



**PROVINSIALE EKSAMEN**  
**NOVEMBER 2022**  
**GRAAD 11**  
**NASIENRIGLYNE**

**WISKUNDE (VRAESTEL 2)**


**INSTRUKSIES EN INLIGTING**

- **A** – AKKURAAATHEID
- **CA** – KONTINUE AKKURAAATHEID
- **S** – BEWERING
- **R** – REDE
- **S/R** – BEWERING met REDE

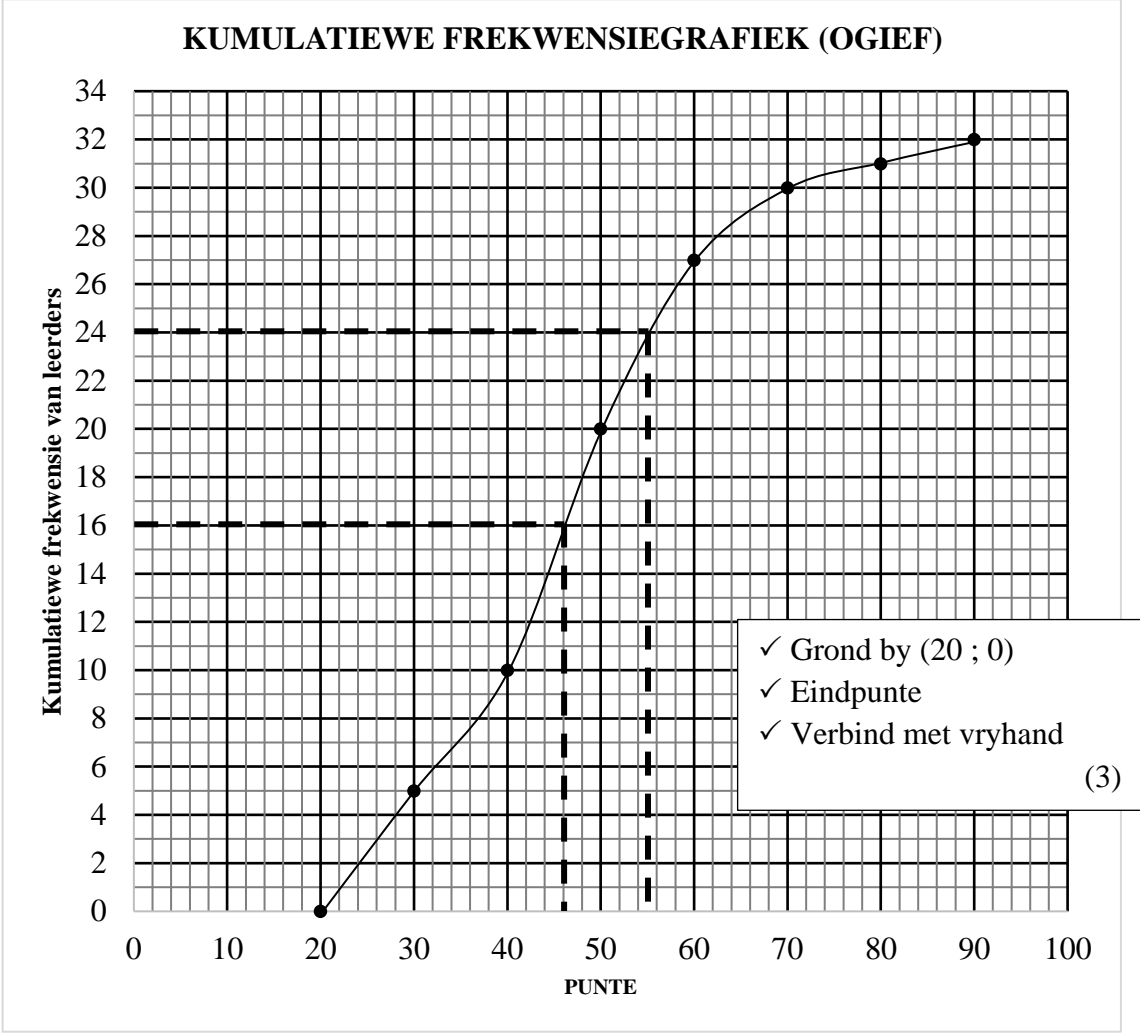
**NOTAS:**

- Indien 'n kandidaat 'n vraag TWEE keer antwoord, merk slegs die EERSTE poging.
- Indien 'n kandidaat 'n antwoord doodgetrek het, en nie die vraag weer gedoen het nie, merk die doodgetrekte poging.
- Deurlopende akkuraatheid moet op ALLE aspekte van die nasienriglyn toegepas word.
- Dit is ONAANVAARBAAR vir kandidate om waardes/antwoorde aan te neem in 'n poging om probleme op te los.

