

Question 1

2.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = 2x^3 - 3ax^2 + bx + 8a$$

where a and b are constants.

Given that $(x - 4)$ is a factor of $f(x)$,

(a) use the factor theorem to show that

$$10a = 32 + b$$

(2)

Given also that $(x - 2)$ is a factor of $f(x)$,

(b) express $f(x)$ in the form

$$f(x) = (2x + k)(x - 4)(x - 2)$$

where k is a constant to be found.

(4)

(c) Hence,

(i) state the number of real roots of the equation $f(x) = 0$

(ii) write down the largest root of the equation $f\left(\frac{1}{3}x\right) = 0$

(2)

Question 2 continued

NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 2 continued

THIS AREA DO NOT WRITE IN THIS AREA

DO NOT WRITE IN

(Total for Question 2 is 8 marks)

Question 2

16. A curve has equation $y = f(x)$, $x \geq 0$

Given that

- $f'(x) = 4x + a\sqrt{x} + b$, where a and b are constants
- the curve has a stationary point at $(4, 3)$
- the curve meets the y -axis at -5

find $f(x)$, giving your answer in simplest form.

(6)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 16 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Lined area for writing answers.

(Total for Question 16 is 6 marks)

Question 3

14. Find, in simplest form, the coefficient of x^5 in the expansion of

$$(5 + 8x^2)\left(3 - \frac{1}{2}x\right)^6$$

(5)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 14 continued

DO NOT WRITE IN THIS AREA

DO

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 14 is 5 marks)

Question 4

12.

In this question you must show detailed reasoning.

Solutions relying entirely on calculator technology are not acceptable.

- (a) Show that the equation

$$4 \tan x = 5 \cos x$$

can be written as

$$5\sin^2 x + 4\sin x - 5 = 0 \quad (3)$$

- (b) Hence solve, for $0 < x \leq 360^\circ$

$$4 \tan x = 5 \cos x$$

giving your answers to one decimal place.

- (c) Hence find the **number of solutions** of the equation

$$4 \tan 3x = 5 \cos 3x$$

in the interval $0 < x \leq 1800^\circ$, explaining briefly the reason for your answer.

OT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 12 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Lined area for writing.

Question 12 continued

Lined area for writing.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 12 continued

THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN

(Total for Question 12 is 9 marks)

Question 5

11. The height, h metres, of a plant, t years after it was first measured, is modelled by the equation

$$h = 2.3 - 1.7e^{-0.2t} \quad t \in \mathbb{R} \quad t \geq 0$$

Using the model,

- (a) find the height of the plant when it was first measured, (2)

- (b) show that, exactly 4 years after it was first measured, the plant was growing at approximately 15.3 cm per year. (3)

According to the model, there is a limit to the height to which this plant can grow.

- (c) Deduce the value of this limit. (1)

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 11 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 11 is 6 marks)

Question 6

9. Using the laws of logarithms, solve the equation

$$2\log_5(3x - 2) - \log_5 x = 2$$

(5)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 9 continued

DO NOT WRITE IN THIS AREA

(Total for Question 9 is 5 marks)

Question 7

- From a full tank of fuel, 40 litres remained in the car's fuel tank after the car had travelled 80 km
- From a full tank of fuel, 25 litres remained in the car's fuel tank after the car had travelled 200 km

(a) find an equation linking V with d .

(4)

- the fuel tank of the car was initially full
- the car continued until it ran out of fuel

(b) (i) the initial volume of fuel that was in the fuel tank of the car,

(ii) the distance that the car travelled on this journey.

(3)

(c) Evaluate the model in light of this information.

(1)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Lined area for writing answers.

(Total for Question 7 is 8 marks)

Question 8

6. The circle C has equation

$$x^2 + y^2 - 6x + 10y + k = 0$$

where k is a constant.

- (a) Find the coordinates of the centre of C .

(2)

Given that C does not cut or touch the x -axis,

- (b) find the range of possible values for k .

