Question 1

Question	Scheme	Marks	AOs
2(a)	$\left(\text{f}(4)=\right)\ 2\times4^3-3a\times4^2+4b+8a=0$	M1	1.1b
	$128+4b=40a\Rightarrow32+b=10a$ *	A1*	1.1b
		(2)	
(b)	$\text{f}(2)=2\times2^3-3a\times2^2+2b+8a=0\Rightarrow8+b=2a$	M1	1.1b
	Solve simultaneously $\Rightarrow a=...$ or $\Rightarrow b=...$	dM1	2.1
	$a=3$ or $b=-2$ or $k=3$	A1	1.1b
	$\left(\text{f}(x)=\right)\ (2x+3)(x-4)(x-2)$	A1	1.1b
		(4)	
(c)(i)	3	B1	1.1b
(ii)	12	B1ft	2.2a
		(2)	
			(8 marks)

Notes

(a)

M1: Attempts $\text{f}(4)=0$ leading to an equation in a and b only. Condone slips. The = 0 may be implied by further work for this mark. Attempts using algebraic division score M0A0.

A1*: Simplifies and rearranges to the given answer with no errors. There must be at least one intermediate stage of working between their first expression or equation and the given answer and = 0 must be correctly seen at some point in their solution or at the start e.g. stating $\text{f}(4)=0$. Isw if they achieve the given answer but then attempt to make a or b the subject.
Minimum acceptable is e.g. $128-48a+4b+8a\Rightarrow128-40a+4b=0\Rightarrow32+b=10a$
Note: $128-48a+4b+8a\Rightarrow128+4b=40a\Rightarrow32+b=10a$ is M1A0* (we do not see = 0 correctly at some point or e.g. $\text{f}(4)=0$)

(b) **Note that there are many different equations which can be formed.**
Sight of $a=3$ or $b=-2$ or $k=3$ scores the first 3 marks BUT answers with no working – send to review

M1: Attempts $\text{f}(2)=0$ to form another equation in a and b. Does not need to be simplified. Condone slips substituting in 2 (but not if there is a clear intention to substitute in -2) and = 0 may be implied by further work. Accept attempts to divide algebraically by $x-2$: e.g. condone slips but they must have a quadratic quotient $2x^2\pm(...a\pm...)x$ and proceed as far as a remainder of the form $...a\pm...b\pm...$ which is then set equal to 0.

$$\begin{array}{rllll}
 & 2x^2 & +(-3a+4)x & +(b-6a+8) & \\
\text{e.g. } x-2\,\big) & 2x^3 & -3ax^2 & +bx & +8a \\
 & 2x^3 & -4x^2 & & \\
 & & (-3a+4)x^2 & +bx & \\
 & & (-3a+4)x^2+(6a-8)x & & \\
 & & & (b-6a+8)x & +8a \\
 & & & (b-6a+8)x-2b+12a-16 & \\
 & & & & -4a+2b+16=0
\end{array}$$

You may also see an attempt at the grid method:

	$2x^2$	$+(4-3a)x$	$+(8-6a+b)$
x	$2x^3$	$+(4-3a)x^2$	$+(8-6a+b)x$
-2	$-4x^2$	$(-8+6a)x$	$-16+12a-2b$

$$-16+12a-2b=8a$$

In this method condone slips but they need to proceed as far as a quadratic quotient of the form $2x^2 \pm (...a \pm ...)x$ and the bottom right cell of the form $...a \pm ..b \pm ...$ which is then set equal to $8a$.

Look out for other valid methods such as

- multiplying out $\mathrm{f}(x) = (2x+k)(x-4)(x-2)$ to achieve a cubic where the coefficients of x^2 and *x*, and the constant term are all in terms of *k*, and equating coefficients to form two more simultaneous equations
- algebraically dividing $2x^3 - 3ax^2 + bx + 8a$ by $x^2 - 6x + 8$ and setting their remainder equal to zero. Look for a quotient of the form $2x \pm ...a \pm ...$
- substituting $b = 10a - 32$ into $\mathrm{f}(x)$ so that the polynomial coefficients are in terms of *a* only (or equivalently all in *b* only) and using $\mathrm{f}(2) = 0$ to form an equation in one variable.

Send to review if you are unsure.

dM1: Attempts to solve their equations simultaneously to find a value for *a* or *b* (or they may proceed directly to finding a value for *k*). This may be done on a calculator. You do not need to check their method for solving. It is dependent on the previous method mark.

A1: $a = 3$ or $b = -2$ or $k = 3$

A1: $\left(\mathrm{f}(x) =\right)(2x+3)(x-4)(x-2)$ (All on one line) (Stating the values of *a*, *b* and *k* alone does not score this mark). Allow to be scored if seen in (c).

Alternative Further Maths method using the sum and product of the roots

M1: $\alpha + \beta + \gamma = \frac{3a}{2} \Rightarrow 4 + 2 - \frac{k}{2} = \frac{3a}{2}$ or $\alpha\beta\gamma = -\frac{8a}{2} \Rightarrow 4 \times 2 \times \left(-\frac{k}{2}\right) = -\frac{8a}{2}$

A1: $\alpha + \beta + \gamma = \frac{3a}{2} \Rightarrow 4 + 2 - \frac{k}{2} = \frac{3a}{2}$ and $\alpha\beta\gamma = -\frac{8a}{2} \Rightarrow 4 \times 2 \times \left(-\frac{k}{2}\right) = -\frac{8a}{2}$

(Less likely but possible to see $\alpha\beta + \beta\gamma + \alpha\gamma = \frac{b}{2} \Rightarrow 4 \times 2 + 2 \times \left(-\frac{k}{2}\right) + 4 \times \left(-\frac{k}{2}\right) = \frac{b}{2}$)

dM1A1A1: See notes above

(c)

(i)

B1: 3 (listing the actual roots only is B0)

(ii)

B1ft: 12 only Follow through on their $2x + k \Rightarrow -\frac{3k}{2}$ if $k < -8$

Question 2

Question	Scheme	Marks	AOs
16	Sets $f'(4) = 0 \Rightarrow 16 + 2a + b = 0$	M1	2.1
	Integrates $f'(x) = 4x + a\sqrt{x} + b \Rightarrow \{f(x) =\} 2x^2 + \frac{2}{3}ax^{\frac{3}{2}} + bx \ \{+c\}$	M1 A1ft	1.1b 1.1b
	Deduces that $c = -5$	B1	2.2a
	Full and complete method using the given information $f'(4) = 0$ and $f(4) = 3$ in order to find values for a and b Note: $a = -15$ and $b = 14$	ddM1	3.1a
	$\{f(x) =\} 2x^2 - 10x^{\frac{3}{2}} + 14x - 5$	A1	1.1b
		(6)	
		(6 marks)	

Notes:

M1: For the key step in setting $f'(4) = 0 \Rightarrow 16 + 2a + b = 0$ to set up an equation in a and b.
Condone slips.

