

Question 1

Q4	Model Solution – 30 Marks	Marking Notes
(a) (i)	Centre = $(h, -3)$ Radius = $\sqrt{12}$ or $2\sqrt{3}$	Scale 10C (0, 4, 7, 10) <i>Low Partial Credit:</i> <ul style="list-style-type: none"> Work of merit towards finding x-ordinate or y-ordinate of the centre <i>High Partial Credit</i> <ul style="list-style-type: none"> Centre or radius correct
(ii)	$\frac{ h - 4(-3) + 7 }{\sqrt{(1)^2 + (-4)^2}} = 5$ $ h + 19 = 5\sqrt{17}$ $h + 19 = 5\sqrt{17} \text{ or } h + 19 = -5\sqrt{17}$ $h = 5\sqrt{17} - 19 \text{ or } h = -5\sqrt{17} - 19$ <p style="text-align: center;">OR</p> $(h + 19)^2 = 425$ $h^2 + 38h - 64 = 0$ $h = 5\sqrt{17} - 19 \text{ or } h = -5\sqrt{17} - 19$	Scale 15D (0, 4, 8, 12, 15) 3 steps: <ol style="list-style-type: none"> $\frac{ h - 4(-3) + 7 }{\sqrt{(1)^2 + (-4)^2}}$ $\frac{ h - 4(-3) + 7 }{\sqrt{(1)^2 + (-4)^2}} = 5$ Find values of h <i>Low Partial Credit:</i> <ul style="list-style-type: none"> Work of merit, for example, some substitution into relevant formula, or draws diagram with relevant figures (5, centre marked, and line) <i>Mid Partial Credit:</i> <ul style="list-style-type: none"> 1 step correct <i>High Partial Credit:</i> <ul style="list-style-type: none"> 2 steps correct
Q4	Model Solution – 30 Marks	Marking Notes
(b)	Centre = (h, k) $k = \frac{3 - 5}{2} = -1$ $(x - h)^2 + (y + 1)^2 = (\sqrt{20})^2$ $x^2 + y^2 - 2hx + h^2 + 2y + 1 = 20$ (8,1) is on the circle... $(8)^2 + (1)^2 - 2h(8) + h^2 + 2(1) + 1 = 20$ $h^2 - 16h + 48 = 0$ $(h - 4)(h - 12) = 0$ $h = 4, h = 12$ $s: (x - 4)^2 + (y + 1)^2 = 20$ <p style="text-align: center;">OR</p> $(a, 3): a^2 + 2ga + 6f + c = -9$ $(a, -5): a^2 + 2ga - 10f + c = -25$ So: $f = 1$ $(8, 1): 16g + 2f + c = -65$ So: $16g + c = -67$... Eqn A $\sqrt{g^2 + f^2 - c} = \sqrt{20}$	Scale 5D (0, 2, 3, 4, 5) <i>Low Partial Credit:</i> <ul style="list-style-type: none"> Some correct substitution of relevant point <i>Mid Partial Credit</i> <ul style="list-style-type: none"> Finds $k = -1$ Finds $f = 1$ 4 independent equations in g, f, c and a <i>High Partial Credit:</i> <ul style="list-style-type: none"> $k = -1$ and quadratic equation in h Finds $f = 1$ and quadratic equation in g <i>Full Credit -1:</i> <ul style="list-style-type: none"> Finds the relevant constants, but equation of circle not stated Finds equation of circles for both values of h but does not select the correct one

So: $g^2 - c = 19$... **Eqn B**

From **A** and **B**:

$$g^2 + 16g + 48 = 0 \quad \text{so } g = -4$$

So $c = -3$

And eqn is $x^2 + y^2 - 8x + 2y - 3 = 0$

Question 2

Q6	Model Solution – 30 Marks	Marking Notes
(a)	Answer: FALSE Justification: <i>Describes or draws any situation where two angles are equal in size without being vertically opposite. Eg. equilateral triangle, isosceles triangle, opposite angles in a parallelogram etc.</i>	Scale 5B (0, 2, 5) Partial Credit: <ul style="list-style-type: none"> • Correct answer with no justification • States True and justifies (ignores “and only if
(b) (i)	<p>1. $\angle EHD = \angle DBC = \theta$... alternate angles $\angle EFD = \angle EHD$... both = θ</p> <p>2. $\angle FED = \angle HED$...rectangle & straight angle $\angle FDE = \angle HDE$... angles in tri. sum to 180**</p> <p>3. $ED = ED$... common side Conclusion: So $FED \equiv HED$... by ASA $FE = EH$</p> <p style="text-align: center;">OR</p> <p>1. $\angle EHD = \angle DBC = \theta$... alternate angles $\angle EFD = \angle EHD$... both = θ $FD = DH$... isosceles triangle</p> <p>2. $\angle FED = \angle HED = 90^\circ$...rectangle & straight angle</p> <p>3. $ED = ED$... common side Conclusion: So $FED \equiv HED$... by RHS $FE = EH$</p> <p><i>Or similar</i></p>	Scale 15D (0, 4, 8, 12, 15) Consider the solution as having 4 elements: 3 steps and a conclusion: Steps 1, 2 & 3: 3 correct statements for congruency (with justifications) and Conclusion (with reason) Note: To prove by SAS, candidates will have to establish that $ \angle FDE = \angle HDE $ Low Partial Credit: <ul style="list-style-type: none"> • Work of merit, for example, $\angle EHD = \angle DBC$ indicated Mid Partial Credit: <ul style="list-style-type: none"> • 2 correct steps (no justifications) High Partial Credit: <ul style="list-style-type: none"> • 3 correct steps (including one side) with at least one justification • All 3 steps correct and conclusion stated (no justifications) Full Credit -1 <ul style="list-style-type: none"> • $FE = EH$ not stated
Q6	Model Solution – 30 Marks	Marking Notes
(b) (ii)	$ FE = \frac{1}{2} AB = 10$ $ FG = 40$ $\tan \theta = \frac{ AG }{ FG } = \frac{90}{40}$ $\theta = 66.037..^\circ$ $\theta = 66^\circ$ [nearest degree] <p style="text-align: center;">OR</p> $ FE = \frac{1}{2} AB = 10$	Scale 10D (0, 3, 5, 8, 10) 4 steps: <ol style="list-style-type: none"> 1. $FE = \frac{1}{2} AB = 10$ 2. Finds 2nd side in a relevant triangle 3. Trigonometric equation set up 4. Finds θ Low Partial Credit:

Triangles FED and BCD are similar

$$\text{So } |ED| = 90/4 = 22.5$$

$$\tan \theta = \frac{|ED|}{|FE|} = \frac{22.5}{10}$$

$$\theta = 66 \cdot 037 \dots^\circ$$

$$\theta = 66^\circ \text{ [nearest degree]}$$

OR

$$|FE| = \frac{1}{2}|AB| = 10$$

Triangles FED and BCD are similar

$$\text{So } |DC| = 90 \times \frac{3}{4} = 67.5$$

$$\tan \theta = \frac{|CD|}{|BC|} = \frac{67.5}{30}$$

$$\theta = 66 \cdot 037 \dots^\circ$$

$$\theta = 66^\circ \text{ [nearest degree]}$$

- Work of merit in finding any side of a relevant triangle

Mid Partial Credit

- 2 steps correct

High Partial Credit:

- 3 steps correct

Full Credit -1

- Incorrect rounding or no rounding

Question 3

Q1	Model Solution –30 Marks	Marking Notes																												
(a)	$P(\text{€6, €9, €6}) = \left[\frac{5}{12} \times \frac{3}{12} \times \frac{5}{12} \right]$ $= \frac{75}{1728} = \frac{25}{576} = 0.04340\dots$ $= 0.0434 \text{ [4 d.p.]}$	<p>Scale 10C (0, 4, 7, 10)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> Any correct relevant probability stated <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> $P(\text{€6}) = \frac{5}{12}$ and $P(\text{€9}) = \frac{3}{12}$ and some multiplication indicated $\frac{5}{12} \times \frac{3}{12} \times \frac{5}{12}$ $\frac{5}{12} \times \frac{3}{11} \times \frac{5}{10}$ and continues <p><i>Full Credit -1</i></p> <ul style="list-style-type: none"> Incorrect rounding or no rounding 																												
(b)	<p>Success = getting a 9</p> $P(\text{success}) = \frac{1}{4}$ <p>Failure = not getting a 9</p> $P(\text{failure}) = \frac{3}{4}$ <p>2 successes in first 7 spins and then success</p> $= \binom{7}{2} \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^5 \times \frac{1}{4}$ $= 0.07786\dots$ $= 0.0779 \text{ [4 d.p.]}$	<p>Scale 10D (0, 3, 5, 8, 10)</p> <p>Consider the solution as being the product of four terms: $\binom{7}{2}$, $\left(\frac{1}{4}\right)^2$, $\left(\frac{3}{4}\right)^5$ and $\frac{1}{4}$</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> $P(\text{success}) = \frac{1}{4}$ $P(\text{failure}) = \frac{3}{4}$ $\binom{7}{2}$ $\frac{1}{4}$ for the last day $\binom{8}{3}$ <p><i>Mid Partial Credit:</i></p> <ul style="list-style-type: none"> Product of two correct terms evaluated Product of three correct terms <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> $\binom{7}{2} \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^5 \times \frac{1}{4}$ <p><i>Full Credit -1</i></p> <ul style="list-style-type: none"> Incorrect rounding or no rounding 																												
Q1	Model Solution –30 Marks	Marking Notes																												
(c)	<p>Will get less than €16 unless 9, 9, so:</p> $1 - P(9, 9) = 1 - \frac{1}{4} \times \frac{1}{4} = \frac{135}{144} \text{ or } \frac{15}{16}$ $= 0.9375$ <p style="text-align: center;">OR</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tbody> <tr> <td style="padding: 2px;">$P(0,0)$</td> <td style="padding: 2px;">$\frac{4}{12} \times \frac{4}{12}$</td> <td rowspan="3" style="padding: 2px; vertical-align: middle;">$P(0 \text{ first})$</td> <td rowspan="3" style="padding: 2px; vertical-align: middle;">$\frac{4}{12}$</td> </tr> <tr> <td style="padding: 2px;">$P(0,6)$</td> <td style="padding: 2px;">$\frac{4}{12} \times \frac{5}{12}$</td> </tr> <tr> <td style="padding: 2px;">$P(0,9)$</td> <td style="padding: 2px;">$\frac{4}{12} \times \frac{3}{12}$</td> </tr> <tr> <td style="padding: 2px;">$P(6,0)$</td> <td style="padding: 2px;">$\frac{5}{12} \times \frac{4}{12}$</td> <td rowspan="3" style="padding: 2px; vertical-align: middle;">$P(6 \text{ first})$</td> <td rowspan="3" style="padding: 2px; vertical-align: middle;">$\frac{5}{12}$</td> </tr> <tr> <td style="padding: 2px;">$P(6,6)$</td> <td style="padding: 2px;">$\frac{5}{12} \times \frac{5}{12}$</td> </tr> <tr> <td style="padding: 2px;">$P(6,9)$</td> <td style="padding: 2px;">$\frac{5}{12} \times \frac{3}{12}$</td> </tr> <tr> <td style="padding: 2px;">$P(9,0)$</td> <td style="padding: 2px;">$\frac{3}{12} \times \frac{4}{12}$</td> <td style="padding: 2px;">$P(9,0)$</td> <td style="padding: 2px;">$\frac{1}{12}$</td> </tr> <tr> <td style="padding: 2px;">$P(9,6)$</td> <td style="padding: 2px;">$\frac{3}{12} \times \frac{5}{12}$</td> <td style="padding: 2px;">$P(9,6)$</td> <td style="padding: 2px;">$\frac{5}{48}$</td> </tr> <tr> <td style="padding: 2px;">TOTAL</td> <td style="padding: 2px;">$= \frac{15}{16}$</td> <td></td> <td style="padding: 2px;">$= \frac{15}{16}$</td> </tr> </tbody> </table>	$P(0,0)$	$\frac{4}{12} \times \frac{4}{12}$	$P(0 \text{ first})$	$\frac{4}{12}$	$P(0,6)$	$\frac{4}{12} \times \frac{5}{12}$	$P(0,9)$	$\frac{4}{12} \times \frac{3}{12}$	$P(6,0)$	$\frac{5}{12} \times \frac{4}{12}$	$P(6 \text{ first})$	$\frac{5}{12}$	$P(6,6)$	$\frac{5}{12} \times \frac{5}{12}$	$P(6,9)$	$\frac{5}{12} \times \frac{3}{12}$	$P(9,0)$	$\frac{3}{12} \times \frac{4}{12}$	$P(9,0)$	$\frac{1}{12}$	$P(9,6)$	$\frac{3}{12} \times \frac{5}{12}$	$P(9,6)$	$\frac{5}{48}$	TOTAL	$= \frac{15}{16}$		$= \frac{15}{16}$	<p>Scale 10D (0, 3, 5, 8, 10)</p> <p>Note:</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> Relevant probability Relevant work on establishing the condition, for example, indicates $P(9,9)$, or lists three that are in line with the condition (e.g., (0,0), (0,6), (0,9)) <p><i>Mid Partial Credit:</i></p> <ul style="list-style-type: none"> $\frac{1}{4} \times \frac{1}{4}$ indicated Probability calculated correctly for three pairs on the table <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> $\frac{1}{16}$ or equivalent Probability calculated correctly for
$P(0,0)$	$\frac{4}{12} \times \frac{4}{12}$	$P(0 \text{ first})$	$\frac{4}{12}$																											
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$P(9,6)$	$\frac{3}{12} \times \frac{5}{12}$	$P(9,6)$	$\frac{5}{48}$																											
TOTAL	$= \frac{15}{16}$		$= \frac{15}{16}$																											

