Question 1

3. The point $P(3, -2)$ lies on the curve with equation $y = f(x), x \in \mathbb{R}$

Find the coordinates of the point to which P is mapped when the curve with equation $y = f(x)$ is transformed to the curve with equation

(i) $y = f(x - 2)$

(ii) $y = f(2x)$

(iii) $y = 3f(-x) + 5$

(4)

Question 3 continued

(Total for Question 3 is 4 marks)

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Question 2

5. Given that θ is small and in radians, use the small angle approximations to find an approximate numerical value of

$$\frac{\theta \tan 2\theta}{1 - \cos 3\theta}$$

(3)

Question 5 continued

(Total for Question 5 is 3 marks)

DO NOT

Question 3

6.

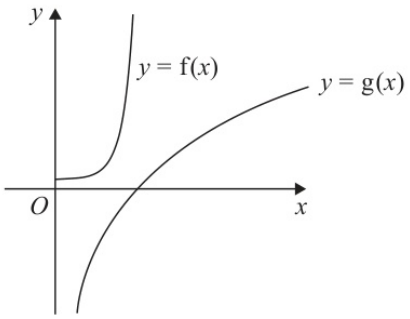

Figure 1

Figure 1 shows a sketch of the curves with equations $y = f(x)$ and $y = g(x)$ where

$$f(x) = e^{4x^2 - 1} \qquad x > 0$$
$$g(x) = 8 \ln x \qquad x > 0$$

(a) Find

(i) $f'(x)$

(ii) $g'(x)$

(2)

Given that $f'(x) = g'(x)$ at $x = \alpha$

(b) show that α satisfies the equation

$$4x^2 + 2 \ln x - 1 = 0$$

(2)

The iterative formula

$$x_{n+1} = \sqrt{\frac{1 - 2 \ln x_n}{4}}$$

is used with $x_1 = 0.6$ to find an approximate value for α

(c) Calculate, giving each answer to 4 decimal places,

(i) the value of x_2

(ii) the value of α

Question 6 continued

Question 6 continued
Question 6 continued

(Total for Question 6 is 7 marks)

Question 4

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\frac{1}{\operatorname{cosec}\theta - 1} + \frac{1}{\operatorname{cosec}\theta + 1} \equiv 2\tan\theta\sec\theta \qquad \theta \neq (90n)^\circ,\ n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $0 < x < 90°$, the equation

$$\frac{1}{\operatorname{cosec} 2x - 1} + \frac{1}{\operatorname{cosec} 2x + 1} = \cot 2x \sec 2x$$

Give each answer, in degrees, to one decimal place.

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 7 marks)

Question 5

12. (a) Express $\dfrac{1}{V(25-V)}$ in partial fractions.

(2)

The volume, V microlitres, of a plant cell t hours after the plant is watered is modelled by the differential equation

$$\frac{dV}{dt} = \frac{1}{10}V(25-V)$$

The plant cell has an initial volume of 20 microlitres.

(b) Find, according to the model, the time taken, in minutes, for the volume of the plant cell to reach 24 microlitres.

(5)

(c) Show that

$$V = \frac{A}{e^{-kt} + B}$$

where A, B and k are constants to be found.

(3)

The model predicts that there is an upper limit, L microlitres, on the volume of the plant cell.

(d) Find the value of L, giving a reason for your answer.

Question 12 continued

Question 12 continued

Question 12 continued

(Total for Question 12 is 12 marks)

Question 6

13. The world human population, P billions, is modelled by the equation

$$P = ab^t$$

where a and b are constants and t is the number of years after 2004

Using the estimated population figures for the years from 2004 to 2007, a graph is plotted of $\log_{10} P$ against t.

The points lie approximately on a straight line with

- gradient 0.0054
- intercept 0.81 on the $\log_{10} P$ axis

(a) Estimate, to 3 decimal places, the value of a and the value of b.

(4)

In the context of the model,

(b) (i) interpret the value of the constant a,

(ii) interpret the value of the constant b.

(2)

(c) Use the model to estimate the world human population in 2030

(2)

(d) Comment on the reliability of the answer to part (c).

(1)

Question 13 continued

Question 13 continued
Question 13 continued

(Total for Question 13 is 9 marks)

Question 7

3. Given that

$$\log_2(x + 3) + \log_2(x + 10) = 2 + 2\log_2 x$$

(a) show that

$$3x^2 - 13x - 30 = 0$$

(3)

(b) (i) Write down the roots of the equation

$$3x^2 - 13x - 30 = 0$$

(ii) Hence state which of the roots in part (b)(i) is not a solution of

$$\log_2(x + 3) + \log_2(x + 10) = 2 + 2\log_2 x$$

giving a reason for your answer.

Question 3 continued

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(Total for Question 3 is 5 marks)

Question 8

7. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$x^3 + 2xy + 3y^2 = 47$$

(a) Find $\frac{dy}{dx}$ in terms of x and y

(4)

The point $P(-2, 5)$ lies on the curve.