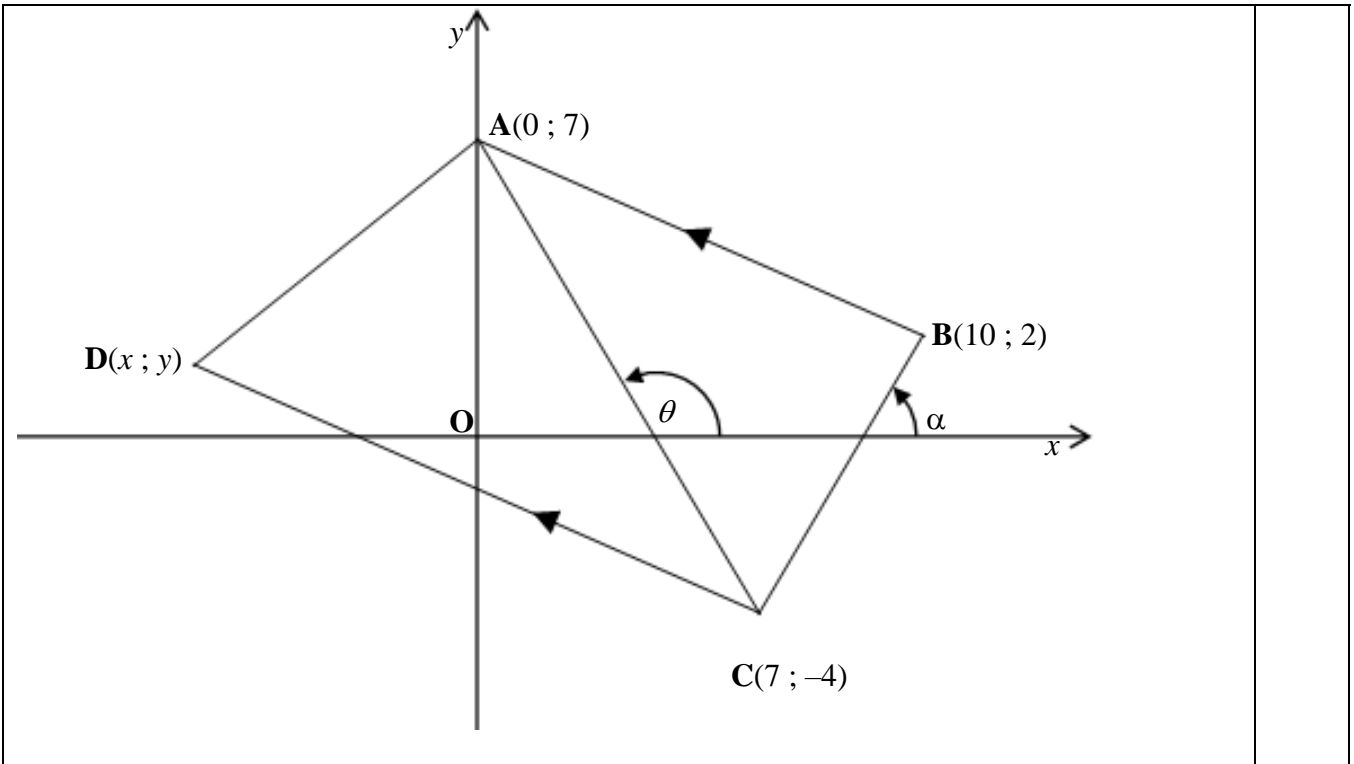
VRAAG 1

1.1		<ul style="list-style-type: none"> <li>✓ Mond (Q1 en Q3)</li> <li>✓ Snor (Min. en Maks.)</li> <li>✓ Mediaan</li> </ul>	(3)
1.2	<p>Die data is simmetries versprei, 'n normale verspreiding.</p> <p style="text-align: center;"><b>OF</b></p> <p>Die data is effens skeef na regs.</p>	<ul style="list-style-type: none"> <li>✓ Verduideliking</li> </ul>	(1)
1.3	$\sigma = 23,77$	<ul style="list-style-type: none"> <li>✓✓ <math>\sigma</math></li> </ul>	(2)
1.4	$(\bar{x} - \sigma; \bar{x} + \sigma)$ $= (287 - 23,77; 287 + 23,77)$ $= (263,23; 310,77)$ $\therefore 6$ munisipaliteite is binne een standaard afwyking.	<ul style="list-style-type: none"> <li>✓ <math>\bar{x} = 287</math></li> <li>✓ Interval</li> <li>✓ 6</li> </ul>	(3)
1.5	$S-IQR = \frac{1}{2} (Q_3 - Q_1)$ $= \frac{1}{2} (306 - 266)$ $= 20$	<ul style="list-style-type: none"> <li>✓ <math>Q_3 - Q_1</math></li> <li>✓ <math>\frac{1}{2} \times</math></li> <li>✓ Antwoord</li> </ul>	(3)
			<b>[12]</b>

VRAAG 2

2.1	$p = 5$ $q = 27$ $r = 1$	✓ $p$ ✓ $q$ ✓ $r$	(3)
2.2	<p style="text-align: center;"><b>KUMULATIEWE FREKWENSIEGRAFIEK (OGIEF)</b></p> 		(3)
2.3	$M = 46$	✓✓ $M = 46(\pm 2)$	(2)
2.4	$32 - 24$ $= 8$ $\therefore 8$ leerdere behaal 'n punt van meer as 55.	✓ $KF = 24(\pm 1)$ ✓ $32 - KF$	(2)
			<b>[10]</b>