M1: For attempting to integrate $f'(x)$. Award for $x^n \to x^{n+1}$ or $b \to bx$
This may come after finding values for a or b or both.

A1ft: $\{f(x) =\} 2x^2 + \frac{2}{3}ax^{\frac{3}{2}} + bx \ \{+c\}$ or, e.g., $\{f(x) =\} 2x^2 + \frac{2}{3}ax^{\frac{3}{2}} + (-16 - 2a)x \ \{+c\}$
Allow ft on their b in terms of a if they substituted in from their $f'(4) = 0 \Rightarrow 16 + 2a + b = 0$
Do not ft if they have a value(s) for a or b
This may be left unsimplified but the indices must be processed.
isw once the mark is awarded. Condone the omission of the $+ c$
This accuracy mark requires only the previous M mark to be scored.

B1: Deduces that the constant term in $f(x)$ is -5.
Note that deducing $b = -5$ is B0. It must be the constant in a changed function.

ddM1: For a complete strategy to find values for both a and b.
Do not be concerned about the logistics of how they solve the simultaneous equations – this may be done on a calculator.
Note: $a = -15$ and $b = 14$
This is dependent on **both** previous method marks and so must include use of both

- $f'(4) = 0$ (their $16 + 2a + b = 0$ o.e.)
- $f(4) = 3$ (their $32 + \frac{16}{3}a + 4b - 5 = 3$ o.e.)

A1: $\{f(x) =\} 2x^2 - 10x^{\frac{3}{2}} + 14x - 5$ or exact simplified equivalent, e.g., use of $x\sqrt{x}$ in place of $x^{\frac{3}{2}}$
Apply isw once a correct expression is seen.

Question 3

Question	Scheme	Marks	AOs
14	Attempts the term in x^3 or the term in x^5 of $\left(3-\frac{1}{2}x\right)^6$ Look for ${}^6C_3 3^3\left(-\frac{1}{2}x\right)^3$ or ${}^6C_5 3^1\left(-\frac{1}{2}x\right)^5$	M1	3.1a
	Correct term in x^3 or correct term in x^5 of $\left(3-\frac{1}{2}x\right)^6$ $-\frac{135}{2}x^3$ **or** $-\frac{9}{16}x^5$	A1	1.1b
	Attempts one of the required terms in x^5 of $\left(5+8x^2\right)\left(3-\frac{1}{2}x\right)^6$ Either $5\times{}^6C_5 3^1\left(-\frac{1}{2}x\right)^5$ **or** $8x^2\times{}^6C_3 3^3\left(-\frac{1}{2}x\right)^3$	M1	1.1b
	Attempts the sum of $5\times{}^6C_5 3^1\left(-\frac{1}{2}x\right)^5$ and $8x^2\times{}^6C_3 3^3\left(-\frac{1}{2}x\right)^3$	dM1	2.1
	Coefficient of x^5 $=-\frac{45}{16}-540=-\frac{8685}{16}$	A1	1.1b
		(5)	

(5 marks)

Notes:

M1: For the key step in attempting to find one of the required terms in the expansion of $\left(3-\frac{1}{2}x\right)^6$ to enable the problem to be solved.

Look for ${}^6C_3 3^3\left(-\frac{1}{2}x\right)^3$ **or** ${}^6C_5 3^1\left(-\frac{1}{2}x\right)^5$ but condone missing brackets and slips in signs.

May be part of a complete expansion but only one of the required terms needs to be of the correct form.

A1: For $-\frac{135}{2}\left\{x^3\right\}$ **or** $-\frac{9}{16}\left\{x^5\right\}$ which may be unsimplified but the ${}^6C_3$ or ${}^6C_5$ must be processed. May be implied by $-540\left\{x^5\right\}$ **or** $-\frac{45}{16}\left\{x^5\right\}$

M1: Attempts one of the required terms in x^5 of the expansion of $\left(5+8x^2\right)\left(3-\frac{1}{2}x\right)^6$

Look for $5\times{}^6C_5 3^1\left(-\frac{1}{2}x\right)^5$ **or** $8x^2\times{}^6C_3 3^3\left(-\frac{1}{2}x\right)^3$ which would also imply the previous M.

The x^5 may be missing as just the coefficient is required.

May be implied by $-540\left\{x^5\right\}$ **or** $-\frac{45}{16}\left\{x^5\right\}$

Condone missing brackets and signs.

You might see candidates make a slip in, e.g., their binomial coefficients, but have an (essentially) correct method to solve the problem.

Note that this M mark is not dependent on the first, so you may be able to award it even if they have made a slip in finding their x^3 or x^5 term in the expansion.

dM1: Attempts the sum of $5\times {}^6C_5 3^1\left(-\frac{1}{2}x\right)^5$ and $8x^2\times {}^6C_3 3^3\left(-\frac{1}{2}x\right)^3$

Dependent on the previous M but may be scored at the same time.

The x^5 may be missing as just the coefficients are required.
Condone missing brackets and signs.

A1: $-\frac{8685}{16}$ or exact equivalent, -542.8125 and apply isw

Condone $-\frac{8685}{16}x^5$ for A1

Note that rounded decimals, e.g., -542.81 will not score the last mark.

Note that full marks can be scored for concise solutions such as:

$$5\times {}^6C_5\times 3\times\left(-\frac{1}{2}\right)^5 \quad + \quad 8\times {}^6C_3\times 3^3\times\left(-\frac{1}{2}\right)^3 \quad = -\frac{8685}{16}$$

Alternative
Attempts via the taking out of the common factor can be scored in the same way.

$$\left(3-\frac{1}{2}x\right)^6 = 3^6\left\{1+6\times\left(-\frac{1}{6}x\right)^1+\frac{6\times5}{2}\left(-\frac{1}{6}x\right)^2+\frac{6\times5\times4}{3!}\left(-\frac{1}{6}x\right)^3+\frac{6\times5\times4\times3}{4!}\left(-\frac{1}{6}x\right)^4+\frac{6\times5\times4\times3\times2}{5!}\left(-\frac{1}{6}x\right)^5+\left(-\frac{1}{6}x\right)^6\right\}$$

For M1 A1 look for $3^6\times\frac{6\times5\times4}{3!}\left(-\frac{1}{6}x\right)^3$ **or** $3^6\times\frac{6\times5\times4\times3\times2}{5!}\left(-\frac{1}{6}x\right)^5$

Score the remaining marks as per the main scheme.

