dfrac{1}{\sin(22,5° - 15°)}}$ $= 2,77$ Answer only: full marks	✓ substitution ✓ answer	(2)
6.2.1	$\tan 2x = 1,01$ $2x = 45,28505....$ $x = 22,6°$ Penalty for incorrect rounding in this question **only**.	✓ $45,28505...$ ✓ answer	(2)
6.2.2	$\dfrac{\sin(2x - 20°)}{3} = 0,099$ $\sin(2x - 20°) = 0,297$ $2x - 20° = 17,2775...$ $x = 18,6°$	✓ $0,297$ ✓ $7,2775...$ ✓ answer	(3)
6.3	$\dfrac{\sin 60°.\sec 30° + \tan^2 60°}{2\tan 45°}$ $= \dfrac{\frac{\sqrt{3}}{2}.\frac{2}{\sqrt{3}} + (\sqrt{3})^2}{2.1}$ $= \dfrac{1+3}{2}$ $= 2$ Answer only: max. 1/5	✓ $\frac{\sqrt{3}}{2}$ ✓ $\frac{2}{\sqrt{3}}$ ✓ $\sqrt{3}$ ✓ $\tan 45° = 1$ ✓ answer	(5)
			[16]

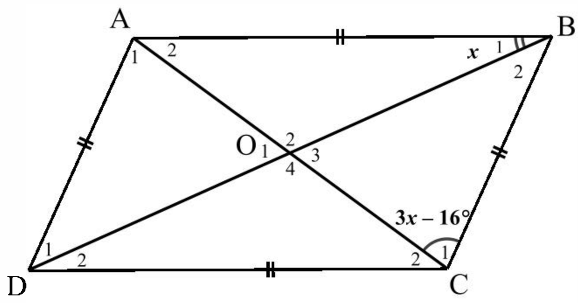
QUESTION 7

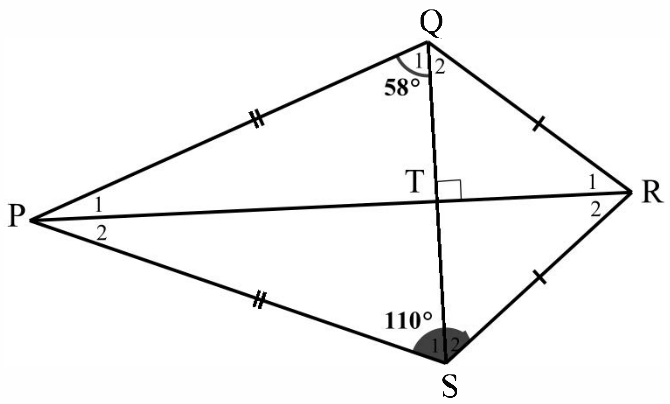


7.1	$a = 2$ $p = -1$	✓ answer ✓ answer	(2)
7.2.1	amplitude of $g = 2$	✓ answer	(1)
7.2.2	$y \in R$ or $y \in (-\infty; \infty)$	✓ answer ✓ answer	(1) (1)
7.2.3	period of $f = 180°$	✓ answer	(1)
7.3	2 solutions	✓ answer	(1)
			[6]

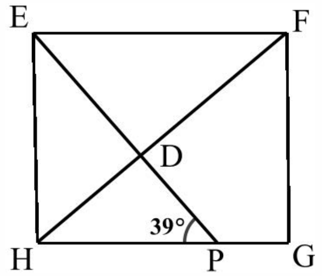
GEOMETRY	
S	**A mark for a correct statement** **(A statement mark is independent of a reason.)**
R	**A mark for a correct reason** **(A reason mark may only be awarded if the statement is correct.)**
S/R	**Award a mark if the statement AND reason are both correct.**

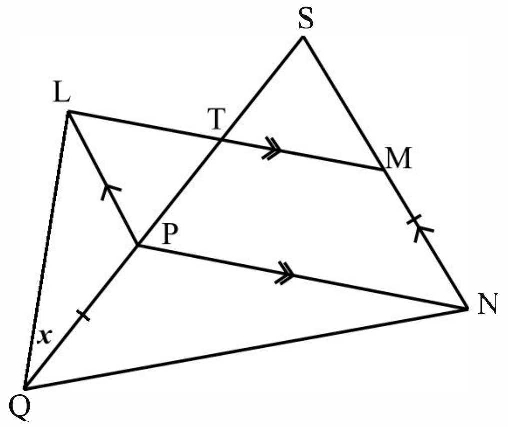
QUESTION 8



8.1.1	$\hat{O}_3 = 90°$ (diags of rhombus)	✓ S	(1)
8.1.2	*In* ΔBOC : $\hat{O}_3 = 90°$ (diags of rhombus) $\hat{B}_1 = \hat{B}_2 = x$ (diags of rhombus) $\hat{C}_1 = 3x - 16°$ (given) $\therefore\ 3x - 16° + x + 90° = 180°$ (sum $\angle$s Δ) $\therefore\ x = 26,5°$	✓ S/R ✓ S/R ✓ S	(3)
8.1.3	$\hat{A}_1 = \hat{C}_1 = 3x - 16°$ (alt $\angle$s; AD // BC) $\therefore \hat{A}_1 = 3(26,5°) - 16° = 63,5°$	✓ S/R ✓ S	(2)
8.2			
8.2.1	*In* ΔPQT : $Q\hat{T}P = 90°$ (diags of kite) $\therefore \hat{P}_1 = 180° - 58° - 90°$ (sum $\angle$s Δ) $\therefore \hat{P}_1 = 32°$	✓ S/R ✓ S/R	(2)
8.2.2	*In* ΔPRS : $P\hat{S}R = 110°$ (given) $\therefore \hat{P}_1 = \hat{P}_2 = 32°$ (diags of kite) $\therefore \hat{R}_2 = 180° - 110° - 32°$ (sum $\angle$s Δ) $\therefore \hat{R}_2 = 38°$	✓ S ✓ R ✓ S/R	(3)
			[11]

QUESTION 9



9.1	In ΔEDF: $F\hat{E}P = 39^\circ$ (alt $\angle$s; EF // HG) $E\hat{F}H = 45^\circ$ (diags of square) $\therefore E\hat{D}F = 180^\circ - 45^\circ - 39^\circ$ (sum $\angle$s Δ) $\therefore E\hat{D}F = 96^\circ$	✓ S ✓ R ✓ S/R ✓ S/R	(4)
			
9.2	*n paralle* log *ram LMNP* $MN = LP$ (*opp sides of parm* *but* $PQ = MN$ $\therefore LP = PQ$ $\therefore Q\hat{L}P = x$ $\therefore L\hat{P}S = 2x$ (*ext* $\angle \Delta LPQ$) $L\hat{P}S = Q\hat{S}N = 2x$ (*alt* $\angle s$; *LP* // *Mn* *and* $L\hat{Q}S = x$ (*given*) $\therefore Q\hat{S}N = 2L\hat{Q}S$	✓ S/R ✓ S ✓ S/R ✓ S ✓ R	(5)
			[9]
		TOTAL:	100