$$= 0.9375$$

six pairs on the table

Full Credit -1

- No rounding or incorrect rounding

Question 4

Q2	Model Solution – 30 Marks	Marking Notes
(a)	$C = \left(\frac{1(8)+4(-1)}{4+1}, \frac{1(-4)+4(3)}{4+1} \right)$ $C = \left(\frac{4}{5}, \frac{8}{5} \right)$ <p style="text-align: center;">OR</p> <p>x: 9 steps back [8 to -1] So $x_C = 8 - \frac{4}{5}(9) = \frac{4}{5}$</p> <p>$y$: 7 steps up [$-4$ to 3] So $y_C = -4 + \frac{4}{5}(7) = \frac{8}{5}$</p> $C = \left(\frac{4}{5}, \frac{8}{5} \right)$	<p>Scale 10C (0,3,7,10)</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> • Correct formula with some substitution • Value from formula explicitly identified (a, b, x_1, y_1, x_2, y_2) • Recognises 9 steps for x or 7 steps for y • Plots both points on a set of axes <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> • Fully correct substitution into formula • $8 + \frac{4}{5}(-1 - 8)$ and $-4 + \frac{4}{5}(3 - (-4))$, or equivalent
(b)	<p>From y-intercept to (q, r):</p> <p>Run = q, so rise = qm, so y-value = $r - qm$</p> <p>Answer: $(0, r - qm)$</p> <p style="text-align: center;">OR</p> <p>$y = mx + c$ So $r = mq + c$ So $c = r - mq$</p> <p>Answer: $(0, -mq + r)$</p> <p style="text-align: center;">OR</p> <p>$y - y_1 = m(x - x_1)$ $y - r = m(x - q)$ $y - r = mx - mq$ $y = mx - mq + r$ $x = 0$, so $y = -mq + r$</p> <p>Answer: $(0, -mq + r)$</p>	<p>Scale 5C (0,2,3,5)</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> • Work of merit, for example, equation of line formula (accept m as substitution), or indicates distance q or r correctly on diagram, or indicates $x = 0$ <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> • States $x = 0$ and work of merit towards finding y • Correct y-value found <p><i>Full Credit –1</i></p> <ul style="list-style-type: none"> • Correct values for x and y, but not given as co-ordinates of a point
Q2	Model Solution – 30 Marks	Marking Notes
(c)	$\tan 30^\circ = \frac{-2-m_2}{1+(-2)m_2}$ <p>So $\frac{1}{\sqrt{3}} = \frac{-2-m_2}{1-2m_2}$</p> $1 - 2m_2 = -2\sqrt{3} - \sqrt{3}m_2$ $m_2 = \frac{1+2\sqrt{3}}{2-\sqrt{3}}$ $m_2 = \frac{1+2\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$ $m_2 = 8 + 5\sqrt{3}$ <p style="text-align: center;">OR</p> $\frac{1}{\sqrt{3}} = \frac{-(-2-m_2)}{1+(-2)m_2}$	<p>Scale 15D (0,4,8,12,15)</p> <ol style="list-style-type: none"> 1. Subs in 30 or -2 2. Fully substituted formula (30 and -2) 3. Eliminate fractions and expands 4. Find slope <p>Note: A solution based on $\tan^{-1}(-2) = 116.5 \dots^\circ$ is awarded at most <i>Low Partial Credit</i></p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> • Work of merit, for example, formula for $\tan \theta$ with substitution for given slope, or $\tan 30^\circ$ indicated • Indicates $\tan^{-1}(-2)$ or $116.5 \dots^\circ$

$$1 - 2m_2 = 2\sqrt{3} + \sqrt{3}m_2$$

$$m_2 = \frac{1-2\sqrt{3}}{2+\sqrt{3}}$$

$$m_2 = \frac{1-2\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$m_2 = 8 - 5\sqrt{3}$$

Mid Partial Credit

- 2 steps correct
- Sets $30 = \frac{-2-m_2}{1-2m_2}$ and finishes correctly

High Partial Credit

- 3 steps correct

Full Credit -1

- Correct value for slope, but not in required form

Question 5

Q1	Model Solution – 30 Marks	Marking Notes																		
(a) (i)	<table border="1" style="margin: auto; border-collapse: collapse;"> <thead> <tr> <th rowspan="2"></th> <th colspan="2">Age (years)</th> <th rowspan="2">Total</th> </tr> <tr> <th>≤ 23</th> <th>≥ 24</th> </tr> </thead> <tbody> <tr> <th>Under.</th> <td style="text-align: center;">12 785</td> <td style="text-align: center;">2922</td> <td style="text-align: center;">15 707</td> </tr> <tr> <th>Post.</th> <td style="text-align: center;">1353</td> <td style="text-align: center;">5654</td> <td style="text-align: center;">7007</td> </tr> <tr> <th>Total</th> <td style="text-align: center;">14 138</td> <td style="text-align: center;">8576</td> <td style="text-align: center;">22 714</td> </tr> </tbody> </table>		Age (years)		Total	≤ 23	≥ 24	Under.	12 785	2922	15 707	Post.	1353	5654	7007	Total	14 138	8576	22 714	<p>Scale 5C (0, 2, 3, 5)</p> <p>Accept correct answers without supporting work</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> • 1 value correct <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> • 2 values correct
	Age (years)		Total																	
	≤ 23	≥ 24																		
Under.	12 785	2922	15 707																	
Post.	1353	5654	7007																	
Total	14 138	8576	22 714																	
(a) (ii)	<p>Conclusion: They are not independent</p> <p>Justification:</p> <p><i>Based on independent iff $P(O) \cdot P(U) = P(O \cap U)$:</i></p> $P(O) = \frac{8576}{22714} \text{ or } \frac{4288}{11357} \text{ or } 0.377..$ $P(U) = \frac{15707}{22714} = 0.6915..$ $P(O) \times P(U) = 0.261 ...$ $P(O \cap U) = \frac{2922}{22714} = 0.1286 \neq P(O) \cdot P(U)$ <p style="text-align: center;">OR</p> <p><i>Based on independent iff $P(O) = P(O U)$:</i></p> $P(O) = \frac{8576}{22714} \text{ or } \frac{4288}{11357} \text{ or } 0.377..$ $P(O U) = \frac{2922}{15707} = 0.186..$ <p style="text-align: center;">OR</p> <p><i>Based on independent iff $P(U) = P(U O)$:</i></p> $P(U) = \frac{15707}{22714} = 0.691$ $P(U O) = \frac{2922}{8576} = 0.3407$ <p style="text-align: center;">OR</p> <p>Relatively few of the undergraduates are 24 or older, compared to the university overall.</p> <p style="text-align: center;">OR</p> <p>Over half the students in the university are undergraduates, but only about one third of students aged 24 or older are undergrads</p>	<p>Scale 10D (0,3,5,8,10)</p> <p>Accept without “$\neq P(O) \cdot P(U)$” or similar, if conclusion is correct</p> <p>Note: calculations that do not involve probabilities are not awarded credit</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> • Work of merit, for example, correct conclusion, or makes a relevant statement, or finds a relevant probability <p><i>Mid Partial Credit</i></p> <ul style="list-style-type: none"> • Finds two of $P(O)$, $P(U)$, $P(O) \times P(U)$, or $P(O \cap U)$ • Finds $P(O U)$ or $P(U O)$ <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> • Enough calculations to support a correct conclusion, but no conclusion • Correct conclusion and finds $P(O) \times P(U)$ • Correct conclusion and finds $P(O)$, $P(U)$, and $P(O \cap U)$ • Correct conclusion and finds $P(O U)$ or $P(U O)$ 																		
(b)	$\frac{1}{7} \times \frac{1}{7} \text{ or } \frac{7}{7} \times \frac{1}{7} \times \frac{1}{7} = \frac{1}{49} \text{ or } 7 \times \left(\frac{1}{7} \times \frac{1}{7} \times \frac{1}{7}\right) = \frac{1}{49}$	<p>Scale 5C (0,2,3,5)</p> <p>Accept correct answer without supporting work</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> • Work of merit, for example, one correct probability 																		

		<p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> $\frac{1}{7} \times \frac{1}{7} \times \frac{1}{7}$ or $\frac{1}{343}$
Q1	Model Solution – 30 Marks	Marking Notes
(c)	$\frac{g}{b+g} = \frac{3}{5} \Rightarrow 3b - 2g = 0 \text{ or } b = \frac{2}{3}g$ $\frac{g+4}{b+g+8} = \frac{4}{7} \Rightarrow -4b + 3g = 4$ $9b - 6g = 0$ $-8b + 6g = 8$ $b = 8$ $3b = 2g \Rightarrow 2g = 24 \Rightarrow g = 12$ <p style="text-align: center;">OR</p> <p>x students in class: $\frac{g}{x} = \frac{3}{5}$ so $g = \frac{3}{5}x$</p> <p>$x + 8$ in class: $\frac{g+4}{x+8} = \frac{4}{7}$ so $7g + 28 = 4x + 32$</p> <p>Sub 1st into 2nd: $7\left(\frac{3}{5}x\right) + 28 = 4x + 32$</p> <p>Solve: $x = 20$, so $g = 12$ and $b = 8$</p> <p style="text-align: center;">OR</p> <p><i>Trial and Improvement:</i></p> <p>2 boys and 3 girls becomes 6 boys and 7 girls, so $P = \frac{7}{13}$ (not correct)</p> <p>4,6 becomes 8, 10, so $P = \frac{10}{18}$ (not correct)</p> <p>8,12 becomes 12, 16, so $P = \frac{16}{28} = \frac{4}{7}$</p> <p>So $b = 8, g = 12$</p>	<p>Scale 10D (0,3,5,8,10)</p> <ol style="list-style-type: none"> Initial equation in b and g Subsequent equation in b and g Finds b Finds g <p>Accept verified correct answer (gives b and g, finds $b + 4$ and $g + 4$, and verifies that probability is correct)</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> Work of merit, for example, indicates $\frac{2}{5}$, or indicates $b + g$, or some part of fraction involving b and g correct, or indicates use of additional 4 boys or girls or total of 8 in a relevant probability <p><i>Mid Partial Credit</i></p> <ul style="list-style-type: none"> 1 step correct 2 or more incorrect possibilities for b and g tested (by adding 4 to each and finding resulting probability) <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> 2 steps correct 2 correct linear equations in 2 unknowns Correct answer without supporting work or verification <p><i>Full Credit –1</i></p> <ul style="list-style-type: none"> Finds one variable (b or g)