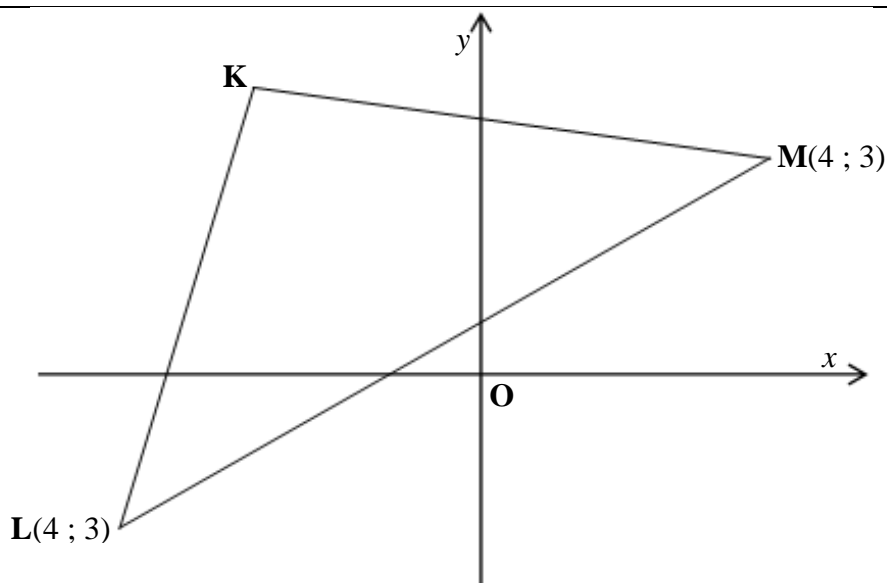
VRAAG 3



3.1	$AB = \sqrt{(0-10)^2 + (7-2)^2}$ $AB = \sqrt{125}$ $AB = 5\sqrt{5}$	✓ Vervang in korrekte formule ✓ AB ✓ Eenvoudigste wortelvorm	(3)
3.2	$m_{AC} = \frac{7 - (-4)}{0 - 7}$ $m_{AC} = -\frac{11}{7}$	✓ Gradiënt substitusie ✓ $m_{AC}$	(2)
3.3	$\tan \theta = -\frac{11}{7}$ $\text{verw.hoek} = 57,528 \dots^\circ$ $\theta = 180^\circ - 57,528 \dots^\circ$ $\theta = 122,47^\circ$	✓ Inklinasie hoek vervang (CA 4.2) ✓ Verw.hoek ✓ $\theta$ (trek verw.hoek af van $180^\circ$ )	(3)

3.4	$\tan \alpha = 2$ $\alpha = 63,434 \dots^\circ$ $\hat{A}CB = 122,47^\circ - 63,434 \dots^\circ$ $\hat{A}CB = 59,04^\circ$ $m_{AB} = \frac{1}{2}$ $\hat{C}AB = 180^\circ - \tan^{-1}\left(\frac{1}{2}\right) - 122,74^\circ$ $\hat{C}AB = 30,69^\circ$ $\hat{A}CD = 30,69^\circ$ [verw. $\angle$ 'e, $AB \parallel CD$ ] $\hat{B}CD = 30,69^\circ + 59,04^\circ$ $\hat{B}CD = 89,73^\circ$ <p style="text-align: center;"><b>OF</b></p> <p><b>Indien <math>\theta</math> nie afgerond is nie</b></p> $\tan \alpha = 2$ $\alpha = 63,434 \dots^\circ$ $\hat{A}CB = 122,4711 \dots^\circ - 63,434 \dots^\circ$ $\hat{A}CB = 59,036 \dots^\circ$ $m_{AB} = \frac{1}{2}$ $\hat{C}AB = 180^\circ - \tan^{-1}\left(\frac{1}{2}\right) - 122,74 \dots^\circ$ $\hat{C}AB = 30,963^\circ$ $\hat{A}CD = 30,963^\circ$ [verw $\angle$ 'e, $AB \parallel CD$ ] $\hat{B}CD = 30,963^\circ + 59,036 \dots^\circ$ $\hat{B}CD = 90^\circ$	$\alpha$ $\hat{A}CB$ $\hat{C}AB$ $\hat{B}CD$ <p style="text-align: center;"><b>OF</b></p> $\checkmark \alpha$ $\checkmark \hat{A}CB$ $\checkmark \hat{C}AB$ $\checkmark \hat{B}CD$	<p style="text-align: right;">(4)</p> <p style="text-align: right;"><b>[12]</b></p>
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VRAAG 4



4.1	$y = 5x + 9$  $y = 5(-1) + 9$ $y = 5$ $\therefore K(-1; 4)$	en $5y + x - 19 = 0$  $5(5x + 9) + x = 19$  $26x = -26$  $x = -1$	✓ Vervang $y$ ✓ Los op vir $x$ ✓ Vervang $x$ terug  GEEN PUNTE vir koördinate nie (gegee)	(3)
4.2	$m_{KL} = 5$  $\therefore m_{KL} \times m_{KM} = -1$  $KL \perp KM$  $\therefore \hat{LKM} = 90^\circ$	en $m_{KM} = -\frac{1}{5}$	✓ $m_{KL}$ ✓ $m_{KL}$  ✓ Definisie van loodregte gradiënte	(3)

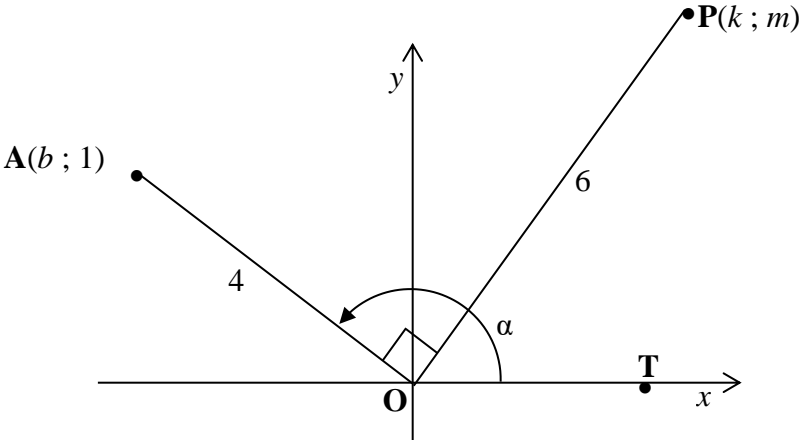
4.3	$KL = \sqrt{((-1 - (-2))^2 + (4 - (-1))^2)}$ $KL = \sqrt{26}$ $KN = \sqrt{(-1 - 4)^2 + (4 - 3)^2}$ $KN = \sqrt{26}$ $\text{Area } \Delta KML = \frac{1}{2} (\sqrt{26})(\sqrt{26})$ $\text{Area } \Delta KML = 13 \text{ eenhede}^2$	<ul style="list-style-type: none"> <li>✓ KL lengte</li> <li>✓ KN lengte</li> <li>✓ Vervang in Oppervlak formule</li> <li>✓ Opp <math>\Delta KML</math></li> </ul>	(4)
4.4	$M \left( \frac{-2 + 4}{2}; \frac{3 + (-1)}{2} \right)$ $M(1; 1)$ $m_{NL} = \frac{3 - (-1)}{4 - (-2)}$ $m_{NL} = \frac{2}{3}$ $y - y_1 = -\frac{2}{3}(x - x_1)$ $y - 1 = -\frac{2}{3}(x - 1)$ $y = -\frac{2}{3}x + \frac{5}{2}$	<ul style="list-style-type: none"> <li>✓ Middelpunt van NL</li> <li>✓ Gradiënt van NL</li> <li>✓ Loodregte gradiënt</li> <li>✓ Vervang Middelpunt</li> <li>✓ Vergelyking</li> </ul>	(5)
4.5	$m_{NL} = m_{NP} = m_{LP}$ $\frac{2}{3} = \frac{y - 3}{7 - 4}$ $y = 5$	<ul style="list-style-type: none"> <li>✓ Definisie van kollineêr</li> <li>✓ Vervang <math>P(7; y)</math></li> </ul>	(2)

4.6	Q(3 ; -2)		✓ $x_Q$ ✓ $y_Q$	(2)
4.7	$\hat{K} = 90^\circ$ [bewys in 4.1] $KL = KN = \sqrt{26}$ [bewys in 4.2] $\therefore$ KLQN is 'n vierkant [’n parallelogram met ’n binnehoek van $90^\circ$ en aangrensende sye gelyk]		✓ Binnehoek van $90^\circ$ ✓ Aangrensende sye gelyk	(2)
				[21]

## VRAAG 5

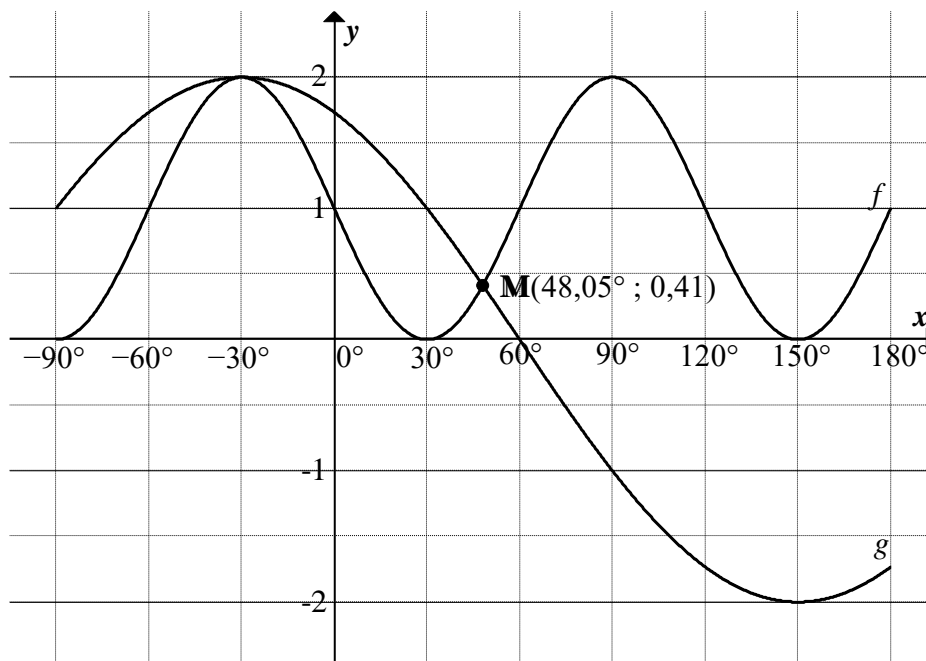
5.1	$\frac{\tan(180^\circ - x) \cos(360^\circ + x)}{2 \cos(90^\circ + x)}$ $= \frac{-\tan x \cdot \cos x}{-2 \sin x}$ $= \frac{-\frac{\sin x}{\cos x} \cdot \cos x}{-2 \sin x}$ $= \frac{-\sin x}{-2 \sin x}$ $= \frac{1}{2}$		✓ $-\tan x$ ✓ $\cos x$ ✓ $-2 \sin x$ ✓ $-\tan x$ identiteit ✓ vereenvoudig ✓ antwoord	(6)
5.2	5.2.1 $LK = \frac{\sin \theta - 2 \sin \theta \cos \theta}{2 \cos^2 \theta + \cos \theta - 1}$ $= \frac{\sin \theta (1 - 2 \cos \theta)}{(2 \cos \theta - 1)(\cos \theta + 1)}$ $= \frac{-\sin \theta (2 \cos \theta - 1)}{(2 \cos \theta - 1)(\cos \theta + 1)}$ $= \frac{-\sin \theta}{\cos \theta + 1}$ $= RK$		✓ $\sin \theta (1 - 2 \cos \theta)$ ✓ $(2 \cos \theta - 1)(\cos \theta + 1)$ ✓ Teken verander/negatiewe faktor ✓ Vereenvoudig <u>met</u> gevolgtrekking	(4)