Question 4

Question	Scheme	Marks	AOs
12(a)	States or uses $\tan x = \dfrac{\sin x}{\cos x}$	B1	1.2
	$4\sin x = 5\cos^2 x \Rightarrow 4\sin x = 5\left(1-\sin^2 x\right)$	M1	1.1b
	$5\sin^2 x + 4\sin x - 5 = 0$ *	A1*	2.1
		(3)	
(b)	Attempts to solve $5\sin^2 x+4\sin x-5=0 \Rightarrow \sin x = ...$	M1	1.1b
	$\sin x = \dfrac{-2\pm\sqrt{29}}{5}$ $(\sin x = \text{awrt } 0.677)$	A1	1.1b
	Takes $\sin^{-1}$ leading to at least one answer in the range	dM1	1.1b
	$x = \text{awrt } 42.6\{°\}$ and $x = \text{awrt } 137.4\{°\}$ only	A1	1.1b
		(4)	
(c)	$15\times"2"=30$ following through on their "2"	B1ft	2.2a
	Explains either "mathematically" by stating $3\times5\times$ their number in range 0 to 360° or 'in words" e.g., stating 3 $\times"2"$ values every 360° and 5 lots of 360°	B1ft	2.4
		(2)	
		(9 marks)	

Notes:

(a) **Allow use of e.g. θ but the final mark requires the equation to be in terms of x**

B1: States or uses $\tan x = \dfrac{\sin x}{\cos x}$ e.g., $4\tan x = 5\cos x \Rightarrow 4\dfrac{\sin x}{\cos x} = 5\cos x$ Allow e.g. $\tan x = \dfrac{\sin\theta}{\cos\theta}$

M1: Multiplies by $\cos x$ and uses $\cos^2 x = 1-\sin^2 x$ to set up a quadratic equation in just $\sin x$
Condone mixed arguments here.

A1*: Proceeds to $5\sin^2 x + 4\sin x - 5 = 0$ with correct notation and algebra, showing all key steps.
The $=0$ must be present in the final answer line.
Condone a single slip in notation, e.g., $\sin x^2$ or $\sin\theta$ seen once.

(b)

M1: Attempts to solve $5\sin^2 x + 4\sin x - 5 = 0 \Rightarrow$ **$\sin x = ...$** using the usual rules.
$\sin x =$ may be implied later.
Allow solution(s) from a calculator but one must be correct (0.6 or 0.7 or –1.4 or –1.5)

A1: Achieves $\sin x = \dfrac{-4\pm\sqrt{116}}{10}$ $(\sin x = \text{awrt } 0.677)$ $\sin x =$ may be implied later.

dM1: Finds one value of x in the range 0 to 360° from their $\sin x =$
May be scored for working in radians. If using $\sin x =$ 0.677 they should have awrt 0.744 or awrt 2.40
If they have made a slip in solving the quadratic, e.g., by the formula, then their values will need checking both in degrees and radians to see if this mark can be implied.

A1:	$x =$ awrt $42.6\{°\}$ and $x =$ awrt $137.4\{°\}$ only. Ignore any values outside of 0 to 360° isw if they round their values to e.g., 3sf after stating acceptable answers. There must be some evidence that the quadratic has been solved.
(c)	
B1ft:	Follow through on 15 multiplied by the number of solutions in (b) in the range 0 to 360° If working in radians in (b), they must state 30 (solutions).
B1ft:	Explains either mathematically **or** in words. See scheme. Note that you might see arguments expanding the range from 1800 to 5400 to account for the stretch parallel to the x axis. $\frac{5400}{360} = 15$ and $15 \times 2 = 30$ which is also acceptable.
Note:	If candidates list 30 values and conclude that there are 30 solutions, score B1ftB1ft There is no need to check their 30 values are correct, but there must be 30.

Question 5

Question	Scheme	Marks	AOs
11(a)	$h = 2.3 - 1.7e^{0}$	M1	3.4
	Either 0.6 {m} or 60 cm	A1	1.1b
		(2)	
(b)	$\left\{\frac{dh}{dt} =\right\} 0.34e^{-0.2t}$	M1	3.1b
	At $t = 4 \Rightarrow$ Rate of growth is $0.34e^{-0.2\times4} = 0.15277...\{\text{m / year}\}$	dM1	3.4
	0.153 {m per year} = 15.3 cm {per year} *	A1*	1.1b
		(3)	
(c)	2.3 (m)	B1	2.2a
		(1)	
			(6 marks)

Notes:

(a)

M1: Substitutes $t = 0$ into $h = 2.3 - 1.7e^{-0.2t}$ Implied by e.g., $h = 2.3 - 1.7e^{-0}$ or $h = 0.6$

A1: Allow 0.6, 0.6 m, or 60 cm and isw after a correct height. Allow $\frac{3}{5}$

The M mark may be implied by A1.

(b)

M1: Links rate of change to gradient and differentiates $h = 2.3 - 1.7e^{-0.2t}$ to $k\,e^{-0.2t}$, $k \neq -1.7$

Accept, e.g., $-0.2 \times -1.7e^{-0.2t}$ Must be seen in (b).

dM1: Substitutes $t = 4$ into $k\,e^{-0.2t}$, $k \neq -1.7$ and calculates its value.

A1*: Fully correct. Requires

- sight of $\left\{\frac{dh}{dt} =\right\} 0.34e^{-0.2t}$ o.e., e.g., $\left\{\frac{dh}{dt} =\right\} \frac{17}{50}e^{-0.2t}$ or $\left\{\frac{dh}{dt} =\right\} -0.2 \times -1.7e^{-0.2t}$
- $\left\{\frac{dh}{dt} =\right\}$ awrt 0.153 {metres per year}
- changing to awrt 15.3 cm {per year}.

Note: Substituting $t = 4$ into $h = 2.3 - 1.7e^{-0.2t}$ gives $h = 1.536...$ scores M0dM0A0 unless differentiation and further correct work is seen separately.

(c)

B1: Allow 2.3, 2.3 m, or 230 cm

$2.2\dot{9}$ and 2.2999… which clearly continues are both acceptable, but 2.29999999 is not.

Question 6

Question	Scheme	Marks	AOs
9	$2\log_5(3x-2)-\log_5 x=2$		
	Uses one correct law e.g. $2\log_5(3x-2)\to\log_5(3x-2)^2$ **or** $2\to\log_5 25$ **or** $\log_5 ...=2\to ...=5^2$	B1	1.1a
	Uses two correct log laws: either $2\log_5(3x-2)\to\log_5(3x-2)^2$ **and** $2\to\log_5 25$ or $2\log_5(3x-2)-\log_5 x\to\log_5\frac{(3x-2)^2}{x}$ leading to an equation without logs	M1	3.1a
	Correct equation without logs, usually $\frac{(3x-2)^2}{x}=25$	A1	1.1b
	$\frac{(3x-2)^2}{x}=25\Rightarrow 9x^2-37x+4=0\Rightarrow(9x-1)(x-4)=0\Rightarrow x=...$	dM1	1.1b
	$x=4$ only	A1 cso	3.2a
		(5)	
			(5 marks)

Notes:

B1: Uses one correct log law. The base does not need to be seen for this mark.
This mark is independent of any other errors they make.

M1: This can be awarded for the overall strategy leading to an equation in x **not involving logs**.
It requires the correct use of two log laws as in the main scheme to reach an equation in x
This mark may **not** be awarded for correct application of two laws following incorrect log work, but numerical slips are condoned.

A1: For a correct unsimplified equation with logs removed and **no incorrect work seen.**
Ignore any incorrect simplification of their equation.

Allow recovery on missing brackets, e.g., $\log_5\frac{3x-2^2}{x}=2\to\frac{9x^2-12x+4}{x}=25$

Correct equations are likely to be $\frac{(3x-2)^2}{x}=25$ or, e.g., $(3x-2)^2=25x$ but you might see

$9x-12+\frac{4}{x}=25$ Sight of a correct equation does **not** imply either the previous M1 or the A1.

Note: $\frac{\log_5(3x-2)^2}{\log_5 x}=2\to\frac{(3x-2)^2}{x}=25$ may be seen and scores B1M0A0.

dM1: For a correct method to solve their equation, **via a 3TQ set = 0**
The 3TQ may be solved by calculator - you may need to check their value(s).
Can be implied by one correct value for their 3TQ set = 0 correct to 1d.p.

A1:	cso $x = 4$ only. If $x = \frac{1}{9}$ is also given it must be rejected. $x = 0$ might also be seen and must be rejected. Ignore any reasoning for rejecting any values. Note that calculators can solve the equation at any stage and so full log work must be shown leading to a 3TQ set = 0.

Question 7

Question	Scheme	Marks	AOs
7 (a)	Uses or implies that $V = ad + b$	B1	3.3
	Uses both $40 = 80a + b$ and $25 = 200a + b$ to get either a or b	M1	3.1b
	Uses both $40 = 80a + b$ and $25 = 200a + b$ to get both a and b	dM1	1.1b
	$\Rightarrow V = -\frac{1}{8}d + 50$ o.e.	A1	1.1b
		(4)	
(b)(i)(ii)	States **either** that the initial volume was 50 {litres} **or** that the distance travelled was 400 {km}	B1 ft	3.4
	Attempts to find **both** answers by solving $0 = -\frac{1}{8}d + 50$ **and** $0 = 400 - 8V$	M1	3.4
	States **both** that the initial volume was 50 litres **and** that the distance travelled was 400 km	A1	3.2b
		(3)	
(c)	States, e.g., "Poor model" as 320km is significantly less than 400 km.	B1ft	3.5a
		(1)	
		(8 marks)	

Notes:

(a)

B1: Attempts a linear model, i.e., uses or implies that $V = ad + b$ or $d = mV + c$ which may be in terms of, e.g., y and x

M1: Awarded for translating the problem in context and starting to solve.
It is scored when both $40 = 80a + b$ and $25 = 200a + b$ are written down and the candidate proceeds to find either a or b
Alternatively, scored when both $200 = 25m + c$ and $80 = 40m + c$ are written down and the candidate proceeds to find either m or c
You may just see $\pm\frac{25-40}{200-80}$ or $\pm\frac{200-80}{25-40}$ or 8km for every litre o.e. so check carefully for attempts at the gradient.

dM1: Uses $40 = 80a + b$ and $25 = 200a + b$ to find both a and b (or m and c)
Alternatively, if the gradient is found, proceeds to use one of the bullet points to find c, with the usual rules applying for straight line (coordinates must be used the correct way round, i.e., they would lead to the correct answer).

A1: $V = -\frac{1}{8}d + 50$ or exact equivalent, e.g., $d = 400 - 8V$ or $d + 8V = 400$ etc.
Withhold this mark if their answer is not stated in terms of V and d

Mark parts (b)(i) and (b)(ii) together. Note that they may restart and not use an equation.

B1ft: States **either** the initial volume was 50 {litres} **or** the distance travelled was 400 {km} but it must be clearly for the correct part, e.g., $V = 50$.
Follow through on their a and b (or m and c). This may be scored from $40 + \frac{80}{8}$ or $\frac{400}{8}$

M1: Complete attempt to find both answers. Must be from a **linear** model.

	Substitutes $V = 0$ and finds d by attempting to solve their $0 = -\frac{1}{8}d + 50$
	and substitutes $d = 0$ and finds V by attempting to solve their $0 = 400 - 8V$
	Note that one (or both) of these attempts may be implied by correct values ft their equations.
A1:	States both 50 litres and 400 km. Units are required to be correct for both values.
	It must be clear which answer applies to each part, which may be simply by correct units.
	Accept l or L for litres.
(c)	
B1ft:	**Main Scheme (comparing (b)(ii) with 320)**
	This mark is only available for answers from (b)(ii) if they are < 290 **or** > 350
	Concludes **poor** model (o.e.) and states that 320 is **significantly** less than "400" (o.e.)
	Note that $320 << 400$ so it is a poor model is acceptable.
	It is not sufficient to say $320 \neq 400$ or $320 < 400$ so it is a poor model.
	Condone "the 400 is **too** far away from 320".
	Alternative (finding remaining fuel after 320 km)
	States **poor** model (o.e.) because after 320 km the model predicts there will be 10 litres left, which is **significantly** more than an empty tank / **much** too high compared to an empty tank (o.e.).