Question 6

Q8	Model Solution – 50 Marks	Marking Notes
<p>(a) (i) (ii)</p>	<p>(i) <i>G and H plotted and labelled correctly</i></p> <p>(ii) <i>Reasonable line of best fit drawn</i></p>	<p>Scale 10C (0,3,7,10)</p> <p>In (i), accept x_H plotted between 15 and 20</p> <p>In (ii), accept line of best fit with some values on each side and with reasonable slope</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> • Work of merit in plotting G or H, for example, one ordinate correct <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> • (i) or (ii) correct <p><i>Full Credit –1</i></p> <ul style="list-style-type: none"> • Points and line correctly plotted, but no or incorrect labels on points
<p>(a) (iii) (iv)</p>	<p>(iii) <i>Answers consistent with candidate's line of best fit</i></p> <p>(iv) Answer: K Reason: L is well beyond all of the given data points</p>	<p>Scale 5C (0,2,3,5)</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> • Work of merit, for example, in (iii): relevant work on the graph, or in (iv): gives K, or reason shows some relevant knowledge <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> • One part correct • Work of merit in both parts <p><i>Full Credit –1</i></p> <ul style="list-style-type: none"> • Correct answers and reason, but without supporting work on the graph for (iii)
Q8	Model Solution – 50 Marks	Marking Notes
<p>(a) (v) (b)</p>	<p>(a)(v) $0.9659 \dots = 0.966$ [3 D.P.]</p> <p>(b) <i>Any valid reason, for example:</i> Correlation is related to linear relationships, and this is quadratic The line of best fit is close to horizontal The data is symmetrical: it is decreasing first, and then increasing in a symmetrical way</p>	<p>Scale 5B (0,2,5)</p> <p>In (a)(v), accept correct answer without supporting work</p> <p><i>Partial Credit</i></p> <ul style="list-style-type: none"> • Work of merit, for example, in (a)(v), fully substituted correct formula; or, in (b), draws axis of symmetry or line of best fit <p><i>Full Credit –1</i></p> <ul style="list-style-type: none"> • (b) correct and 0.965 or 0.96 or 0.97 given in (a)(v)
<p>(c)</p>	<p>Mean = $\frac{534 + S + M}{13} = 52$ $534 + S + M = 676$ $S + M = 142$</p>	<p>Scale 10D (0,3,5,8,10)</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> • Work of merit, for example, finds

Median = 54, so the least value S could be is 55
Greatest value of $S = 142 - 55 = 87$

total of the scores (including or excluding S and/or M), or work towards finding the median (for example, rewriting the values in ascending order)

Mid Partial Credit

- Correct equation in S and M
- Least value = 55

High Partial Credit

- Correct equation in S and M **and** indicates effort to find values greater than 54
- Finds answers of 54 and 88, 56 and 86, etc. (that is, $S + M = 142$ and incorrect values, both ≥ 54 , found)
- Correct answers (55 and 87) with insufficient supporting work

Question 7

Q9	Model Solution – 50 Marks	Marking Notes
(a) (i)	$(x - 1)^2 + (y - 17)^2 = 144$	Scale 5B (0, 2, 5) <i>Partial Credit:</i> <ul style="list-style-type: none"> Work of merit, for example, some correct substitution into equation of circle; or indication of radius on the diagram
(a)(ii)	$(a - 1)^2 + (8 - 17)^2 = 144$ $(a - 1)^2 = 63$ $a - 1 = \sqrt{63}$ (as $a > 0$) $= 1 + \sqrt{63}$	Scale 5C (0, 2, 3, 5) <i>Low Partial Credit:</i> <ul style="list-style-type: none"> Work of merit, for example, some correct substitution into equation <i>High Partial Credit:</i> <ul style="list-style-type: none"> $a^2 - 2a - 62 = 0$ $(a - 1)^2 = 63$ <i>Full Credit-1</i> <ul style="list-style-type: none"> Apply a * if $1 - \sqrt{63}$ is also presented as a solution
(a)(iii)	$ PC = \sqrt{(10 - 1)^2 + (6 - 17)^2} = \sqrt{202}$ $ PC - \text{radius} = \sqrt{202} - 12 = 2.2126 \dots$ Shortest distance = $2.2126 \dots \times 100$ $= 221.26 \dots$ $= 221$ [m] [nearest m]	Scale 10C (0, 4, 6, 10) <i>Low Partial Credit:</i> <ul style="list-style-type: none"> Work of merit, for example, some correct substitution into distance/perpendicular distance formula; finds slope of CP; <i>High Partial Credit:</i> <ul style="list-style-type: none"> $PC - \text{radius} = \sqrt{202} - 12$ or equivalent <i>Full Credit -1</i> <ul style="list-style-type: none"> Apply a * for no rounding or incorrect rounding
(b)	$x = 13$	Scale 5B (0, 2, 5) <i>Partial Credit:</i> <ul style="list-style-type: none"> Work of merit, for example, some indication that equation is of form $x = \text{constant}$, or draws in relevant tangent or normal

Q9	Model Solution – 50 Marks	Marking Notes
(c)	Step 1: Slope of $w = \frac{1}{3}$ Step 2: \perp slope = -3 Step 3: $y - 6 = -3(x - 10)$ Step 4: $3x + y = 36$ $3x + y = 36$ ($\times 3$) $x - 3y = 9$ $9x + 3y = 108$ $x - 3y = 9$ $10x = 117$ $x = 11.7$	Scale 10D (0, 3, 5, 7, 10) Consider the solution involving 5 steps: <ol style="list-style-type: none"> Find the slope of w Find the perpendicular slope Find the equation of the line through P that is perpendicular to w Find one co-ordinate of the point of intersection of the two lines Find the other co-ordinate of the point of intersection of the two lines <p>Note: $w: x - 3y = 9$, therefore eq. of \perp line is $3x + y + c = 0$ (Step 1 & Step 2)</p>

Step 5: $11 \cdot 7 - 3y = 9$

$y = 0 \cdot 9$

$(11 \cdot 7, 0 \cdot 9)$

Low Partial Credit:


- Work of merit, for example, $-3y = -x + 9$

Mid Partial Credit

- 2 steps correct and work of merit in another step

High Partial Credit

- 4 steps correct

Q9	Model Solution – 50 Marks	Marking Notes
(d)	<p> $y^2 + (3y)^2 = (12)^2$ $10y^2 = 144$ $y = \sqrt{14 \cdot 4} = 3 \cdot 8 \dots$ $x = 9 + 3\sqrt{14 \cdot 4} = 20 \cdot 4 \dots$ $(20 \cdot 4, 3 \cdot 8)$ [1 DP] </p> <p style="text-align: center;">OR</p> <p> $x^2 = 3^2 + 1^2$ $x^2 = 10$ $x = \sqrt{10}$ </p> <p> $1200\text{m} = 12 \text{ units}$ Factor increase = $\frac{12}{\sqrt{10}} = 3 \cdot 794 \dots$ $x = 3 \times 3 \cdot 794 + 9 = 20 \cdot 38 \dots$ $y = 3 \cdot 794 \dots$ $(20 \cdot 4, 3 \cdot 8)$ [1 DP] </p> <p style="text-align: center;">OR</p> <p>  </p> <p> Slope of line $w = \frac{1}{3}$ Angle between line w and the $x - axis =$ $\tan^{-1} \frac{1}{3} = 18 \cdot 435$ Then, $\sin 18 \cdot 435 = \frac{y}{12}$ $y = 3 \cdot 794 \dots$ $x = 3 \cdot 794 \times 3 + 9 = 20 \cdot 384 \dots$ $(20 \cdot 4, 3 \cdot 8)$ [1 DP] </p>	<p>Scale 15D (0, 4, 6, 8, 15)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> • Work of merit in identifying inputs for Pythagoras; slope of $w = \frac{1}{3}$; $1200\text{m} = 12 \text{ units}$ <p><i>Mid Partial Credit:</i></p> <ul style="list-style-type: none"> • $y^2 + (3y)^2 = (12)^2$ or equivalent • Finds angle between line w and the positive direction of the $x - axis$ <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> • Finds x or y value <p><i>Full Credit-1</i></p> <ul style="list-style-type: none"> • Apply a * for no rounding or incorrect rounding, once only