	<p>5.2.2 Ongedefinieerd wanneer</p> $2\cos\theta - 1 = 0$ $\cos\theta = \frac{1}{2}$ <p>Verw. hoek = <math>60^\circ</math></p> <p>QI : <math>\theta = 60^\circ + k.360^\circ ; k \in Z</math></p> <p>QIV : <math>\theta = 300^\circ + k.360^\circ</math></p> <p><b>OF</b> wanneer</p> $\cos\theta = -1$ $\theta = 180^\circ + k.360^\circ ; k \in Z$ <p><b>NOTA: Aanvaar ander ekwivalente algemene oplossings</b></p> $\cos\theta - 1 = 0$ $\theta = 180^\circ + k.360^\circ ; k \in Z$	<p>✓ <math>2\cos\theta - 1 = 0</math></p> <p>✓ QI algemene oplossing</p> <p>✓ QIV algemene oplossing</p> <p>✓ <math>k \in Z</math></p> <p>✓ <math>\cos\theta = -1</math> algemene oplossing</p>	(5)
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5.3				
5.3.1	$b^2 + 1^2 = 4^2$ $b = \pm\sqrt{15}$ $b = -\sqrt{15}$ (QII)	✓ Pythagoras substitusie  ✓ $b = -\sqrt{15}$	(2)	
5.3.2	(a)	$\tan \alpha = -\frac{1}{\sqrt{15}}$	✓ $\frac{y}{x}$ ✓ CA $x$	(2)
	(b)	$\sin(-\alpha - 180^\circ) + \cos(-\alpha)$ $= \sin \alpha + \cos \alpha$ $= \left(\frac{1}{4}\right) + \left(\frac{-\sqrt{15}}{4}\right)$ $= \left(\frac{1-\sqrt{15}}{4}\right)$	✓ $\sin \alpha$ ✓ $\cos \alpha$ ✓ Vervang $x ; y$ en $r$	(3)

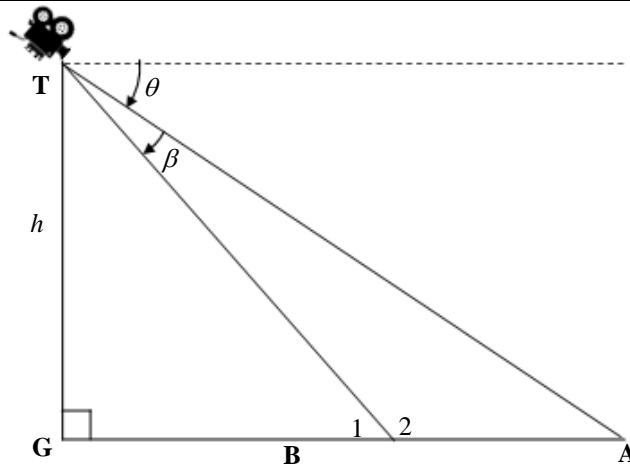
	<p>5.3.3</p>	$\cos(\alpha - 90) = \frac{k}{6}$ $\sin \alpha = \frac{k}{6}$ $\therefore \frac{k}{6} = \frac{1}{4}$ $\therefore k = \frac{3}{2}$	$\sin(\alpha - 90) = \frac{m}{6}$ $-\cos \alpha = \frac{m}{6}$ $\cos \alpha = -\frac{m}{6}$ $\therefore \frac{m}{6} = \left( \frac{-\sqrt{15}}{4} \right)$ $\therefore m = \frac{3\sqrt{15}}{2}$ <p style="text-align: center;"><b>OF</b></p> <p>P lê in QI, daarom is <math>k</math> en <math>m</math> positief</p> <p>'n <math>90^\circ</math> rotasie beteken <math>(x ; y) \rightarrow (y ; x)</math></p> $\frac{r_2}{r_1}$ $\frac{6}{4}$ $\frac{3}{2}$ $\therefore P\left(\frac{3}{2}; \frac{3}{2} \cdot \sqrt{15}\right) = P(k ; m)$ $\therefore k = \frac{3}{2} \text{ en } m = \frac{3\sqrt{15}}{2}$	$\checkmark \cos(\alpha - 90) = \frac{k}{6}$ $\checkmark \sin(\alpha - 90) = \frac{m}{6}$ <p><math>\checkmark</math> Komplementêre herleiding</p> <p><math>\checkmark</math> Verhoudings gelyk stel</p> <p><math>\checkmark</math> Waardes van <math>k</math> en <math>m</math></p> <p style="text-align: center;"><b>OF</b></p> <p><math>\checkmark</math> P in QI</p> <p><math>\checkmark</math> <math>(x ; y) \rightarrow (y ; x)</math> idee</p> <p><math>\checkmark</math> Verhouding van radii</p> <p><math>\checkmark</math> Pas skaal toe op <math>x</math> en <math>y</math></p> <p><math>\checkmark</math> Waardes van <math>k</math> en <math>m</math></p> <p><b>Antwoord alleen MAG NIE volpunte kry NIE.</b></p>	<p style="text-align: right;">(5) [27]</p>
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VRAAG 6



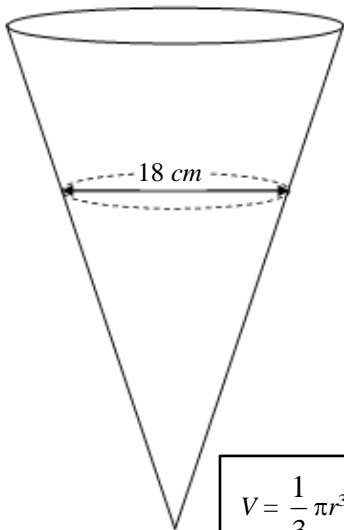
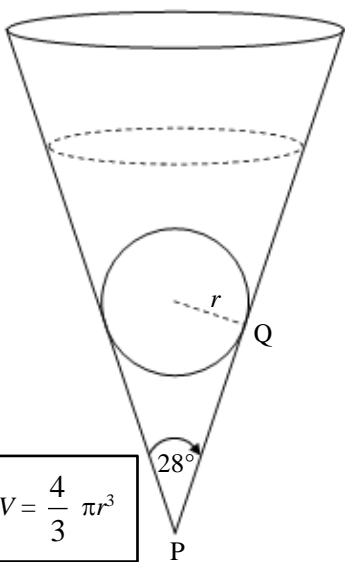
6.1	$A = 2$	✓ Antwoord	(1)
6.2	$120^\circ$	✓ Antwoord	(1)
6.3	$a = -1$ $b = 3$ $p = 30^\circ$	✓ $a$ ✓ $b$ ✓ $p$	(3)
6.4	6.4.1	$-90^\circ \leq x < 60^\circ$	✓ Interval (1)
	6.4.2	$x = -30^\circ$ of $48,05^\circ \leq x \leq 180^\circ$	✓ Interval ✓ $x = -30^\circ$ (2)
	6.4.3	$60^\circ < x < 150^\circ$ of $150^\circ < x \leq 180^\circ$	✓ $60^\circ$ en $180^\circ$ ✓ Nie ingesl. $150^\circ$ maar $180^\circ$ ingesl. (2)
6.5	$t = -30^\circ + k \cdot 360^\circ ; k \in \mathbb{Z}$		✓✓ $t = -30^\circ$ ✓ $k \cdot 360^\circ$ (3)
			<b>[13]</b>

