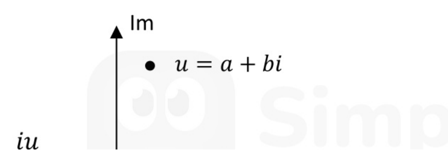
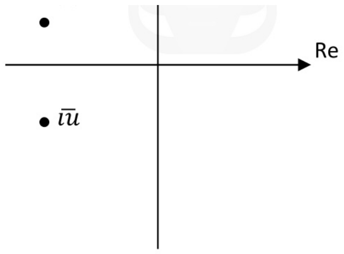
Question 1

Q4	Model Solution – 30 Marks	Marking Notes
(a)	**Method 1** $(1+i)^2+(3-2i)(1+i)+p=0$ $1+2i+i^2+3+i-2(i)^2+p=0$ $5+3i+p=0$ $p=-5-3i$ **Method 2** Let the second root $= z_2$ Sum of roots: $1+i+z_2=-3+2i$ $z_2=-4+i$ Product of roots: $(1+i)(-4+i)=p$ $p=-5-3i$ **Method 3** $z=\frac{-(3-2i)\pm\sqrt{(3-2i)^2-4p}}{2}$ $2z=-(3-2i)\pm\sqrt{(3-2i)^2-4p}$ $2z+3-2i=\pm\sqrt{(3-2i)^2-4p}$ $[2z+3-2i]^2=(3-2i)^2-4p$ $z=1+i$ satisfies this equation $[2(1+i)+3-2i]^2=(3-2i)^2-4p$ $5^2=(3-2i)^2-4p$ $4p=-20-12i$ $p=\frac{-20-12i}{4}$ $=-5-3i$ **Method 4** $z+(4-i)$ $z-1-i\,/\,\overline{z^2+(3-2i)z+p}$ $\underline{z^2-(1+i)z}$ $(4-i)z+p$ $\underline{(4-i)z-5-3i}$ $p+5+3i$ $p+5+3i=0$ $p=-5-3i$	**Scale 5C (0, 2, 3, 5)** Note: Any attempt involving the conjugate of $\mathbf{1}+\boldsymbol{i}$, award Low Partial Credit at most. **Method 1** *Low Partial Credit:* • Work of merit, for example, some correct substitution or some correct multiplication *High Partial Credit*: • Fully correct substitution and multiplication **Method 2** *Low Partial Credit:* • $z^2-(\text{sum})z+\text{product}$ • States p is the product of the roots • Sum of the roots = $-3+2i$ *High Partial Credit:* • Finds 2nd root • States sum of the roots = $3-2i$, but finishes correctly **Method 3** *Low Partial Credit:* • Some correct substitution in the quadratic formula *High Partial Credit:* • Formula fully substituted and $1+i$ substituted for z • Formula fully substituted and set equal to $1+i$ **Method 4** *Low Partial Credit:* Sets up long division but divisor must be of the form $z-a+bi$, where $a=1$ and $b=-1$ (Accept $b=1$ here) *High Partial Credit:* • First cycle of long division done correctly.

Q4	Model Solution – 30 Marks	Marking Notes
(b)	Reference Angle: $\alpha = \tan^{-1}\frac{\sqrt{3}}{1} = 60° \ \left(\frac{\pi}{3} \text{ rads}\right)$ Argument: $\theta = 180^0 - 60^0 = 120^0 \left(\frac{2\pi}{3} \text{ rads}\right)$ Modulus: $r = \sqrt{(-1)^2 + \left(\sqrt{3}\right)^2}$ $= \sqrt{4}$ $= 2$ General Polar Form: $2\left(\cos\left(\frac{2\pi}{3} + 2n\pi\right) + i\sin\left(\frac{2\pi}{3} + 2n\pi\right)\right)$ $w^2 = 2\left(\cos\left(\frac{2\pi}{3} + 2n\pi\right) + i\sin\left(\frac{2\pi}{3} + 2n\pi\right)\right)$ $w = \left[2\left(\cos\left(\frac{2\pi}{3} + 2n\pi\right) + i\sin\left(\frac{2\pi}{3} + 2n\pi\right)\right)\right]^{\frac{1}{2}}$ De Moivre: $w = 2^{\frac{1}{2}}\left[\left(\cos\frac{1}{2}\left(\frac{2\pi}{3} + 2n\pi\right) + i\sin\frac{1}{2}\left(\frac{2\pi}{3} + 2n\pi\right)\right)\right]$ $= 2^{\frac{1}{2}}\left[\cos\left(\frac{\pi}{3} + n\pi\right) + i\sin\left(\frac{\pi}{3} + n\pi\right)\right]$ $n = 0$: $w = \sqrt{2}\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)$ $= \sqrt{2}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$ $= \frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}i$ $n = 1$: $w = \sqrt{2}\left(\cos\left(\frac{\pi}{3} + \pi\right) + i\sin\left(\frac{\pi}{3} + \pi\right)\right)$ $= \sqrt{2}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$ $= -\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{2}i$	**Scale 15D (0, 4, 8, 12, 15)** Note: polar form must be used to achieve any credit. Note: Accept correct polar form without work (i.e., finding $\boldsymbol{r}$ and $\boldsymbol{\theta}$) Note: if $(\boldsymbol{w^2})^2$ is found, award *Mid Partial Credit* at most. Note: general polar form is not required to find the roots. Note: Accept solution in decimal form. **4 steps:** **1**. Finds θ **2**. Finds r **3.** One root evaluated from De Moivre's expression **4.** 2nd root found *Low Partial Credit:* • Work of merit, for example, plots $-1 + \sqrt{3}i$ • Work of merit towards finding r or θ • $w = \left(-1 + \sqrt{3}i\right)^{\frac{1}{2}}$ *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Full Credit –1* • Roots found correctly, but one or both in polar form