Question 8

<p>(a) (i)</p>	<p>$P(2 \text{ from the 1st 9 are O}) \times P(10\text{th is O})$ $= \binom{9}{2} \left(\frac{8}{100}\right)^2 \left(\frac{92}{100}\right)^7 \times \frac{8}{100}$ $= 0.01028 \dots = 0.0103 \text{ [4 D.P.]}$</p>	<p>Scale 10D (0, 3, 5, 8, 10)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> $\frac{8}{100}$ or $\frac{92}{100}$ or $\binom{9}{2}$ First line of solution indicated (accept with "and" instead of \times) <p><i>Mid Partial Credit:</i></p> <ul style="list-style-type: none"> $\binom{9}{2} \left(\frac{8}{100}\right)^2 \left(\frac{92}{100}\right)^7$ $\binom{10}{3} \left(\frac{8}{100}\right)^3 \left(\frac{92}{100}\right)^7$ evaluated <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> $\binom{9}{2} \left(\frac{8}{100}\right)^2 \left(\frac{92}{100}\right)^7 \frac{8}{100}$ $\binom{10}{3} \left(\frac{8}{100}\right)^3 \left(\frac{92}{100}\right)^7 \frac{8}{100}$ evaluated
<p>(a) (ii)</p>	<p>$1 - P(\text{none are O})$ $= 1 - \left(\frac{92}{100}\right)^5$ $= 0.34091 \dots = 0.3409 \text{ [4 D.P.]}$</p> <p style="text-align: center;">OR</p> <p>$P(1 \text{ or } 2 \text{ or } 3 \text{ or } 4 \text{ or } 5 \text{ are O})$ $= \binom{5}{1} \left(\frac{8}{100}\right)^1 \left(\frac{92}{100}\right)^4 + \binom{5}{2} \left(\frac{8}{100}\right)^2 \left(\frac{92}{100}\right)^3$ $+ \binom{5}{3} \left(\frac{8}{100}\right)^3 \left(\frac{92}{100}\right)^2 + \binom{5}{4} \left(\frac{8}{100}\right)^4 \left(\frac{92}{100}\right)^1$ $+ \left(\frac{8}{100}\right)^5$ $= 0.34091 \dots = 0.3409 \text{ [4 D.P.]}$</p>	<p>Scale 10C (0, 3, 7, 10)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> $\left(\frac{92}{100}\right)^a$ where $0 < a < 5$ First line of either solution <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> $\left(\frac{92}{100}\right)^5$ Three terms in second solution
<p>(a) (iii)</p>	<p>$1 - 0.92^k > 0.97$ so $0.92^k < 0.03$ Find where $0.92^k = 0.03$ i.e. $k(\ln(0.92)) = \ln(0.03)$ so $k = \frac{\ln(0.03)}{\ln(0.92)} = 42.05 \dots$ so least $k = 43$</p>	<p>Scale 10D (0, 3, 5, 8, 10)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> 0.92^k 0.03 <p><i>Mid Partial Credit:</i></p> <ul style="list-style-type: none"> $1 - 0.92^k > 0.97$ or $= 0.97$ $k(\ln(0.92))$ <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> Equation in k without indices (logs handled correctly)
<p>(b)</p>	<p>Interpretation 1: initial €70 charged regardless $0.8(70) + 0.2(70 + 150 + 80)$ $= 0.8(70) + 0.2(300) = \text{€}116$</p> <p>Interpretation 2: initial €70 not charged if not successful $0.8(70) + 0.2(150 + 80)$</p>	<p>Scale 10C (0, 3, 7, 10)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> A correct calculation, for example, $0.8(70)$ or 300 or 230 <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> $0.8(70)$ and $0.2(300)$ or $0.8(70)$ and $0.2(230)$

(c)

$$= 0.8(70) + 0.2(230) = \text{€}102$$

Average pay-out per customer:
 $120\,000(0.0001) + 40\,000(0.002)$
 $= \text{€}92$

Target profit per customer:
 $\frac{900\,000}{18\,000} = \text{€}50$

Required premium: $50 + 92 = \text{€}142$

OR

Average pay-out per customer:
 $120\,000(0.0001) + 40\,000(0.002)$
 $= \text{€}92$

Total expected payout:
 $\text{€}92 \times 18\,000 = \text{€}1\,656\,000$

Total revenue required:
 $\text{€}1\,656\,000 + \text{€}900\,000 = \text{€}2\,556\,000$

Required premium:
 $\text{€}2\,556\,000 \div 18\,000 = \text{€}142$

Scale 10D (0, 3, 5, 8, 10)

Low Partial Credit:

- One relevant calculation, for example,
 $120\,000(0.0001)$ or $\frac{900\,000}{18\,000}$

Mid Partial Credit:

- Finds €92 (average payout per person)

High Partial Credit:

- Finds €2 556 000 (total revenue)
- Finds €92 and €50