VRAAG 7



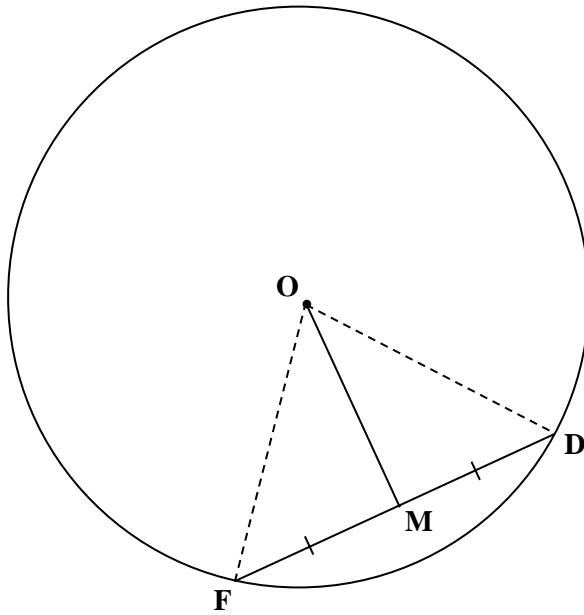
7.1	$\hat{B}_1 = \theta = \beta$	✓ Antwoord	(1)
7.2	<p>In <math>\Delta TBG</math></p> $\sin(\theta + \beta) = \frac{h}{BT}$ $BT = \frac{h}{\sin(\theta + \beta)}$ <p>In <math>\Delta ABT</math></p> $\hat{A} = \theta$ $\frac{x}{\sin \beta} = \frac{BT}{\sin \theta}$ $\frac{x}{\sin \beta} = \frac{h}{\sin(\theta + \beta) \sin \theta}$ $x = \frac{h \sin \beta}{\sin(\theta + \beta) \sin \theta}$	<p>✓ Korrekte verhouding in <math>\Delta TBG</math></p> <p>✓ BT</p> <p>✓ Sin reël substitusie</p> <p>✓ BT substitusie</p> <p>✓ vereenvoudiging</p>	(5)
7.3	$x = \frac{h \sin \beta}{\sin(\theta + \beta) \sin \theta}$ $x = \frac{20 \sin 37^\circ}{\sin(65^\circ + 37^\circ) \sin 65^\circ}$ $x = 13,58 \text{ m}$ <p>Die kat stap 13,58 m</p>	<p>✓ Substitusie</p> <p>✓ Antwoord</p>	(2)
			<b>[8]</b>

VRAAG 8

FIGUUR A		FIGUUR B	
			
$V = \frac{1}{3} \pi r^3$ of $V = \frac{4}{3} \pi r^3$			
8.1	$V = \frac{1}{3} \pi (9)^3$  $V = 763,41 \text{ cm}^3$	✓ Substitusie ( $r = 9$ )  ✓ Antwoord	(2)
8.2	$V = \frac{4}{3} \pi r^3$  $268,08 = \frac{4}{3} \pi r^3$  $\sqrt[3]{\frac{3 \times 268,08}{4\pi}} = r$  $r = 4$  $\tan 14^\circ = \frac{4}{PQ}$  $PQ = \frac{4}{\tan 14^\circ}$  $PQ = 16,04 \text{ cm}$	✓ Vervang volume in die korrekte formule  ✓ Onderwerp van die formule  ✓ $r$  ✓ $\tan 14^\circ$  ✓ $PQ$	(5)
			[7]

VRAAG 9

9.1



✓ Konstruksie

Konstruksie: **Verbind OF en OD**

In  $\triangle OFM$  en  $\triangle ODM$

i.  $FM = MD$  [GEGEE]

ii.  $OM$  is 'n gemeenskaplike sy

iii.  $OF = OD$  [*Radii*]

$\therefore \triangle OFM \equiv \triangle ODM$  [SSS]

✓ R

$\hat{M}_1 = \hat{M}_2$  [ $\triangle OFM \equiv \triangle ODM$ ]

✓ R

Maar  $\hat{M}_1 + \hat{M}_2 = 180^\circ$  [ $\angle$ 'e op 'n reguit lyn]