(3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

DO NOT WRITE IN THIS AREA

[illegible]

Question 9

2.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Using the substitution $u = \sqrt{x}$ or otherwise, solve

$$6x + 7\sqrt{x} - 20 = 0$$

(4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

DO NOT WRITE IN THIS AREA

Lined area for writing answers.

(Total for Question 2 is 4 marks)

Question 10

3.

Given that the equation $f(x) = 0$ has a single root α

- (a) show that α lies in the interval $[3.6, 3.7]$ (2)
- (b) Find $f'(x)$ (2)
- (c) Using 3.7 as a first approximation for α , apply the Newton–Raphson method once to obtain a second approximation for α . Give your answer to 3 decimal places. (2)

Question 3 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

NOT WRITE IN THIS AREA

DO

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 3 is 6 marks)

Question 11

12.

$$y = \sin x$$

where x is measured in radians.

Use differentiation from first principles to show that

$$\frac{dy}{dx} = \cos x$$

You may

- use without proof the formula for $\sin(A \pm B)$
- assume that as $h \rightarrow 0$, $\frac{\sin h}{h} \rightarrow 1$ and $\frac{\cos h - 1}{h} \rightarrow 0$

(5)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 12 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Lined area for writing answers.

(Total for Question 12 is 5 marks)

Question 12

7. The function f is defined by

$$f(x) = 3 + \sqrt{x-2} \quad x \in \mathbb{R} \quad x > 2$$

- (a) State the range of f

(1)

- (b) Find f^{-1}

(3)

The function g is defined by

$$g(x) = \frac{15}{x-3} \quad x \in \mathbb{R} \quad x \neq 3$$

- (c) Find $gf(6)$

(2)

- (d) Find the exact value of the constant a for which

$$f(a^2 + 2) = g(a)$$

(2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Lined area for writing answers.

Question 7 continued

Lined area for writing answers.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

DO NOT WRITE IN

(Total for Question 7 is 8 marks)

Question 13

5. A continuous curve has equation $y = f(x)$.

The table shows corresponding values of x and y for this curve, where a and b are constants.

x	3	3.2	3.4	3.6	3.8	4
y	a	16.8	b	20.2	18.7	13.5

The trapezium rule is used, with all the y values in the table, to find an approximate area under the curve between $x = 3$ and $x = 4$

Given that this area is 17.59

- (a) show that $a + 2b = 51$

(3)

Given also that the sum of all the y values in the table is 97.2

- (b) find the value of a and the value of b

(3)

Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

[illegible]

Question 14

3. Relative to a fixed origin O

- the point A has position vector $5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$
- the point B has position vector $2\mathbf{i} + 4\mathbf{j} + a\mathbf{k}$

where a is a positive integer.

(a) Show that $|\vec{OA}| = \sqrt{38}$

(1)

(b) Find the smallest value of a for which

$$|\vec{OB}| > |\vec{OA}|$$

(2)

DO NOT WRITE IN THIS AREA

Question 3 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Lined area for writing answers.

(Total for Question 3 is 3 marks)

Question 15

2.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = 4x^3 + 5x^2 - 10x + 4a \quad x \in \mathbb{R}$$

where a is a positive constant.

Given $(x - a)$ is a factor of $f(x)$,

(a) show that

$$a(4a^2 + 5a - 6) = 0 \quad (2)$$

(b) Hence

(i) find the value of a

(ii) use algebra to find the exact solutions of the equation

$$f(x) = 3 \quad (4)$$

Question 2 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 2 is 6 marks)

Question 16

1. Find

$$\int \frac{x^{\frac{1}{2}}(2x-5)}{3} dx$$

writing each term in simplest form.

(4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 1 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Lined area for writing answers.

(Total for Question 1 is 4 marks)

Question 17

9. The first 3 terms of a geometric sequence are

$$3^{4k-5} \quad 9^{7-2k} \quad 3^{2(k-1)}$$

where k is a constant.

- (a) Using algebra and making your reasoning clear, prove that $k = \frac{5}{2}$ (3)
- (b) Hence find the sum to infinity of the geometric sequence. (3)

Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA