



GAUTENGSE DEPARTEMENT VAN ONDERWYS
PROVINSIALE EKSAMEN
JUNIE 2017
GRAAD 11

WISKUNDE
VRAESTEL 2

MEMORANDUM

10 bladsye

**GAUTENGSE DEPARTEMENT VAN ONDERWYS
PROVINSIALE EKSAMEN****WISKUNDE Vraestel 2****MEMORANDUM**

INSTRUKSIES EN INLIGTING

A – Akkuraatheid

CA – Volgehoue Akkuraatheid

S – Stelling

R – Rede

SR- Stelling/Rede

NOTA:

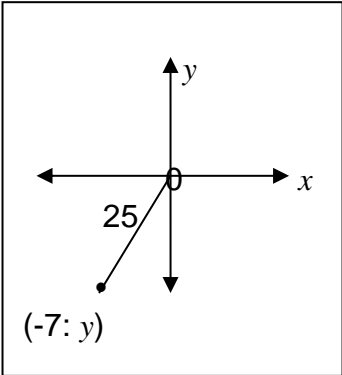
- Indien 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.
- Indien 'n kandidaat 'n antwoord DOODGETREK het, maar dit nie oorgedoen het nie, merk die doodgetrekte antwoord.
- Volgehoue akkuraatheid word in ALLE aspekte van die memorandum toegepas. Hou op nasien by die tweede berekeningsfout.
- Om antwoorde/waardes te veronderstel om 'n probleem op te los, word NIE toegelaat NIE.

VRAAG 1		PUNTE: 25	
1.1	$M_{MR} = \frac{6-0}{4-0}$ $= \frac{6}{4} = \frac{3}{2}$ Vergelyking van lyn MR is: $y = \frac{3}{2}x$	✓ vervang in korrekte formule ✓ $m_{MR} = \frac{3}{2}$ ✓ vergelyking van MR	(3)
1.2	$y - 5x + 14 = 0$ $y = 5x - 14$ MS \parallel PR $\therefore m_{PR} = 5$ Vergelyking van lyn PR: $y - y_1 = m(x - x_1)$ $y - 4 = 5(x + 2)$ $y = 5x + 14$	✓ $m_{MS} = 5$ ✓ $m_{PR} = 5$ ✓ verv. $(-2; 4)$ ✓ antwoord	(4)
1.3	$m_{PR} = 5$ $\therefore \tan \alpha = 5$ $\alpha = 78,69^\circ$ $m_{MR} = \frac{3}{2}$ $\tan \beta = \frac{3}{2}$ $\beta = 56,31^\circ$ $\therefore \theta = \alpha - \beta$ $\theta = 22,38^\circ$	✓ $\tan \alpha = 5$ ✓ $78,69^\circ$ ✓ $56,31^\circ$ ✓ $\therefore \theta = (\alpha - \beta)$..(som van hoeke v. Δ) ✓ $22,38^\circ$	(5)
1.4	$y = \frac{3}{2}x$ en $y = 5x + 14$ $5x + 14 = \frac{3}{2}x$ $10x + 28 = 3x$ $7x = -28$ $x = -4$ $y = -6$ R $(-4; -6)$	✓ stel $5x + 14 = \frac{3}{2}x$ ✓ $7x = -28$ ✓ $x = -4$ ✓ $y = -6$	(4)

1.5	$d_{MR} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(4 + 4)^2 + (6 + 6)^2}$ $= \sqrt{64 + 144}$ $= 4\sqrt{13}$	✓ verv. in korrekte formule ✓ antwoord	(2)
1.6	$d_{PR} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(-4 + 2)^2 + (-6 - 4)^2}$ $= \sqrt{104} \text{ or } 2\sqrt{26}$ <p>Oppvkl van ΔPMR</p> $= \frac{1}{2} PR \cdot MR \cdot \sin \theta$ $= \frac{1}{2} 2\sqrt{26} \cdot 4\sqrt{13} \cdot \sin 22,38^\circ \text{ OF}$ $\frac{1}{2} \sqrt{104} \cdot 4\sqrt{13} \cdot \sin 22,38^\circ$ $= 28 \text{ eenhede}^2$	✓ verv. in korrekte formule ✓ $\sqrt{104}$ OF $2\sqrt{26}$ $\checkmark \frac{1}{2} 2\sqrt{26} \cdot 4\sqrt{13} \cdot \sin 22,38^\circ$ OF $\frac{1}{2} \sqrt{104} \cdot 4\sqrt{13} \cdot \sin 22,38^\circ$ (✓ afronding) antwoord ✓	(5)
1.7	S(2 ; -4)	✓ $x = 2$ ✓ $y = -4$	(2)

VRAAG 2

PUNTE: 26

2.1	$y^2 = r^2 - x^2$ $\therefore y^2 = (25)^2 - (-7)^2$ $\therefore y^2 = 576$ $\therefore y = -24$ $\therefore 14 \tan \theta = 14 \left(\frac{-24}{-7} \right)$ $= 48$		✓ diagram in korrekte kwadrant ✓ $y = -24$ ✓ $\tan \theta = \frac{-24}{-7}$ ✓ antwoord	(4)
2.2	$\frac{\cos(90^\circ + x) \cdot \sin(180^\circ + x)}{\tan 225^\circ - \cos^2(-x)}$ $= \frac{(-\sin x)(-\sin x)}{\tan 45^\circ - \cos^2 x}$ $= \frac{\sin^2 x}{\sin^2 x}$ $= \frac{1 - \cos^2 x}{\sin^2 x}$ $= \frac{\sin^2 x}{\sin^2 x}$ $= 1$	✓ $(-\sin x)(-\sin x)$ ✓ $\tan 45^\circ$ ✓ $\cos^2 x$ ✓ $\tan 45^\circ = 1$ ✓ $1 - \cos^2 x = \sin^2 x$ ✓ antwoord	(6)	

2.3	$2 \cos 2\theta = -0,44$ $\therefore \cos 2\theta = -0,22$ $\therefore \text{Verw. hoek} = 77,2909 \dots^\circ$ $2\theta = 180^\circ - 77,2909 \dots^\circ + k \cdot 360^\circ$ $\therefore \theta = 51,35^\circ + k \cdot 180, k \in Z \text{ or}$ $2\theta = 180^\circ + 77,2909 \dots^\circ + k \cdot 360^\circ$ $\therefore \theta = 128,65^\circ + k \cdot 180^\circ, k \in Z$	$\checkmark \cos 2\theta = -0,22$ $\checkmark 77,2909^\circ$ $\checkmark 2\theta = 180^\circ - 77,2909^\circ + k \cdot 360^\circ$ $\checkmark \theta = 51,35^\circ + k \cdot 180, k \in Z$ $\checkmark 2\theta = 180^\circ + 77,2909^\circ + k \cdot 360^\circ$ $\checkmark \therefore \theta = 128,65^\circ + k \cdot 180^\circ, k \in Z$	(6)
2.4	$\therefore \text{LHS} = \frac{\tan \theta - \sin \theta}{1 - \cos \theta}$ $= \frac{\frac{\sin \theta}{\cos \theta} - \sin \theta}{1 - \cos \theta}$ $= \frac{1 - \cos \theta}{\sin \theta - \sin \theta \cdot \cos \theta}$ $= \frac{1 - \cos \theta}{\cos \theta}$ $= \frac{\sin \theta (1 - \cos \theta)}{\cos \theta} \times \frac{1}{1 - \cos \theta}$ $= \frac{\sin \theta}{\cos \theta}$ $= \tan \theta$	$\checkmark \tan \theta = \frac{\sin \theta}{\cos \theta}$ $\checkmark \frac{\sin \theta - \sin \theta \cdot \cos \theta}{\cos \theta}$ $\checkmark \sin \theta (1 - \cos \theta)$ $\checkmark \times \frac{1}{1 - \cos \theta}$ $\checkmark \frac{\sin \theta}{\cos \theta} = \tan \theta$	(5)
2.5	$\alpha = 90^\circ - \beta$ $\therefore \frac{\cos 20^\circ}{\sin 70^\circ} - \frac{\sin(90^\circ - \beta)}{\sin(90^\circ - \beta)}$ $= \frac{\sin 70^\circ}{\sin 70^\circ} - 1$ $= 1 - 1$ $= 0$	$\checkmark \cos 20^\circ$ $\checkmark \sin \alpha = \sin(90^\circ - \beta)$ $\checkmark \cos 20^\circ = \sin 70^\circ$ $\checkmark 1 - 1$ $\checkmark \text{antwoord}$	(5)