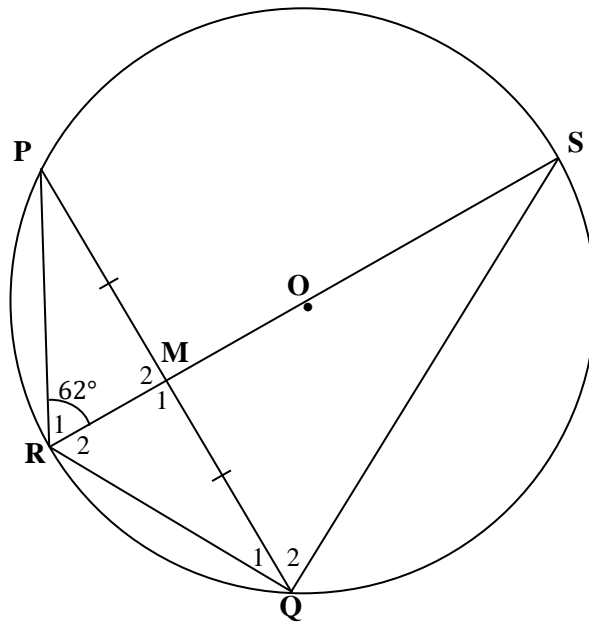
✓ S

$\therefore \hat{M}_1 = \hat{M}_2 = 90^\circ$

$\therefore OM \perp FD$

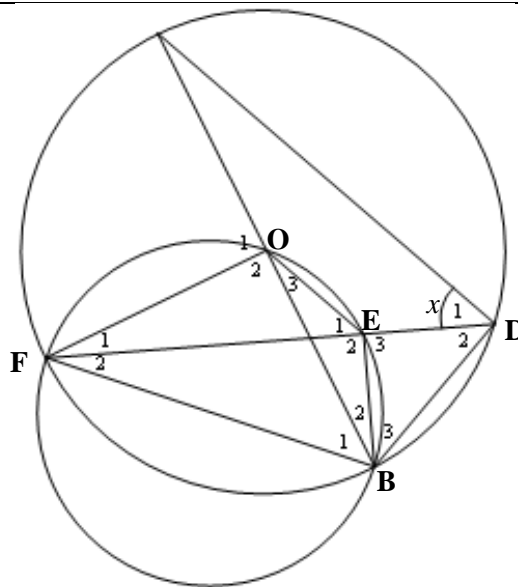
(4)

9.2



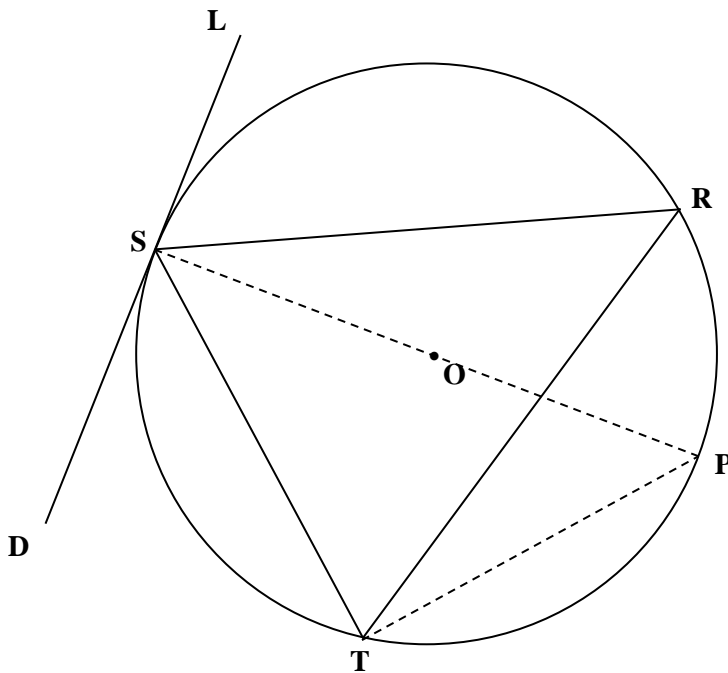
9.2.1	$\hat{M}_2 = 90^\circ$ [lyn van midpunt sirkel, tot midpunt koord]	✓ S ✓ R	(2)	
9.2.2	$\hat{Q}_1 + \hat{Q}_2 = 90^\circ$ [ $\angle$ 'e in 'n halwe sirkel]	✓ S ✓ R	(2)	
9.2.3	$\hat{P} = 28^\circ$ [binne $\angle$ 'e van 'n $\Delta$ ]	✓ S/R		
	$\hat{S} = \hat{P} = 28^\circ$ [ $\angle$ 'e in dieselfde sirkelsegment]	✓ R	(2)	
			<b>[10]</b>	

VRAAG 10



10.1	$\hat{B}_1 = \hat{D}_1 = x$ [∠'e in dieselfde sirkelsegment]	✓ S	
	$\hat{B}_1 = \hat{E}_1 = x$ [∠'e in dieselfde sirkelsegment]	✓ S	(2)
10.2	10.2.1 $\hat{O}_1 = 2x$ [midpunts∠ = 2 × omtreks∠]	✓ S ✓ R	(2)
	10.2.2 $\hat{O}_1 = \hat{O}FB + \hat{B}_1$ [buite ∠ van Δ] $2x = \hat{O}FB + x$ $\hat{O}FB = x$	✓ R ✓ $\hat{O}FB = x$	(2)
	10.2.3 $\hat{D}_2 = 90^\circ - x$ [∠'e in 'n halwe sirkel]	✓ S ✓ R	(2)
	10.2.4 $\hat{O}_2 = 180^\circ - 2x$ [∠'e op 'n reguitlyn] $\hat{O}_2 = \hat{E}_2 = 180^\circ - 2x$ [∠'e in dieselfde sirkelsegment]	✓ S ✓ R ✓ S ✓ R	(4)
10.3	$\hat{E}_2 = \hat{B}_3 + \hat{D}_2$ [buite ∠ van Δ] $\therefore \hat{B}_3 = 90^\circ - x$ $\therefore \hat{B}_3 = \hat{D}_2$ [albei $90^\circ - x$ ]	✓ R ✓ $\hat{B}_3 = 90^\circ - x$ ✓ S	
	$\therefore EB = ED$ [sye teenoor gelyke ∠'e]	✓ R	(4)
			<b>[16]</b>

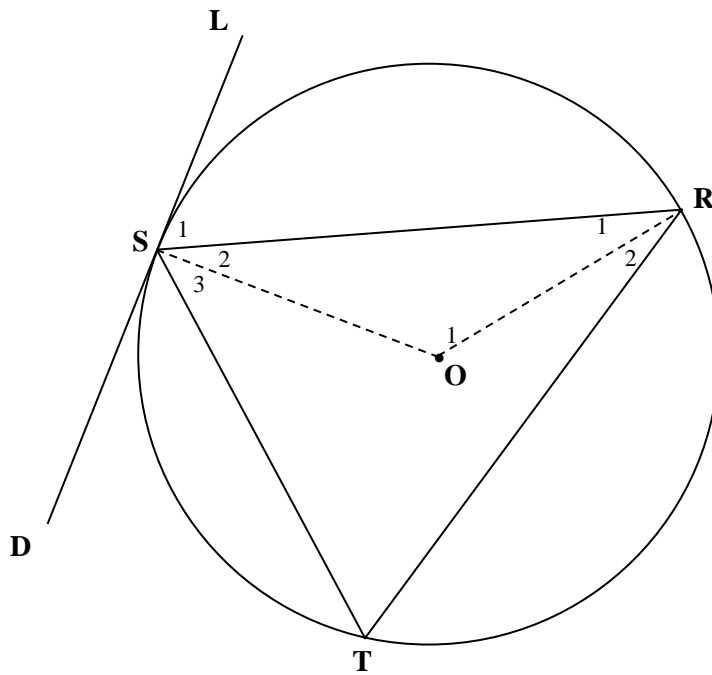
VRAAG 11



11.1	Konstruksie: Trek middellyn SP en verbind PT	✓ Konstruksie
	$\hat{L}SR = 90^\circ - \hat{R}SP$ [rklyn $\perp$ rad]	✓ S ✓ R
	$\hat{S}TR = 90^\circ - \hat{R}TP$ [ $\angle$ 'e in 'n halwe sirkel]	✓ S ✓ R
	$\hat{R}SP = \hat{R}TP$ [ $\angle$ 'e in dieselfde sirkelsegment]	
	$\therefore \hat{L}SR = \hat{S}TR$	✓ S ✓ R



Of



Konstruksie: Verbind OS en OR

$$\hat{S}_1 + \hat{S}_2 = 90^\circ$$

[rklyn  $\perp$  radius]

✓ Konstruksie

✓ S ✓ R

$$\hat{S}_2 = 90^\circ - \hat{S}_1$$

$$\hat{R}_1 = \hat{S}_2 = 90^\circ - \hat{S}_1$$

[ $\angle$ 'e teenoor gelyke radii]

✓ S/R

$$\hat{O}_1 = 180^\circ - 2[90^\circ - \hat{S}_1] = 2\hat{S}_1$$

[binne  $\angle$ 'e van  $\Delta$ ]

✓ S

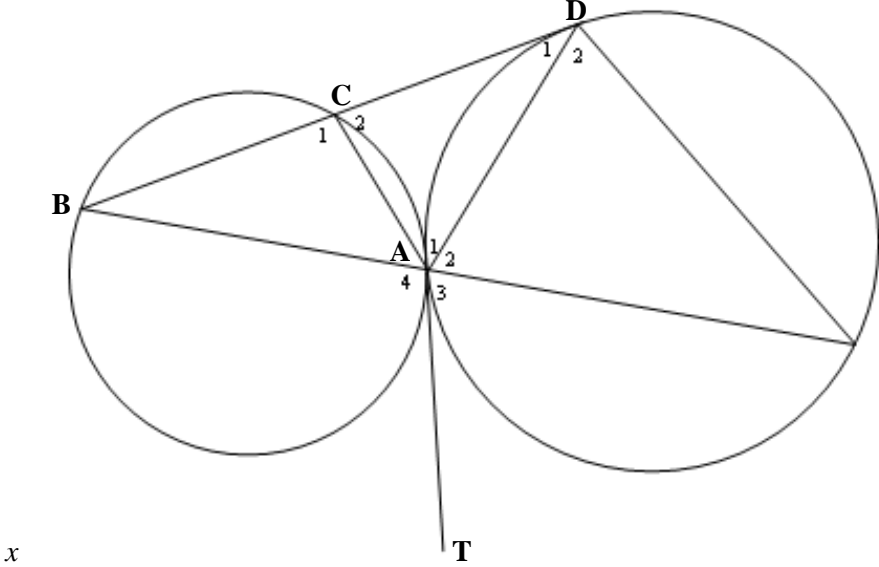
$$\hat{O}_1 = 2\hat{T}$$

[midpunts $\angle = 2 \times$  omtreks $\angle$ ]

✓ S ✓ R

$$\therefore \hat{L}\hat{S}\hat{R} = \hat{S}\hat{T}\hat{R}$$

(6)

11.2				
11.2.1	$\angle$ tussen rklyn en koord	✓ R	(1)	
11.2.2	$\hat{A}_4 = 180^\circ - \hat{A}_3$ [ $\angle$ 'e op 'n reguitlyn] $\hat{C}_1 = 180^\circ - \hat{A}_3$ [ $\angle$ tussen rklyn en koord] $\hat{C}_2 = \hat{A}_3$ [ $\angle$ 'e op 'n reguitlyn] $\hat{D}_2 = \hat{A}_3$ [ $\angle$ tussen rklyn en koord] $\therefore \hat{C}_2 = \hat{D}_2$ [albei gelyk aan $\hat{A}_3$ ] $\hat{D}_1 = \hat{F}$ [ $\angle$ tussen rklyn en koord /bewys in 11.2.1] $\hat{A}_1 = \hat{A}_2$ [binne $\angle$ 'e van $\Delta/3^{de}$ $\angle$ in $\Delta$ ]	✓ SR ✓ S ✓ R ✓ SR ✓ SR ✓ S ✓ R	(7)	
			<b>[14]</b>	
<b>TOTAAL:</b>			<b>150</b>	