Q4	Model Solution – 30 Marks	Marking Notes
(c) **(i)** **(ii)** **(iii)**	**(i)** $iu = i(a + bi)$ $= ai + bi^2$ $= -b + ai$ $\overline{iu} = -b - ai$ **(ii)** 	**Scale 10D (0, 3, 5, 8, 10)** Note: 4 elements required: iu, $\overline{iu}$, plot, transformation Note: Accept conjugate plot from either candidate's work in (i) or by reflection of their $\overline{iu}$ in the real axis *Low Partial Credit:* • Work of merit in one element, for example, $i(a + bi)$ *Mid Partial Credit*

	 **(iii)** $90°$ counterclockwise rotation about the origin followed by axial symmetry in Re axis (x axis) Axial symmetry in the Im axis (y axis) followed by a $90°$ counterclockwise rotation about the origin. Axial symmetry in a line through the origin with slope -1 or similar	• 1 element correct **and** work of merit in a 2^{nd} element *High Partial Credit* • 2 elements correct **and** work of merit in a 3^{rd} element *Full Credit -1* • Diagram not labelled, otherwise correct

Question 2

Q3	Model Solution – 30 Marks	Marking Notes
(a) (i)	$z - iz = 6 + 2i - i(6 + 2i)$ $= 6 + 2i - 6i - 2i^2$ $= 8 - 4i$ **OR** $z(1 - i) = 8 - 4i$ $z = \frac{8-4i}{1-i}$ $z = \frac{(8-4i)(1+i)}{(1-i)(1+i)}$ $z = \frac{12+4i}{2} = 6 + 2i$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit* • Work of merit, for example, some correct substitution, or $z(1 - i)$ *High Partial Credit* • Mishandles $-2i^2$, otherwise correct • Multiplies numerator and denominator by conjugate
(a) (ii)	$\lvert z\rvert^2 = 6^2 + 2^2 = 40$ $\lvert iz\rvert^2 = 2^2 + 6^2 = 40$ $\lvert z - iz\rvert^2 = 8^2 + 4^2 = 80$ $40 + 40 = 80$ **OR** $\lvert x + iy\rvert^2 + \lvert -y + xi\rvert^2 = \lvert (x + y) + (y - x)i\rvert^2$ $x^2 + y^2 + y^2 + x^2$ $= x^2 + 2xy + y^2 + x^2 - 2xy + y^2$ $2x^2 + 2y^2 = 2x^2 + 2y^2$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit* • Work of merit, for example, correct formula with some substitution *Mid Partial Credit* • Two correct values found, from $\lvert z\rvert$, $\lvert iz\rvert$ and $\lvert z - iz\rvert$ *High Partial Credit* • Two correct values found, from $\lvert z\rvert^2$, $\lvert iz\rvert^2$, and $\lvert z - iz\rvert^2$ *Full Credit –1* • Finds $\lvert z\rvert^2$, $\lvert iz\rvert^2$, and $\lvert z - iz\rvert^2$, but no conclusion of equality
(a) (iii)	Radius $= \sqrt{80} \div 2 = \sqrt{20}$ Area $= \pi\left(\sqrt{20}\right)^2 = 20\pi$ square units **OR** Centre $= \frac{6+2i-2+6i}{2} = 2 + 4i$ Radius $= \sqrt{(6-2)^2 + (2-4)^2} = \sqrt{20}$ Area $= \pi\left(\sqrt{20}\right)^2 = 20\pi$ square units (Accept without units)	**Scale 5C (0, 2, 3, 5)** Allow solution treating problem as being in the real 2D co-ordinate plane rather than the complex plane *Low Partial Credit* • Some work of merit, for example, some substitution into a relevant formula *High Partial Credit* • Finds radius • Some work of merit in finding radius, and finds area based on incorrect radius

Q3	Model Solution – 30 Marks	Marking Notes
(b)	$\tan A = \frac{1}{\sqrt{3}}$, so $A = 30°$, so $\theta = 330°$ $r = \sqrt{1^2 + (\sqrt{3})^2} = 2$ $(\sqrt{3} - i)^9 = 2^9(\cos 9(330) + i \sin 9(330))$ $= 512(\cos 2970 + i \sin 2970)$ $= 0 + 512i$ $a = 0,\ b = 512$	**Scale 10D (0, 3, 5, 8, 10)** Note: polar form must be used to achieve any credit Note: Accept $0 + 512i$ for *Full Credit* Note that argument may also be given as $\theta = -30°$, etc., or $\theta = \frac{11\pi}{6}$, etc. **4 steps:** **1.** Finds r **2.** Finds θ **3.** Subs into de Moivre's Theorem **4.** Evaluates *Low Partial Credit* • Work of merit, for example, plots $\sqrt{3} - i$, or some correct substitution into de Moivre's Theorem *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Full Credit –1* • a and b not explicitly stated and solution given as $512i$

Question 3

Q4	Model Solution – 30 Marks	Marking Notes
(a)	$u_3 = \sqrt{\frac{u_2}{u_1}} = \sqrt{\frac{64}{2}} = \sqrt{32} = (2^5)^{\frac{1}{2}} = 2^{\frac{5}{2}}$	**Scale 10C (0, 3, 7, 10)** **3 steps:** **1.** Substitutes u_1 and u_2 into u_3 **2.** Writes 64 or 32 as a power of 2 **3.** Finishes (deals with square root) *Low Partial Credit* • Work of merit, for example, some correct substitution into u_3 *High Partial Credit* • 2 steps correct
(b) (i)	$[5e^k - 13 = 13 - 5e^{-k}]$ $5y - 13 = 13 - \frac{5}{y}$ $5y^2 - 13y = 13y - 5$ $5y^2 - 26y + 5 = 0$ **OR** $T_2 - T_1 = T_3 - T_2$ $T_1 + T_3 = 2T_2$ $5(e^k)^2 - 26(e^k) + 5 = 0$ $5e^{2k} - 26e^k + 5 = 0$ $5e^k + 5e^{-k} = 26$ $T_1 + T_3 = 2T_2$ **OR** $a = \frac{5}{y}$ $\frac{5}{y} + d = 13$ $d = 13 - \frac{5}{y}$ $13 + \left(13 - \frac{5}{y}\right) = 5y$ $26y - 5 = 5y^2$ $5y^2 - 26y + 5 = 0$	**Scale 10C (0, 3, 7, 10)** Each method shown has 3 steps. *Method 1:* **1.** Equates common differences **2.** Replaces e^k with y **and** e^{-k} with $\frac{1}{y}$ or y^{-1} **3.** Writes in required form *Method 2:* **1.** Shows $T_1 + T_3 = 2T_2$ for any arithmetic sequence **2.** Replaces y with e^k and simplifies **3.** Divides by e^k to show $T_1 + T_3 = 2T_2$ *Method 3:* **1.** Finds the common difference in terms of y **2.** Finds equation in y **3.** Writes in required form *Low Partial Credit* • Work of merit, for example, finds one common difference, or replaces e^k with y **or** y with e^k, or states $T_3 - T_2 = T_2 - T_1$ *High Partial Credit* • 2 steps correct
Q4	**Model Solution – 30 Marks**	**Marking Notes**
(b) (ii)	$(y - 5)(5y - 1) = 0$ $y = 5$ or $\frac{1}{5}$ $e^k = 5$ or $e^k = \frac{1}{5}$ $k = \ln 5$ or $k = \ln\frac{1}{5} = \ln(5^{-1}) = -\ln 5$	**Scale 10D (0, 3, 5, 8, 10)** **3 steps:** **1.** Fully substituted quadratic formula **OR** factors found **2.** Solves for y **3.** Solves for k, in correct form *Low Partial Credit* • Work of merit, for example,

			effort at factorisation, or identifies a, b, or c *Mid Partial Credit* • 1 step correct *High Partial Credit* • 2 steps correct *Full Credit –1* • 2 values of e^k correctly found, only 1 value of k correctly found

Question 4

Q6	Model Solution – 30 Marks	Marking Notes
(a)	$h'(x) = a(x+1)(x-3)$ $= a(x^2 - 2x - 3)$ $(0, 6) \in h'(x)$ $\therefore 6 = a(0 + 0 - 3)$ $\Rightarrow a = -2$ $h'(x) = -2(x^2 - 2x - 3)$ $h'(x) = -2x^2 + 4x + 6$ **OR** $h'(x) = -2x^2 + 4x + 6$ y-intercept $= h'(0) = 6$ $h'(x) = -2(x+1)(x-3)$ $\therefore$ Roots = –1 and 3 **OR** $h'(x) = ax^2 + bx + c$ $h'(0) = c = 6$ So $h'(-1) = a - b + 6 = 0$ and $h'(3) = 9a + 3b + 6 = 0$ $3 \times h'(1) = 3a - 3b + 18 = 0$ So $12a + 24 = 0$ $\therefore a = -2$ and $b = 4$ i.e. $h'(x) = -2x^2 + 4x + 6$	**Scale 10D (0, 3, 5, 8, 10)** *Note*: Accept 3 points from graph verified as belonging to given equation of $h'(x)$ *Note:* three points identified from graph is *Low Partial Credit*. *Low Partial Credit:* • Identifies or uses a relevant value from the graph, for example, $(x+1)$ or 3 • A factorisation of given $h'(x)$, for example $2(-x^2 + 2x + 3)$ *Mid Partial Credit:* • Generates $(x+1)(x-3)$ from graph • Correctly verifies one point from the graph into the given $h'(x)$ • Full factorisation of given $h'(x)$ • Using simultaneous equations, finds one value (a, b, or c) *High Partial Credit:* • Generates $x^2 - 2x - 3$ from graph • From given $h'(x)$, finds two roots • Correctly verifies two points from the graph into the given $h'(x)$ • From given $h'(x)$, shows that y-intercept is 6 **and** fully factorises • Using simultaneous equations, finds two values (from a, b, and c)

Q6	Model Solution – 30 Marks	Marking Notes
(b)	$h''(x) = -4x + 4 = 0$ at max/min of $h'(x)$ $\therefore x = 1$ $h'''(x) = -4 < 0$, i.e. max $h'(1) = -2(1)^2 + 4(1) + 6 = 8$ **OR** [*Quadratic with negative x^2, so max occurs halfway between the roots:*] $x = \frac{-1+3}{2} = 1$ $h'(1) = -2(1)^2 + 4(1) + 6 = 8$ **OR** $h'(x) = -2(x^2 - 2x - 3)$ $= -2(x^2 - 2x + 1 - 1 - 3)$ $= -2((x-1)^2 - 4)$	**Scale 10C(0, 3, 7, 10)** *Note*: It is possible to accept for *Full Credit* without $h'''(x) < 0$ *Low Partial Credit:* • Some correct differentiation of $h'(x)$ • Finds $h''(x)$ • $h'(x) = -2(x^2 - 2x - 3)$ • Indicates axis of symmetry on graph *High Partial Credit:* • $x = 1$ • $h'(x) = -2((x-1)^2 - 4)$

	$= -2(x-1)^2 + 8$ $\therefore$ max positive slope$= 8$	
(c)	$h(x) = \int h'(x)\,dx$ $h(x) = -\frac{2x^3}{3} + \frac{4x^2}{2} + 6x + C$ $(0, -2) \in h(x)$: $-2 = -0 + 0 + 0 + C$ $\Rightarrow C = -2$ $\therefore h(x) = -\frac{2x^3}{3} + 2x^2 + 6x - 2$	**Scale 10D (0, 3, 5, 8, 10)** *Note:* Accept correct answer without work. *Low Partial Credit:* • Any indication of integration *Mid Partial Credit:* • Integration of 3 terms fully correct *High Partial Credit:* • Relevant equation in C (with substitution)

Question 5

(a)(i)	**Method 1** $F(60) = 0{\cdot}05(60)^2 - 8{\cdot}5(60) + 800$ $= 470$ $F(110) = 0{\cdot}05(110)^2 - 8{\cdot}5(110) + 800$ $= 470$ $[= F(60)]$ **Method 2** $F'(c) = 0{\cdot}1c - 8{\cdot}5$ $0{\cdot}1c - 8{\cdot}5 = 0$ at local min $c = 85$ [axis of symmetry] $\frac{60 + 110}{2} = 85$ $\Rightarrow F(60) = F(110)$[by symmetry]	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit, for example, substitutes in 60 or 110 for c, or finds $F'(c)$ *High Partial Credit:* • Evaluates $F(60)$ or $F(110)$ • Finds $c = 85$ when $F'(c) = 0$
(a)(ii) (iii)	**(ii)** $\frac{dF}{dc} = 0{\cdot}1c - 8{\cdot}5$ **(iii)** At $t = 7$: $c = 78 + 9\ln(7^2)$ $= 113{\cdot}02 \ldots$ $\frac{dF}{dc} = (0{\cdot}1c - 8{\cdot}5)$ $= (0{\cdot}1(113{\cdot}02 \ldots) - 8{\cdot}5)$ $= 2{\cdot}80 \ldots$ $= 2{\cdot}8$ [1 D.P.]	**Scale 10D (0, 2, 4, 6, 10)** Consider solution as consisting of 4 steps: **Step 1**. **(ii)** correct **Step 2**. In **(iii)**, subs $t = 7$ into c **Step 3**. In **(iii)**, subs c (with $t = 7$) into $\frac{dF}{dc}$ **Step 4**. In **(iii)**, evaluates $\frac{dF}{dc}$ If c is evaluated at $t = 7$ and this value is subbed into $\frac{dF}{dc}$, and the resulting expression is evaluated, then consider all the evaluating as comprising Step 4. So, if there are errors in evaluating both c and $\frac{dF}{dc}$ these are both treated as errors in Step 4, and up to *HPC* can still be awarded for 3 steps correct. *Low Partial Credit:* • Work of merit, for example, in **(ii)**, some correct differentiation; in **(iii)**, 7^2 evaluated *Mid Partial Credit:* • 2 steps correct *High Partial Credit:* • 3 steps correct *Full Credit –1:* • Apply a * for Incorrect rounding
(a)(iv)	**Method 1** $\frac{dc}{dt} = 9\left(\frac{1}{t^2}\right)(2t)$ $= \frac{18t}{t^2}$ $= \frac{18}{t}$	**Scale 5B (0, 2, 5)** *Partial Credit:* • Some correct differentiation, for example, $\frac{1}{t^2}$ • $c = 78 + 18\ln t$

	Method 2 $c = 78 + 18\ln t$ $\dfrac{dc}{dt} = \dfrac{18}{t}$	
(a)(v)	$\dfrac{dF}{dt} = \dfrac{dF}{dc} \times \dfrac{dc}{dt}$ $= (2{\cdot}8) \times \left(\dfrac{18}{t}\right)$ $= (2{\cdot}8) \times \left(\dfrac{18}{7}\right)$ $= 7{\cdot}2$ or $\frac{36}{5}$ [(litres/10000 km) / minute]	**Scale 10D (0, 2, 4, 6, 10)** Accept correct answer without unit Consider solution as consisting of 4 steps: **Step 1**. $\frac{dF}{dt} = \frac{dF}{dc} \times \frac{dc}{dt}$ **Step 2**. Fills in value for $\frac{dF}{dc}$ **and** expression for $\frac{dc}{dt}$ **Step 3**. Fills in $t = 7$ in $\frac{dc}{dt}$ **Step 4**. Evaluates $\frac{dF}{dt}$ Steps can happen in different orders. Step 1 does not need to be explicitly stated. As in **(a)(iii)**, all evaluating is considered to be part of Step 4, and all errors in evaluating are considered to be restricted to Step 4. *Low Partial Credit:* • 1 step correct *Mid Partial Credit:* • 2 steps correct *High Partial Credit:* • 3 steps correct

Q7	Model Solution – 50 Marks	Marking Notes
(a)	$v(0) = \frac{2}{3}(0)^3 - 6(0)^2 + 13(0) + 109$ $v(0) = 109$ km/hr	**Scale 5B (0, 2, 5)** Accept correct answer without work. *Partial Credit:* • Some correct substitution • Identifies $t = 0$ • Substitutes $t = 5$ *Full Credit -1* • No unit or incorrect unit
(b)	$v'(t) = 2t^2 - 12t + 13$ $v'(5) = 2(5)^2 - 12(5) + 13$ $= 3$ [km/hr/min]	**Scale 10C (0, 4, 7, 10)** Accept correct answer without unit *Low Partial Credit:* • Some correct differentiation *High Partial Credit* • Substitution into correct derivative • At most one error in differentiation and finishes correctly

Q7	Model Solution – 50 Marks	Marking Notes
(c)	Maximum speed when $v'(t) = 0$ $2t^2 - 12t + 13 = 0$ $t = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(2)(13)}}{2(2)}$ $t = 4 \cdot 58$ or $t = 1 \cdot 42$ Maximum at $t = 1 \cdot 42$ [as coefficient of $t^3 > 0$ and domain of interest is $[0, 4]$, with local min at $t = 4 \cdot 58$] **OR** [$v''(t) = 4t - 12$ $v''(1.42) < 0$ $\Rightarrow$ maximum at $t = 1.42$]	**Scale 10C (0, 4, 7, 10)** Note: Accept candidate's derivative from part (b) Note: If candidate's derivative is linear, award *Low Partial Credit* at most 3 steps: **1.** $\boldsymbol{v'(t) = 0}$ **2.** Substitutes into formula **3**. Evaluates for $\boldsymbol{t}$ *Low Partial Credit:* • Work of merit in finding $v'(t)$ or brings $v'(t)$ from (b) • States $v'(t) = 0$ or similar • $v''(t)$ appears *High Partial Credit:* • 2 steps correct *Full Credit –1* • $t = 4.58$ written down and not explicitly excluded • Rounded incorrectly or no rounding, otherwise correct

Q7	Model Solution – 50 Marks	Marking Notes
(d)	$\frac{1}{5-0}\left[\int_0^5 \left(\frac{2}{3}t^3 - 6t^2 + 13t + 109\right) dt\right]$ $= \frac{1}{5}\left[\frac{t^4}{6} - 2t^3 + \frac{13t^2}{2} + 109t\right]_0^5$ $= \frac{1}{5}\left[\left(\frac{(5)^4}{6} - 2(5)^3 + \frac{13(5)^2}{2} + 109(5)\right) - (0)\right]$ $= 112 \cdot 333 \ldots$ km/hr $= 112 \cdot 33$ km/hr [2 d.p.]	**Scale 10D (0, 3, 5, 8, 10)** Note: Indication of integration is required to be awarded any credit **4 steps:** Note: If $\frac{1}{5}$ is omitted, treat step 1 as not fully correct, but all other steps can be accepted as correct Note: If speed is treated as $\boldsymbol{v'(t)}$ in (a) a correct solution must include the line, $\frac{1}{5}\left[\int_0^5 \boldsymbol{v'(t)dt}\right]$ **1.** $\frac{1}{5}\left[\int_0^5 v(t)dt\right]$ **2.** Integrates correctly **3.** Subs in limits **4.** Evaluates correctly *Low Partial Credit:* • Work of merit, for example, integration indicated *Mid Partial Credit:* • 2 steps correct *High Partial Credit* • 3 steps correct *Full Credit -1* • Rounded incorrectly or no rounding, otherwise correct • Incorrect unit or no unit, otherwise correct

Q7	Model Solution – 50 Marks	Marking Notes
(e)	Answer: B Justification: $v'(1) > 0$: Function is increasing so the slope is positive. $v''(1) < 0$: Rate of increase is slowing. **OR** The slope is decreasing.	**Scale 5C (0, 2, 3, 5)** Note: Justification needs to explicitly deal with both $\boldsymbol{v'}$ and $\boldsymbol{v''}$, but can be a single combined sentence. Note: Substitution into $\boldsymbol{v'(t)}$ or $\boldsymbol{v''(t)}$ is not considered work of merit. Note: Accept "the function is concave down", or similar as a justification using $\boldsymbol{v''(1) < 0}$ 3 elements required: **1.** Answer B **2.** Justification for $\boldsymbol{v'(1) > 0}$ **3.** Justification for $\boldsymbol{v''(1) < 0}$ *Low Partial Credit:* • Answer correct • Work of merit in either justification *High Partial Credit:* • 2 elements correct • Answer given as B or D, justified correctly using $v'(1)$

		• Answer given as B justified correctly using $v''(1)$ • Answer given as A justified correctly using $v''(1)$ • No answer given, but 2 justifications are correct
(f)	Time = Distance/Speed $= \frac{10}{100} = 6$ [minutes]	**Scale 5B (0, 2, 5)** Note: Accept correct answer without units *Partial Credit:* • Work of merit in finding time *Full Credit –1* • $1/10$ (i.e. incorrect unit)
(g)	<u>120 km/hr for 2 minutes:</u> $\text{Distance} = 120 \times \frac{2}{60} = 4\text{km}$ $10 - 4 = 6$ km remaining to get to B <u>Average Speed:</u> $\frac{10}{\text{total time}} = 100$ $\text{total time} = \frac{1}{10}\text{hrs}$ $= 6 \text{ minutes}$ ⇒ 4 minutes remaining to get to B <u>Average speed for last 6 km:</u> $\text{Avg Speed} = 6 \div \left(\frac{1}{15}\right) = 90 \text{ km/hr}$ $\frac{120+v}{2} = 90$, where v is the speed at B $v = 60\ km/hr$ Decelerates from 120 to 60 over 4 minutes. So, deceleration = 15 km/hr per minute **OR** Average speed for last 6 km: $\frac{120+v}{2}$, where v is the speed at B $\text{Distance} = \left(\frac{120+v}{2}\right) \times \text{time}$ $6 = \left(\frac{(120+v)}{2}\right) \times \frac{4}{60}$ $v = 60$ Decelerates from 120 to 60 over 4 minutes. Deceleration = 15 km/hr per minute **OR** Average speed for the last 6km: $\frac{1}{4}\int_0^4 (120 - at)dt = 90$ $\frac{1}{4}\left[120t - \frac{1}{2}at^2\right]_0^4 = 90$ $\frac{1}{4}\left[120(4) - \frac{1}{2}a(4)^2\right] = 90$ $120 - 2a = 90$ $a = 15\text{kmh}^{-1} \text{ per minute}$	**Scale 5D (0, 2, 3, 4, 5)** Accept -15km/hr/min *Low Partial Credit:* • Some correct substitution in relation to distance for first 2 minutes or time / distance for remaining part • Indicates integration *Mid Partial Credit:* • Identifies 4 km, 6 km or 4 minutes. *High Partial Credit* • Calculates 60 km/hr after the extra 4 minutes or 90km/hr after extra 2 minutes. *Full Credit -1* • Answer in km/hr/hr

Question 7

Q8	Model Solution – 50 Marks	Marking Notes
(a)	$F = 3000(1+0.024)^5$ $= €3377.70$	**Scale 5B (0, 2, 5)** *Partial Credit:* • Work of merit, for example, some correct substitution into relevant formula; finds 2.4% as a decimal
(b) (i), (ii)	**(i)** It is the amount that should be invested today to amount to €1000 in 1 years' time at the particular interest rate. **(ii)** $4000 = P(1+0.024)^6$ $\frac{4000}{1.024^6} = P$ $P = €3469 \cdot 45$	**Scale 10D (0, 3, 5, 8, 10)** Note: In (i) Accept $\boldsymbol{P} = \frac{\mathbf{1000}}{(\mathbf{1}+\boldsymbol{i})}$ *Low Partial Credit:* • Work of merit in (i) or (ii), for example, formula in (i), correct substitution into relevant formula in (ii) • 2.4% written as a decimal *Mid Partial Credit:* • (i) or (ii) correct • Work of merit in both parts *High Partial Credit* • One part correct and work of merit in the other part
(c)	$1 \cdot 024 = (1+i)^4$ $(1.024)^{\frac{1}{4}} = 1+i$ $(1.024)^{\frac{1}{4}} - 1 = i$ $0 \cdot 005947.. = i$ Rate $= 0 \cdot 59\%$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Some correct substitution into relevant formula • 2.4% written as a decimal *High Partial Credit:* • $(1.024)^{\frac{1}{4}} = 1+i$ • Evaluates correctly $i = (1.024)^4 - 1$ • Uses 3 or 12 instead of 4, but otherwise correct *Full Credit -1* • Answer given as a decimal

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(d) (i) (ii)	**(i)** $A(1 \cdot 0011)^{36} + A(1 \cdot 0011)^{35} + \cdots$ $\ldots + A(1 \cdot 0011)^2 + A(1 \cdot 0011)$ **OR** $= A[(1 \cdot 0011)^{36} + (1 \cdot 0011)^{35} + \cdots$ $\ldots + (1 \cdot 0011)^2 + (1 \cdot 0011)]$ **(ii)** $A[1{\cdot}0011 + (1 \cdot 0011)^2 + \cdots$ $\ldots + (1 \cdot 0011)^{35} + (1 \cdot 0011)^{36}]$ $a = 1 \cdot 0011,\ \ r = 1 \cdot 0011,\ \ \ n = 36$ $A\left[\frac{1 \cdot 0011\,(1 - 1 \cdot 0011^{36})}{1 - 1 \cdot 0011}\right] = 12000$ $A = \frac{12000}{1 \cdot 0011(1 - 1 \cdot 0011^{36})}$	**Scale 15D (0, 4, 8, 12, 15)** Consider as requiring 3 steps: **1.** Finds geometric series **2.** Substitutes into geometric formula **3.** Finds A *Low Partial Credit:* • Work of merit in either part, for example, in **(i)**Writes 0.11% as a decimal; in **(ii)**, sets answer in **(i)** equal to 12000 *Mid Partial Credit:* • 1 step correct • Substantial work of merit in both parts *High Partial Credit:* • 2 steps correct

	$1 - 1 \cdot 0011$ $A = €326 \cdot 60$ [2 D.P.]	*Full Credit –1:* • Correct solution, but excludes second and/or second last term • Investments made at the end of each month, otherwise correct
(e)	$E(x) = 11(0 \cdot 52) + (x - 5)(0 \cdot 15) + x(0 \cdot 33) = 13 \cdot 85$ $0{\cdot}15x + 0{\cdot}33x = 13{\cdot}85 - 5{\cdot}72 + 0{\cdot}75$ $0 \cdot 48x = 8 \cdot 88$ $x = €18 \cdot 50$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • Work of merit, for example, some correct term in $E(x)$ *High Partial Credit* • Fully correct equation
(f)	Cost Price = 82% of Selling Price Profit = 18% of Selling Price Mark-up = $\frac{0{\cdot}18}{0{\cdot}82} \times 100$ $= 0 \cdot 2195 = 22\%$ [nearest percent] **OR** Let x = selling price and y = cost price $\frac{x - y}{x} = 0.18 \rightarrow y = 0.82x$ Mark up: $\frac{x-y}{y} = \frac{x-0.82x}{0.82x} = \frac{9}{41}$ Mark up: $\frac{9}{41} \times 100 = 21.95$ = 22% [nearest percent]	**Scale 5B (0, 2, 5)** *Partial Credit:* • Work of merit, for example, states CP = 82% of SP • Mentions 82% • Finds 18% of a number

Question 8

Q9	Model Solution – 50 Marks	Marking Notes
(a)	$15(0\cdot6^{2\cdot5}) = 4\cdot182\ldots = 4\cdot18$ [mg] [2 DP]	**Scale 5B (0, 2, 5)** Accept correct answer without work *Low Partial Credit* • Work of merit, for example, correct substitution into given expression
(b)	$15(0\cdot6^t) = 1$ $0\cdot6^t = \frac{1}{15}$ $\ln 0\cdot6^t = t\ln 0\cdot6 = \ln\frac{1}{15}$ $t = \frac{\ln\frac{1}{15}}{\ln 0\cdot6} = 5\cdot30\ldots = 5\cdot3$ [days] **OR** $t = \log_{0.6}\frac{1}{15} = 5.30\ldots$ $= 5.3$ [days] [1 DP]	**Scale 10C (0, 3, 7, 10)** **1.** Isolates $0\cdot6^t$ **2.** Converts to log equation (not necessarily to base e) **3.** Solves for t *Low Partial Credit* • Work of merit, for example, $15(0\cdot6^t) = 1$ • 5.3 days by trial and improvement *High Partial Credit* • 2 steps correct
(c)	15: amount from injection just given $15(0\cdot6)$: amount from injection 1 day ago $15(0\cdot6^2)$: amount from injection 2 days ago $15(0\cdot6^3)$: amount from injection 3 days ago **OR** There is 15 mg from the injection just given, and the amount from each previous injection has reduced by 40% each day	**Scale 5A (0, 5)**
(d)	$15 + 15(0\cdot6) + \cdots + 15(0\cdot6^9)$ $S_n = \frac{a(1-r^n)}{1-r} = \frac{15(1-0\cdot6^{10})}{1-0\cdot6}$ $= 37\cdot273\ldots = 37\cdot27$ [mg] [2 DP]	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit* • Work of merit, for example, indicates sum of two or more relevant terms, identifies a or r, or S_n formula with some substitution *High Partial Credit* • S_n formula fully substituted

Q9	Model Solution – 50 Marks	Marking Notes
(e)	$S_\infty = \frac{a}{1-r} = \frac{15}{1-0{\cdot}6} = 37{\cdot}5$ [mg]	**Scale 5B (0, 2, 5)** *Partial Credit* • S_∞ formula with some substitution • Identifies a or r
(f) (i)	Amount immediately after n th injection: $d + d(0 \cdot 85) + \cdots + d(0 \cdot 85^{n-1})$ $= \frac{a(1-r^n)}{1-r} = \frac{d(1-0{\cdot}85^n)}{1-0{\cdot}85}$ $= \frac{20d(1-0{\cdot}85^n)}{3}$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit* • Work of merit, for example, identifies a or r, or one term in series (other than d) *High Partial Credit* • Fully correct substitution into formula • One error in substitution and finishes correctly
(f) (ii)	$\frac{20d(1-0{\cdot}85^7)}{3} = 50$ $d = \frac{50\times3}{20(1-0{\cdot}85^7)} = 11{\cdot}03 \ldots = 11$ [mg] [$\in \mathbb{N}$]	**Scale 5C (0, 2, 3, 5)** **2 steps:** **1.** Sets up equation in d **2.** Finds value of d *Low Partial Credit* • Work of merit, for example, some correct substitution into formula for amount of drug, or identifies a or r *High Partial Credit* • 1 step correct