VRAAG 3

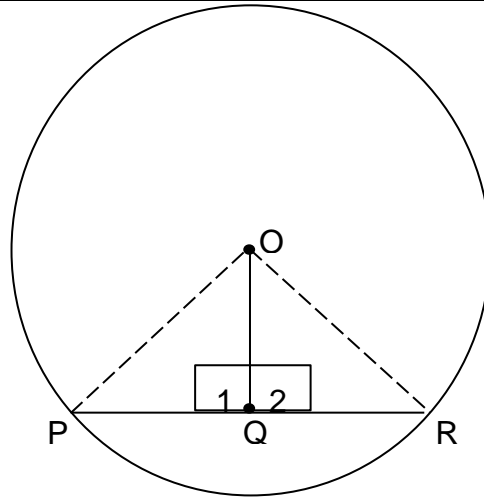
PUNTE:14

3.1		<ul style="list-style-type: none"> ✓ y-afsnit van f ✓ y-afsnit van g ✓✓ beide draaipunte van g ✓ beide x-afsnitte van f ✓ beide draaipunte van f 	(6)
3.2	2	✓✓ antwoord	(2)
3.3	$2 \cos x + 1 = 1 - \sin x$ $\therefore \sin x = -2 \cos x$ $\therefore \tan x = -2$ $\therefore \text{Verw. hoek} = 63,4349\dots^\circ$ $\therefore x = 180^\circ - 63,4349\dots^\circ + k \cdot 180^\circ$ $\therefore x = 116,57^\circ + k \cdot 180^\circ \text{ met } k \in \mathbb{Z}$ $\therefore x = -63,43^\circ \text{ OF } 116,57^\circ \text{ OF } 296,57^\circ$	<ul style="list-style-type: none"> ✓✓ antwoord ✓ $\tan x = -2$ ✓ $63,4349^\circ$ ✓ $x = 116,57^\circ + k \cdot 180^\circ \text{ met } k \in \mathbb{Z}$ ✓ $-63,43^\circ$ ✓ $116,57^\circ$ ✓ $296,57^\circ$ 	(6)

VRAAG 4

PUNTE: 13

4.1



Konstruksie: Verbind PO en OR

In $\triangle POQ$ and $\triangle ROQ$

- 1) $OQ = OQ$ gemeenskaplik
- 2) $PO = OR$ radii
- 3) $\hat{Q}_1 = \hat{Q}_2 = 90^\circ$... gegee

$\therefore \triangle POQ \equiv \triangle ROQ$... 90° HS
 $\therefore PQ = RQ$

✓ konstruksie(radii, OP en OR)

✓ SR ia ($PO = OR$ radii)

✓ S ia ($\hat{Q}_1 = \hat{Q}_2 = 90^\circ$)

✓ S R

✓ $PQ = RQ$

(5)

4.2.1

$\hat{A}_1 = 90^\circ$... lyn deur middel punt van \odot na middel van koord

$\hat{B} = 180^\circ - (\hat{O}_1 + \hat{A}_1)$ som van hoeke van \triangle
 $= 50^\circ$

✓ S R

✓ antwoord

(2)

4.2.2	$\hat{C} = 90^\circ$ hoek in semi \odot $\hat{D} = 180^\circ - (\hat{C} + \hat{B})$ som van hoeke van Δ $= 40^\circ$	✓ S R ✓ antwoord (2)
4.2.3	$BD = \sqrt{CD^2 + BC^2}$Pyth. $= \sqrt{40^2 + 30^2}$ $= 50$ $BO = \frac{1}{2}BD$radii $= 25$ $OA = \frac{1}{2}CD$...Midpt. St. $= 20$ $AE = OE - OA$ $= 25 - 20$ $= 5$ eenhede	✓ $BD = 50$ ✓ $BO = 25$ ✓ SR ie ($OA = 20$.. Midpt.St) ✓ $AE = 5$ (4)

VRAAG 5		PUNTE : 10
5.1	$\hat{L} = \hat{J}_1 = x \dots \dots \dots$ hoeke teenoor gelyke sye $\hat{N}_1 = \hat{L} = x \dots \dots \dots =$ koorde onderspan gelyke hoeke	\checkmark S R \checkmark S \checkmark R (3)
5.2	$\hat{M} = \hat{J}_3 = y \dots \dots \dots$ hoeke teenoor = sye $\hat{K}_1 = \hat{M} = y \dots \dots \dots =$ koorde onderspan gelyke hoeke $\hat{Q}_2 = \hat{K}_1 + \hat{J}_1 \dots$ buite hoek van Δ $= x + y$ $\hat{P}_2 = \hat{N}_1 + \hat{J}_3 \dots$ buite hoek van Δ $= x + y$ $\therefore \hat{Q}_2 = \hat{P}_2 = x + y$	\checkmark S R \checkmark S R \checkmark S R \checkmark S R (4)
5.3	$\hat{P}_4 = \hat{P}_2 \dots \dots$ regoorstde. hoeke $\hat{Q}_4 = \hat{Q}_2 \dots \dots$ regoorstde. hoeke but $\hat{Q}_2 = \hat{P}_2 \dots \dots \dots$ bewys in Vr 5.2 $\therefore \hat{P}_4 = \hat{Q}_4$ $\therefore JQ = JP \dots \dots$ sye teenoor = hoeke OF gelykb. Δ	\checkmark S R vir $\hat{P}_4 = \hat{P}_2$ OF $\hat{Q}_4 = \hat{Q}_2$ $\checkmark \hat{P}_4 = \hat{Q}_4$ \checkmark R (3)

VRAAG 6		PUNTE: 12
6.1	$\widehat{K}_2 = \widehat{T}_1$ hoeke teenoor = sye OF gelyke radii $\widehat{T}_1 = \widehat{C}$ hoeke in selfde segment $\therefore \widehat{K}_2 = \widehat{C}$	✓ S R ✓ $\widehat{T} = \widehat{C}$ ✓ R (3)
6.2	$\widehat{T}_2 = \widehat{A}$ hoeke teenoor = sye OF gelyke radii $\widehat{P}_2 = \widehat{T}_2 + \widehat{A}$... buite hoek van Δ $= 2\widehat{T}_2$ $\widehat{K}_1 = \widehat{P}_2$ hoeke in selfde segment $\widehat{K}_1 = 2\widehat{T}_2$	✓ S R ✓ S R ✓ $\widehat{K}_1 = \widehat{P}_2$ ✓ R (4)
6.3	$\widehat{P}_4 = 2\widehat{T}$ midpts. hoek = 2 x omtreks hoek $\widehat{T} = \widehat{T}_1 + \widehat{T}_2$ $\widehat{T}_1 = \widehat{C}$ hoeke in selfde segment $\widehat{T}_2 = \frac{1}{2} \widehat{K}_1$ bewys in Vr.6.2 $\widehat{P}_4 = 2(\widehat{C} + \frac{1}{2} \widehat{K}_1)$ $= 2\widehat{C} + \widehat{K}_1$	✓ $\widehat{P}_4 = 2\widehat{T}$ ✓ R ✓ S R ✓ $\widehat{T}_2 = \frac{1}{2} \widehat{K}_1$ ✓ $\widehat{P}_4 = 2(\widehat{C} + \frac{1}{2} \widehat{K}_1)$ (5)
		TOTAAL : 100