(b) Find the equation of the normal to the curve at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

Question 7 continued

DO NOT W

(Total for Question 7 is 7 marks)

Question 9

8. (a) Express $2\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants,

$R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

The first three terms of an arithmetic sequence are

$$\cos x \qquad \cos x + \sin x \qquad \cos x + 2\sin x \qquad x \neq n\pi$$

Given that S_9 represents the sum of the first 9 terms of this sequence as x varies,

(b) (i) find the exact maximum value of S_9

(ii) deduce the smallest positive value of x at which this maximum value of S_9 occurs.

(3)

Question 8 continued

Question 8 continued

Question 8 continued

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(Total for Question 8 is 6 marks)

Question 10

11.

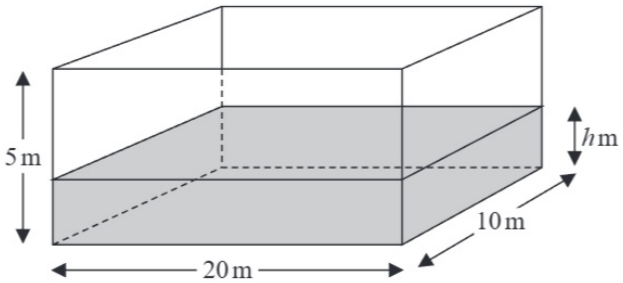

Figure 1

A tank in the shape of a cuboid is being filled with water.

The base of the tank measures 20 m by 10 m and the height of the tank is 5 m, as shown in Figure 1.

At time t minutes after water started flowing into the tank the height of the water was h m and the volume of water in the tank was V m^3

In a model of this situation

- the sides of the tank have negligible thickness
- the rate of change of V is inversely proportional to the square root of h

(a) Show that

$$\frac{dh}{dt} = \frac{\lambda}{\sqrt{h}}$$

where λ is a constant.

(3)

Given that

- initially the height of the water in the tank was 1.44 m
- exactly 8 minutes after water started flowing into the tank the height of the water was 3.24 m

(b) use the model to find an equation linking h with t, giving your answer in the form

$$h^{\frac{3}{2}} = At + B$$

where A and B are constants to be found.

(5)

(c) Hence find the time taken, from when water started flowing into the tank, for the tank to be completely full.

(2)

(Total for Question 11 is 10 marks)

Question 11

12.

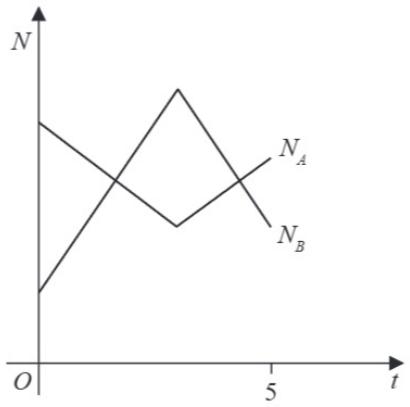

Figure 2

The number of subscribers to two different music streaming companies is being monitored.

The number of subscribers, N_A, in thousands, to **company A** is modelled by the equation

$$N_A = |t - 3| + 4 \qquad t \geqslant 0$$

where t is the time in years since monitoring began.

The number of subscribers, N_B, in thousands, to **company B** is modelled by the equation

$$N_B = 8 - |2t - 6| \qquad t \geqslant 0$$

where t is the time in years since monitoring began.

Figure 2 shows a sketch of the graph of N_A and the graph of N_B over a 5-year period.

Use the equations of the models to answer parts (a), (b), (c) and (d).

(a) Find the initial difference between the number of subscribers to **company A** and the number of subscribers to **company B**.

(2)

When $t = T$ **company A** reduced its subscription prices and the number of subscribers increased.

(b) Suggest a value for T, giving a reason for your answer.

(2)

(c) Find the range of values of t for which $N_A > N_B$ giving your answer in set notation.

(5)

(d) State a limitation of the model used for **company B**.

(1)

(Total for Question 12 is 10 marks)

Question 12

13. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$(3 + x)^{-2}$$

writing each term in simplest form.

(4)

(b) Using the answer to part (a) and using algebraic integration, estimate the value of

$$\int_{0.2}^{0.4} \frac{6x}{(3 + x)^2} \, dx$$

giving your answer to 4 significant figures.

(4)

(c) Find, using algebraic integration, the exact value of

$$\int_{0.2}^{0.4} \frac{6x}{(3 + x)^2} \, dx$$

giving your answer in the form $a \ln b + c$, where a, b and c are constants to be found.

(5)

(Total for Question 13 is 13 marks)

Question 13

14. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2\tan\theta(8\cos\theta + 23\sin^2\theta) = 8\sin 2\theta(1 + \tan^2\theta)$$

may be written as

$$\sin 2\theta(A\cos^2\theta + B\cos\theta + C) = 0$$

where A, B and C are constants to be found.

(b) Hence, solve for $360° \leqslant x \leqslant 540°$

$$2\tan x(8\cos x + 23\sin^2 x) = 8\sin 2x(1 + \tan^2 x) \qquad x \in \mathbb{R} \qquad x \neq 450°$$

(4)

Question 14 continued

Question 14 continued

Question 14 continued

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(Total for Question 14 is 7 marks)