Question 1

Question 4 **(30 marks)**

In this question, $i^2 = -1$.

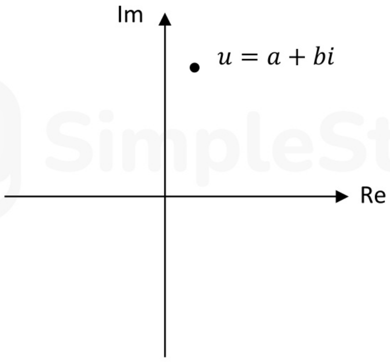
(a) The complex number $z_1 = 1 + i$ is a root of the equation $z^2 + (3 - 2i)z + p = 0$.

Find the value of p, where $p = a + bi$, with $a, b \in \mathbb{Z}$.

(b) Use **De Moivre's Theorem** to find the values of w for which $w^2 = -1 + \sqrt{3}i$.

Give each value of w in the form $a + bi$, with $a, b \in \mathbb{R}$.

(c) The Argand diagram below shows the complex number $u = a + bi$, where $a, b \in \mathbb{R}$.



(i) Write the complex numbers iu and $\overline{iu}$ in their simplest form, in terms of a and b, where $\overline{iu}$ is the complex conjugate of iu.

iu:

$\overline{iu}$:

(ii) **Plot** and **label** the complex numbers iu **and** $\overline{iu}$ on the diagram above, as accurately as possible.

(iii) State a transformation, or series of transformations, that would send u to $\overline{iu}$. Do **not** include a translation in your answer.

Question 3 **(30 marks)**

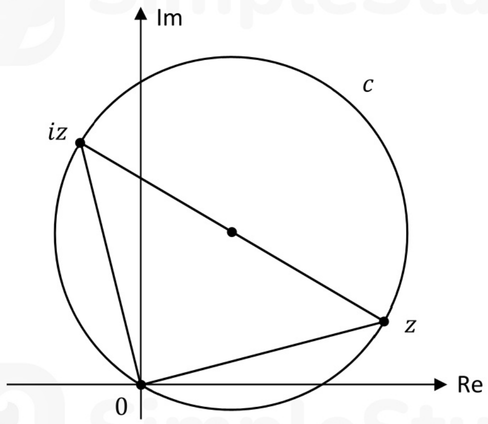
(a) $z = 6 + 2i$, where $i^2 = -1$.

(i) Show that $z - iz = 8 - 4i$.

(ii) Show that $|z|^2 + |iz|^2 = |z - iz|^2$

(iii) The circle c passes through the points z, iz, and 0, as shown in the diagram below (not to scale). z and iz are endpoints of a diameter of the circle.

Find the area of the circle c in terms of π.
There is space on the next page for your solution.



*Space for **part (a)(iii)**.*

(b) $(\sqrt{3} - i)^9$ can be written in the form $a + ib$, where $a, b \in \mathbb{Z}$ and $i^2 = -1$.

Use de Moivre's Theorem to find the value of a and the value of b.

$a =$____________ $b =$____________

Question 4 **(30 marks)**

(a) A sequence u_1 , u_2 , u_3 , ... is defined as follows, for $n \in \mathbb{N}$:

$$u_1 = 2, \quad u_2 = 64, \quad u_{n+1} = \sqrt{\frac{u_n}{u_{n-1}}}$$

Write u_3 in the form 2^p , where $p \in \mathbb{R}$.

(b) The first three terms in an **arithmetic** sequence are as follows, where $k \in \mathbb{R}$:

$$5e^{-k}, \quad 13, \quad 5e^k$$

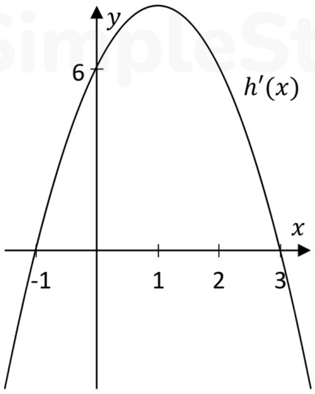
(i) By letting $y = e^k$ in this arithmetic sequence, show that:

$$5y^2 - 26y + 5 = 0$$

(ii) Use the equation in y in **part (b)(i)** to find the two possible values of k.
Give each value in the form $\ln p$ or $-\ln p$, where $p \in \mathbb{N}$.

Question 6 **(30 marks)**

The diagram below shows the graph of $h'(x)$ the derivative of a cubic function $h(x)$.



(a) Show that $h'(x) = -2x^2 + 4x + 6$.

(b) Use $h'(x)$ to find the maximum positive value of the **slope** of a tangent to $h(x)$.

(c) The graph of $h(x)$ passes through the point $(0, -2)$.
Find the equation of $h(x)$.

Question 5

Dani drives a car.

(a) The fuel consumption, F, of Dani's car depends on the speed of the car, c.
For one particular journey, F is given by:

$$F(c) = 0{\cdot}05\,c^2 - 8{\cdot}5\,c + 800$$

where F is in litres per 10 000 km, and c is in km/hour, with $40 \le c \le 120$.

(i) Show that there is no difference between the fuel consumption (F) when Dani's car is travelling at 60 km/hour and when it is travelling at 110 km/hour.

(ii) Find an expression for $\frac{dF}{dc}$, the rate of change of fuel consumption with respect to speed.

During part of the journey, the speed, c, of Dani's car at time t is given by:

$$c = 78 + 9\ln(t^2)$$

where t is the time in minutes, $1 \le t \le 10$, and c is in km/hour.

(iii) Use this, and your answer to part **(a)(ii)**, to find the value of $\frac{dF}{dc}$ when $t = 7$.
Give your answer correct to 1 decimal place.

(iv) Show that the rate of change of the car's speed with respect to time is given by:

$$\frac{dc}{dt} = \frac{18}{t}$$

(v) Use your answers to parts **(a)(iii)** and **(a)(iv)** to find the rate of change of the car's fuel consumption, F, with respect to time, at the instant when $t = 7$ minutes.

Give your answer in (litres per $10\ 000$ km) per minute.

Question 7 **(50 marks)**

Fiona is driving on a motorway. She passes a point **A** on the motorway. Her speed is given by:

$$v(t) = \frac{2}{3}t^3 - 6t^2 + 13t + 109$$

where v is her speed in km/hour t minutes after passing the point **A**, for $0 \leq t \leq 5$ and $t \in \mathbb{R}$.

(a) Work out Fiona's speed when she passes the point **A**.

(b) Work out Fiona's acceleration (that is, the rate at which her speed is increasing) 5 minutes after she passes the point **A**. Give your answer in km/hour per minute.

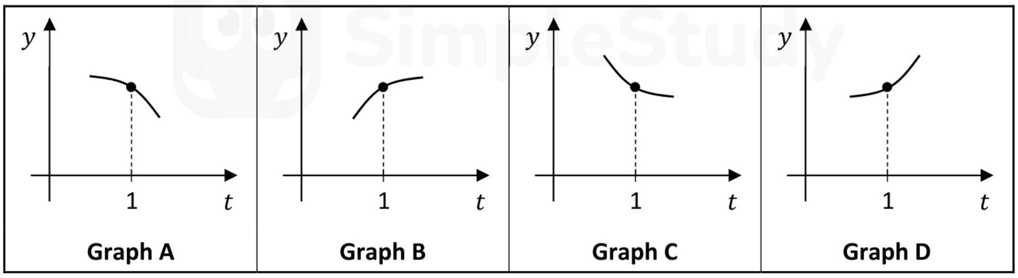
(c) Find the time (value of t) at which Fiona reaches her maximum speed, during the first 4 minutes after she passes the point **A**. Give your answer correct to 2 decimal places.

(d) Use integration to work out Fiona's average speed over the 5 minutes after she passes the point **A**. Give your answer correct to 2 decimal places.

(e) Taking $v'(t)$ to be the derivative of v, and $v''(t)$ to be the second derivative of v:

$$v'(1) > 0 \text{ and } v''(1) < 0$$

Four graphs, **A**, **B**, **C**, and **D**, are shown below.



Close to where $t = 1$, the graph of $y = v(t)$ must look like one of the four graphs given above. Write down which graph this is. Justify your answer, using both $v'(1)$ **and** $v''(1)$.

Answer (**A**, **B**, **C**, or **D**):

Using $v'(1) > 0$:

Using $v''(1) < 0$:

This question continues on the next page.

There is an **Average Speed Zone** on the motorway, starting at the point **A** and ending at point **B**.
The distance from **A** to **B** along the motorway is 10 km.
Cameras record the time taken for each car to travel from the point **A** to the point **B**.
Each car's average speed from **A** to **B** is then calculated.

(f) Work out the **minimum** time, in minutes, that a driver could get from **A** to **B**, while not driving above 100 km/hour.

(g) Rohan drives from **A** to **B**.

He passes the point **A** driving at a constant speed of 120 km/hour. After 2 minutes driving at this speed, he starts to decelerate (reduce his speed) at a constant rate, until he reaches the point **B**. Overall, his average speed in driving from **A** to **B** is 100 km/hour.

Work out Rohan's deceleration. Give your answer in km/hour per minute.

Question 8 **(50 marks)**

Olga, Chen, Fiona, and Rohan all have bank accounts.

(a) Olga puts €3000 in a savings account. Interest is added annually at a rate of 2·4% per year. Work out the amount in Olga's account after 5 years, correct to the nearest cent.

(b) **(i)** Explain what is meant by the "present value" of a payment of €1000 in 1 year's time, at a particular interest rate.

(ii) Chen puts a different amount in a savings account with the same interest rate (2·4% per year). After 6 years, Chen has €4000 in the account.

Work out how much money Chen put in the account initially, correct to the nearest cent.

(c) Fiona is taking out a loan at the same annual interest rate (2·4% per year).
Fiona makes payments quarterly (that is, 4 times per year).
Work out the quarterly interest rate that would be equivalent to an APR of 2·4%.
Give your answer as a **percentage**, correct to 2 decimal places.

(d) Rohan wants to put the same amount of money in a savings account at the start of each month for 36 months so that, at the end of 3 years, he will have a total of €12 000 in the account. Interest is calculated at a rate of $0{\cdot}11\%$ per month.

(i) Taking €A to be the amount Rohan puts in his account at the start of each month, write down a geometric series in €A to show the total amount of money in the account at the end of the 3 years. Include the first two and the last two terms.

(ii) Hence, find the value of €A that will give a total of €$12\ 000$ in the account after 3 years. Give your answer correct to the nearest cent.

(e) A park sells three types of ticket: child, student, and adult. The table below gives information on the price of each ticket and the percentage of tickets sold.
For example, 15% of all tickets sold are student tickets.

Type of ticket	**Child**	**Student**	**Adult**
Price of ticket	€11	€5 less than an adult ticket	€ x
Percentage	52%	15%	33%

The expected value of the price of a ticket is €$13{\cdot}85$.
Work out the value of x, the price of an adult ticket.

(f) When an item is being sold:

- the **mark up** is the profit as a percentage of the cost price, and

- the **margin** is the profit as a percentage of the selling price.

A shop sells an item with a **margin** of 18%. Work out the **mark up** for this item.
Give your answer as a percentage, correct to the nearest percent.

Question 9 **(50 marks)**

Alex gets injections of a medicinal drug. Each injection has 15 mg of the drug.
Each day, the amount of the drug left in Alex's body from an injection **decreases** by 40%.
So, the amount of the drug (in mg) left in Alex's body t days after a single injection is given by:

$$15(0{\cdot}6)^t$$

where $t \in \mathbb{R}$.

(a) Find the amount of the drug left in Alex's body $2{\cdot}5$ days after a single 15 mg injection. Give your answer in mg, correct to 2 decimal places.

(b) How long after a single 15 mg injection will there be exactly 1 mg of the drug left in Alex's body? Give your answer in days, correct to 1 decimal place.

Alex is given a 15 mg injection of the drug at the same time **every** day for a long period of time.

(c) Explain why the total amount of the drug, in mg, in Alex's body immediately after the 4th injection is given by:

$$15 + 15(0{\cdot}6) + 15(0{\cdot}6)^2 + 15(0{\cdot}6)^3$$

(d) Find the total amount of the drug in Alex's body immediately after the 10th injection.
Give your answer in mg, correct to 2 decimal places.

(e) Use the formula for the sum to infinity of a geometric series to estimate the amount of the drug (in mg) in Alex's body, after a long period of time during which he gets daily injections.

This question continues on the next page.

(f) Jessica also gets daily injections of a medicinal drug at the same time every day.
She gets d mg of the drug in each injection, where $d \in \mathbb{R}$.
Each day, the amount of the drug left in Jessica's body from an injection **decreases** by 15%.

(i) Use the sum of a geometric series to show that the total amount of the drug (in mg) in Jessica's body immediately after the nth injection, where $n \in \mathbb{N}$, is:

$$\frac{20d(1 - 0{\cdot}85^n)}{3}$$

(ii) Immediately after the 7th injection, there are 50 mg of the drug in Jessica's body.

Find the amount of the drug in one of Jessica's daily injections.
Give your answer correct to the nearest mg.