Question 8

Question	Scheme	Marks	AOs
6 (a)	$x^2+y^2-6x+10y+k=0$		
	$(x-3)^2+(y+5)^2 \pm ... = ...$	M1	1.1b
	Centre $(3,-5)$	A1	1.1b
		(2)	
(b)	Deduces that $k=9$ is a critical point	B1ft	2.2a
	Recognises that radius > 0 $"9"+"25"-k>0$	M1	3.1a
	$9<k<34$	A1	1.1b
		(3)	
			(5 marks)

Notes:

(a)

M1: For sight of $(x\pm3)^2 \pm (y\pm5)^2 \pm ... = ...$ or one coordinate for centre from $(\pm3, \pm5)$

A1: Centre $(3,-5)$

(b)

B1ft: Deduces that $k...9$ is a critical point. Allow this to come from their $("5")^2$ Condone $\frac{36}{4}$

Note that this might come from setting $y=0$ and using the discriminant on $x^2-6x+k=0$

M1: $(x\pm3)^2+(y\pm5)^2=("3")^2+("5")^2-k$ **and** recognises that the radius^2 must be positive so $("3")^2+("5")^2-k>0$ but condone $("3")^2+("5")^2-k \geqslant 0$

$k<34$ **or** $k \leqslant 34$ would imply this method mark.

Note: they may have incorrectly calculated $("3")^2+("5")^2$ in (a) so allow their value for this in place of $("3")^2+("5")^2$ as long as the intention is clear.

A1: $9<k<34$ but condone $9<k\leqslant 34$. Allow inequalities to be separate, i.e., $k>9, k<34$

Set notation may be seen $\{k:k>9\}\cap\{k:k<34\}$ **or** $k\in(9,34)$

Condone $\{k:k>9\}\cap\{k:k\leqslant 34\}$ **or** $k\in(9,34]$ **or** $k>9$ and $k\leqslant 34$

Must not be combined incorrectly, e.g., $\{k:k>9\}\cup\{k:k<34\}$

Question 9

Question	Scheme	Marks	AOs
2	Let $u=\sqrt{x}$ $\quad 6x+7\sqrt{x}-20=0 \Rightarrow 6u^2+7u-20=0$ $\Rightarrow (3u-4)(2u+5)\{=0\}$	M1A1	1.1b 1.1b
	Attempts $\sqrt{x}=\text{"}\frac{4}{3}\text{"},\text{"}-\frac{5}{2}\text{"} \Rightarrow x=\ldots$	M1	1.1b
	$x=\frac{16}{9}$ only	A1 cso	2.3
		(4)	
			(4 marks)
Alt 1	$6x+7\sqrt{x}-20=0 \Rightarrow 7\sqrt{x}=20-6x \Rightarrow 49x=(20-6x)^2$ $\Rightarrow 49x=400-240x+36x^2$	M1	1.1b
	$36x^2-289x+400\{=0\}$	A1	1.1b
	$(9x-16)(4x-25)=0$	M1	1.1b
	$x=\frac{16}{9}$ only	A1 cso	2.3
		(4)	
Alt 2	$6x+7\sqrt{x}-20=0 \Rightarrow (3\sqrt{x}-4)(2\sqrt{x}+5)=0$	M1 A1	1.1b 1.1b
	Attempts $\sqrt{x}=\text{"}\frac{4}{3}\text{"},\text{"}-\frac{5}{2}\text{"} \Rightarrow x=\ldots$	M1	1.1b
	$x=\frac{16}{9}$ only	A1 cso	2.3
		(4)	

Notes:

M1: Attempts a valid method that enables the problem to be solved. See General Principles for Pure Mathematics Marking at the front of the mark scheme for guidance. Score for either letting $u=\sqrt{x}$ and attempting to factorise to $(au\pm c)(bu\pm d)$ with $ab=6, cd=20$

or making $7\sqrt{x}$ the subject and attempting to square both sides.

or attempting to factorise to $(a\sqrt{x}\pm c)(b\sqrt{x}\pm d)$ with $ab=6, cd=20$

or by quadratic formula or completing the square following usual rules.

A1: $(3u-4)(2u+5)\{=0\}$ **or** $36x^2-289x+400\{=0\}$ **or** $(3\sqrt{x}-4)(2\sqrt{x}+5)\{=0\}$

If they use the formula, it must be correct e.g., $u\ \{\text{or}\ \sqrt{x}\}=\frac{-7\pm\sqrt{7^2-4(6)(-20)}}{12}$ followed by $u\ \{\text{or}\ \sqrt{x}\}=\frac{4}{3}$ or equivalent e.g., $\frac{16}{12}$. Ignore if they have $u\ \{\text{or}\ \sqrt{x}\}=-\frac{5}{2}$ or not.

If they complete the square, they must have $\left(u+\frac{7}{12}\right)^2=\frac{529}{144}$ followed by $u\ \{\text{or}\ \sqrt{x}\}=\frac{4}{3}$ or equivalent e.g., $\frac{16}{12}$. Ignore if they have $u\ \{\text{or}\ \sqrt{x}\}=-\frac{5}{2}$ or not.

M1: Correct method from $p\sqrt{x}\pm q=0$ leading to $x=\ldots$ by squaring

In Alt 1, it is for solving their quadratic using the General Principles for Pure Mathematics Marking. There must be a method shown, i.e., the solutions should not come straight from a calculator. If attempting to factorise, it must be to $(ax\pm c)(bx\pm d)$ with $ab=36, cd=400$

In Alt 2, it is for squaring their value(s) for u to get $x=\ldots$

A1: **cso** $x=\frac{16}{9}$ only. $x=\frac{25}{4}$ must be discarded. Note 0011 is not possible.

Allow "incorrect" $x=-\frac{16}{9}$ or $x=-\frac{25}{4}$ to be seen as long as they are discarded.

Ignore any reason they give for rejecting solutions.

Note that a method to solve their quadratic must be seen – solutions must not come directly from a calculator. Simply stating the quadratic formula (without substitution) is insufficient.

Question 10

Question	Scheme	Marks	AOs
3(a)	$\{f(3.6)=\}\ 3.6+\tan\left(\frac{1}{2}(3.6)\right) = -0.686\ldots < 0$ **and** $\{f(3.7)=\}\ 3.7+\tan\left(\frac{1}{2}(3.7)\right) = 0.211\ldots > 0$	M1	1.1b
	<u>Change of sign</u> and function is <u>continuous</u> in the interval $\Rightarrow$ <u>conclusion</u> e.g. "there is a root in [3.6, 3.7]" *	A1*	2.4
		(2)	
(b)	Use of $\tan\left(\frac{1}{2}x\right) \to \ldots\sec^2\left(\frac{1}{2}x\right)$	M1	1.1b
	$\{f'(x)=\}\ 1+\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$	A1	1.1b
		(2)	
(c)	Attempts $3.7 - \dfrac{3.7+\tan\left(\frac{1}{2}(3.7)\right)}{"1+\frac{1}{2}\sec^2\left(\frac{1}{2}(3.7)\right)"} = \ldots$ (N.B. $f(3.7) = 0.211\ldots$ and $f'(3.7) = 7.58\ldots$)	M1	1.1b
	α = awrt 3.672	A1	1.1b
		(2)	
			(6 marks)

Notes

(a)

M1: Attempts both f(3.6) and f(3.7) or a narrower interval that contains the root 3.672…
(which may be implied by sight of f(3.6) = … and f(3.7) = … with at least one correct)
and obtains at least one correct to 1 significant figure (rounded or truncated) for their interval
and considers their signs. Use of degrees is M0.
Some examples for consideration of sign (which are also sufficient for the change of sign part of reasoning for the A1):

- f(3.6) = –0.6 < 0 and f(3.7) = 0.2 > 0
- $f(3.6)\times f(3.7) < 0$
- f(3.6) = –0.7, f(3.7) = 0.2 "change in sign"

For reference f(3.6) = –0.68626… and f(3.7) = 0.21194…

A1*: This mark requires:

- both f(3.6) and f(3.7) correct to 1 significant figure (rounded or truncated) (or their values correct to 1 significant figure if using a narrower interval)
- a reference to sign change
- a reference to continuity {of f(x)}
- a (minimal) conclusion, e.g. "hence root", "proved", ✓, #, QED, $3.6 < \alpha < 3.7$

Accept as a minimum, "change of sign, continuous, root".
Do not condone "change in sign therefore continuous" or other incorrect statements such as "x is continuous", "the interval is continuous" – these score A0.
Condone "the graph is continuous".
Condone reference to x in place of α in their conclusion, e.g. "hence x lies in the interval".
Condone statements such as "there is at least one root" in place of their conclusion.

(b)	**Note:** Their answer to (b) may be seen in part (c) provided that they have not clearly attempted part (b) incorrectly, e.g., an attempt at $f^{-1}(x)$ in (b).
M1:	For $\tan\left(\frac{1}{2}x\right) \to ...\sec^2\left(\frac{1}{2}x\right)$ o.e. The brackets are not required. You may see attempts at the quotient rule but the method should be correct and they should reach something equivalent to $...\sec^2\left(\frac{1}{2}x\right)$. e.g. $\tan\left(\frac{1}{2}x\right) = \dfrac{\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)} \to \dfrac{k\cos\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right) - -k\sin\left(\frac{1}{2}x\right)\sin\left(\frac{1}{2}x\right)}{\cos^2\left(\frac{1}{2}x\right)}$ where k is a **positive** constant scores M1. If the formula is seen it must be correct.
A1:	$\{f'(x) =\}\ 1 + \frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$ o.e. which may be unsimplified and apply isw. The brackets are not required. There is no need for $f'(x) =$ just look for the expression. Note that $\{f'(x) =\}\ \frac{3}{2} + \frac{1}{2}\tan^2\left(\frac{1}{2}x\right)$ is correct and appears occasionally. $\{f'(x) =\}\ 1 + \frac{1}{2}\sec\frac{1}{2}x^2$ is condoned for M1A0 only but $1 + \frac{1}{2}\left(\sec\frac{1}{2}x\right)^2$ scores M1A1.
(c)	
M1:	Attempts $3.7 - \dfrac{f(3.7)}{f'(3.7)}$ and obtains a value following through on their $f'(x)$ as long as it is a "changed" function in terms of x. Just stating $3.7 - \dfrac{f(3.7)}{f'(3.7)} = ...$ without evidence of use of 3.7 in f(x) (note that this evidence might come from part (a)) **and** in their $f'(x)$ is M0 unless implied by a correct value for both f(x) **and** $f'(x)$ **or** by their final answer. Must be a correct N-R formula used – you may need to check their values – accuracy of at least 3s.f. rounded or truncated required. Allow if attempted in degrees. For reference in degrees f (3.7) = 3.73… and $f'(3.7)$ = 1.50… and gives α = 1.21… Note that the full N-R accuracy is 3.672051617. For reference, the value of α is approximately 3.673194406… and scores M0A0 without other valid work.
A1:	For awrt 3.672 Ignore any subsequent iterations.

Question 11

Question	Scheme	Marks	AOs
12	$\dfrac{\sin(x+h)-\sin x}{h}$	B1	2.1
	$\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}$	M1 A1	1.1b 1.1b
	(As $h\to 0$), $\sin x\left(\dfrac{\cos h-1}{h}\right)+\cos x\left(\dfrac{\sin h}{h}\right)\to 0\times\sin x+1\times\cos x$	dM1	2.1
	so $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\cos x$ *	A1*	2.5
			(5 marks)

Notes

Throughout the question allow the use of $h=\delta x$ if used consistently

There is no requirement to see "gradient of chord" written down.

B1: Gives the correct fraction such as $\dfrac{\sin(x+h)-\sin x}{x+h-x}$ or $\dfrac{\sin x-\sin(x+h)}{-h}$ or $\dfrac{\sin(x+h)-\sin(x-h)}{2h}$ or $\dfrac{\sin(x-h)-\sin x}{x-h-x}$. Condone invisible brackets. May be implied by $\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}$

M1: Uses the compound angle formula for $\sin(x\pm h)$ to give $\sin x\cos h\pm\cos x\sin h$

A1: Achieves $\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}$ or equivalent (may be implied by further work).

Allow invisible brackets to be recovered.

dM1: **It is dependent on both the B and the M marks being awarded.**

Complete attempt to apply the given limits to the gradient of their chord. They must isolate $\left(\dfrac{\cos h-1}{h}\right)$ and replace with 0 and isolate $\left(\dfrac{\sin h}{h}\right)$ and replace with 1.

e.g. $\sin x\left(\dfrac{\cos h-1}{h}\right)+\cos x\left(\dfrac{\sin h}{h}\right)=\sin x\times 0+\cos x\times 1$

Accept as a minimum $\sin x\left(\dfrac{\cos h-1}{h}\right)+\cos x\left(\dfrac{\sin h}{h}\right)=\cos x$ (implying the application of the limits)

If they do not fully show $\left(\dfrac{\cos h-1}{h}\right)$ and $\left(\dfrac{\sin h}{h}\right)$ being isolated but proceed from

e.g. $\dfrac{\sin x(\cos h-1)+\cos x\sin h}{h}$ to $0\times\sin x+\cos x$ (or e.g. $0+\cos x$) then this can be implied and score dM1

$\dfrac{\sin x(\cos h-1)+\cos x\sin h}{h}=\cos x$ is dM0

Condone if limit notation remains within their expression after the limits have been applied.

e.g. $\lim\limits_{h\to 0}(\sin x\times 0+\cos x\times 1)$

Alternatively, condone use of the small angle approximations such that

$\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}\to\dfrac{-\dfrac{h^2}{2}\sin x+h\cos x}{h}=-\dfrac{h}{2}\sin x+\cos x$ and replaces $\dfrac{h}{2}$ with 0

A1*: Uses correct mathematical language of limiting arguments to show that $\frac{dy}{dx} = \cos x$ with no errors seen. (cso)

We need to see $h \to 0$ at some point in their solution and linking $\frac{dy}{dx}$ with $\cos x$ e.g.

- $\frac{dy}{dx} = ... = \lim_{h \to 0}\left(\sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right)\right) = \cos x$
- $\frac{dy}{dx} = ... = \lim_{h \to 0}\left(-\frac{h}{2}\sin x + \cos x\right) = 0 \times \sin x + \cos x = \cos x$ (using small angle approximations)
- $\frac{dy}{dx} = ... = \frac{\sin x(\cos h - 1) + \cos x \sin h}{h} = \sin x \times 0 + 1 \times \cos x = \cos x$ as $h \to 0$

Condone $f'(x)$ or y' in place of $\frac{dy}{dx}$

Give final A0 for no evidence of limiting arguments**:**

e.g. when $h = 0$ $\frac{dy}{dx} = ... = \sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right) = \sin x \times 0 + \cos x \times 1 = \cos x$ is A0

Do not allow the final A1 for just stating $\frac{\sin h}{h} = 1$ and $\frac{\cos h - 1}{h} = 0$ and attempting to apply these (without seeing e.g. $h \to 0$ at some point in their solution)

If they work in another variable (e.g. θ) then withhold the final mark. If they have mixed variables within some of their statements, then allow recovery but withhold the final mark.

Withhold this mark if there has been incorrect bracketing or invisible brackets when isolating $\sin x(\cos h - 1)$ e.g. $\frac{\sin x \cos h - 1 + \cos x \sin h}{h}$ but accept terms written as e.g. $\sin x \frac{\cos h - 1}{h}$ which do not require brackets. Condone a missing trailing bracket if the intention is clear.

Question 12

Question	Scheme	Marks	AOs
7(a)	$f(x) > 3$	B1	1.1b
		(1)	
(b)	$y = 3+\sqrt{x-2} \Rightarrow x = ...$	M1	1.1b
	$f^{-1}(x) = (x-3)^2 + 2$	A1	1.1b
	$x > 3$	B1ft	2.2a
		(3)	
(c)	$f(6) = 3+\sqrt{6-2} = 5 \Rightarrow g("5") = \dfrac{15}{"5"-3} = ...$	M1	1.1b
	$= \dfrac{15}{2}$	A1	1.1b
		(2)	
(d)	$3+\sqrt{a^2+2-2} = \dfrac{15}{a-3} \Rightarrow "a^2 - 9 = 15"$	M1	1.1b
	$a = 2\sqrt{6}$	A1	2.2a
		(2)	
			(8 marks)

Notes

(a)

B1: $f(x) > 3$ o.e.

e.g. $y > 3$, range > 3, $f(x) \in (3, \infty)$, $\{f(x) : f(x) > 3\}$, $f > 3$ **but not e.g.** $x > 3$, $f(x) \geqslant 3$, $[3, \infty)$

(b)

M1: Sets $y = 3+\sqrt{x-2}$ and attempts to make x the subject (or vice versa). Look for the correct order of operations so score for an expression of the form $(x =)\ (y \pm 3)^2 \pm 2$ or $(y =)\ (x \pm 3)^2 \pm 2$

A1: $f^{-1}(x) = (x-3)^2 + 2$ Also accept $f^{-1} : x \to (x-3)^2 + 2$. Condone $f^{-1} = (x-3)^2 + 2$ (or $f^{-1} = y = (x-3)^2 + 2$) but do not allow just $y = ...$ or $f^{-1} : y =$

Also accept other equivalent expressions such as $f^{-1}(x) = x^2 - 6x + 11$ (simplified or unsimplified)

B1ft: $x > 3$ or follow through on their part (a). The omission of $x \in \mathbb{R}$ is condoned.

Allow equivalent answers such as $x \in ("3", \infty)$ or $\{x : x > "3"\}$

Note: It is also acceptable to define f^{-1} in any variable e.g. as $f^{-1}(t) = (t-3)^2 + 2$ $\quad t > 3$ as long as the variable is used consistently to score M1A1B1. If another variable is used other than x it must be fully defined e.g. $f^{-1}(t) = ...$ not just $f^{-1} = ...$

(c)

M1: Substitutes $x = 6$ into f and substitutes the result into g to find a value for $gf(6)$.

Allow an attempt to substitute $x = 6$ into $gf(x) = \dfrac{15}{\sqrt{x-2}}$ condoning slips. They must proceed to find a value. Condone arithmetical slips and bracket errors/omissions. Condone for M1 attempts where when dealing with $\sqrt{x-2}$ leads to two different answers e.g. $\dfrac{15}{\sqrt{6-2}} \to \pm\dfrac{15}{2}$

A1: $\dfrac{15}{2}$ only oe isw once a correct answer is seen

(d)

M1: Attempts to form the equation $3+\sqrt{a^2+2-2} = \dfrac{15}{a-3}$, and proceeds to a quadratic in a (usually $a^2 = k$ or $a^2 - k = 15$ but condone arithmetical, miscopying and sign slips. Condone equations which would lead to complex roots.

May be implied by a correct exact answer.

Alternatively, they attempt to form the equation $a^2+2=\mathrm{f}^{-1}\mathrm{g}(a) \Rightarrow a^2+2=\left(\dfrac{15}{a-3}-3\right)^2+2$

$\Rightarrow (a+3)(a-3)=15 \Rightarrow a^2-9=15$ (condone slips)

They should be square rooting both sides so that $\sqrt{a^2+2-2} \to a$, before multiplying both sides by $a-3$ and rearranging so that the a^2 term comes from their "$(a+3)(a-3)$"

May be implied by a correct exact answer for their quadratic in *a* but a correct decimal answer does not imply this mark.

A1: $(a=)\ 2\sqrt{6}$ or accept $\sqrt{24}$ (they must reject the negative solution if found as $\mathrm{f}\left(a^2+2\right) \neq \mathrm{g}(a)$ when $a=-2\sqrt{6}$) $\sqrt{6}\times\sqrt{4}$ is A0

isw $\sqrt{24}$ followed by $4\sqrt{6}$ (incorrect manipulation of the surd) but not followed by $\pm\sqrt{24}$ o.e.

A decimal answer on its own or multiple answers e.g. $\pm\sqrt{24}$ score A0.

Question 13

Question	Scheme	Marks	AOs
5(a)	$h = 0.2$	B1	1.1b
	$\frac{1}{2}\times\text{"}0.2\text{"}\times\{a+13.5+2(16.8+b+20.2+18.7)\}=17.59$	M1	1.1b
	e.g. $\Rightarrow a+13.5+2b+111.4=175.9 \Rightarrow a+2b=51*$	A1*	2.1
		(3)	
(b)	$a+16.8+b+20.2+18.7+13.5=97.2 \Rightarrow a+b=28 \Rightarrow a=...$ (or $b=...$)	M1	3.1a
	$a=5$ or $b=23$	A1	1.1b
	$a=5$ and $b=23$	A1	1.1b
		(3)	
			(6 marks)

Notes

(a)

B1: States or uses $h = 0.2$ o.e.

M1: Forms the equation $\frac{1}{2}\times\text{"}0.2\text{"}\times\{a+13.5+2(16.8+b+20.2+18.7)\}=17.59$ o.e. but condone copying slips. They may have added some of the y values together so as a minimum accept e.g. $\text{"}0.1\text{"}\times\{a+13.5+2(55.7+b)\}=17.59$

Condone invisible brackets as long as they are recovered or implied in further work before achieving the given answer. Condone the use of $\approx$ for this mark.

Allow this mark if they add the areas of individual trapezia e.g.

$$\frac{\text{their } h}{2}\{a+16.8\}+\frac{\text{their } h}{2}\{16.8+b\}+\frac{\text{their } h}{2}\{b+20.2\}+\frac{\text{their } h}{2}\{20.2+18.7\}+\frac{\text{their } h}{2}\{18.7+13.5\}$$

Condone copying slips but it must be a complete method using all the trapezia. h must be numerical but condone $h = 1$

A1*: A rigorous argument leading to $a+2b=51$ from correct working and no errors seen including brackets, although do not penalise a missing trailing bracket at the end e.g.

$\frac{1}{2}\times\text{"}0.2\text{"}\times\{a+13.5+2(16.8+b+20.2+18.7)=17.59 \Rightarrow ... \Rightarrow a+2b=51$ could score B1M1A1but

$\frac{1}{2}\times\text{"}0.2\text{"}\times a+13.5+2(16.8+b+20.2+18.7)=17.59 \Rightarrow ... \Rightarrow a+2b=51$ could score max B1M1A0

provided later work implied correct brackets.

Both sets of brackets must be dealt with correctly before proceeding to the final answer such that

e.g. $... \Rightarrow a+2b+124.9=175.9 \Rightarrow a+2b=51$ is M1A1*

$... \Rightarrow a+13.5+33.6+2b+40.4+37.4=175.9 \Rightarrow a+2b=51$ is M1A1*

$... \Rightarrow 0.1a+1.35+3.36+0.2b+4.04+3.74=17.59 \Rightarrow a+2b=51$ is M1A1*

Note that $a+2b\approx 51$ as the **final** answer is A0*

(b)

M1: Attempts to form the equation $a+16.8+b+20.2+18.7+13.5=97.2$, condoning copying errors, (may just be stated as e.g. $a+b=28$ o.e.) and attempts to solve their equation simultaneously with the given equation (or condone their equation from part (a)). Do not be too concerned with the process here as calculators may be used. Score if values for a or b are reached from a pair of simultaneous equations.

A1: for $a=5$ or $b=23$

A1: for both $a=5$ and $b=23$

Question 14

Question	Scheme	Marks	AOs
3(a)	$\left(\lvert\overrightarrow{OA}\rvert =\right)\sqrt{5^2+3^2+2^2} = \sqrt{38}$ *	B1*	1.1b
		(1)	
(b)	$\lvert\overrightarrow{OB}\rvert = \sqrt{2^2+4^2+a^2} = \sqrt{20+a^2}$ so when $a=5$ $\lvert\overrightarrow{OB}\rvert = \sqrt{20+25} = \sqrt{45}$	M1	1.1b
	$= 5$	A1cso	2.3
		(2)	
			(3 marks)

Notes

(a)

B1*: Shows the magnitude of $\lvert\overrightarrow{OA}\rvert$ is $\sqrt{38}$. Must see $\sqrt{5^2+3^2+2^2}$ or e.g. $\sqrt{25+9+4}$. We need to see how the value 38 or $\sqrt{38}$ is formed using the three components. Withhold this mark for incorrect working such as $\lvert\overrightarrow{OA}\rvert = 5^2+3^2+2^2 = 38 \Rightarrow \lvert\overrightarrow{OA}\rvert = \sqrt{38}$ but do not penalise poor notation to denote vectors or the magnitude as long as the intention is clear as to what they are finding (OA instead of $\lvert\overrightarrow{OA}\rvert$ is fine). Do not penalise if their square root does not go fully over all three terms as long as the intention is clear. May find $\lvert\overrightarrow{AO}\rvert$ instead which is acceptable.

$\left(\lvert\overrightarrow{OA}\rvert^2 =\right) 25+9+4=38 \Rightarrow \left(\lvert\overrightarrow{OA}\rvert =\right)\sqrt{38}$ scores B1 (we see how 38 is found)

$\left(\lvert\overrightarrow{OA}\rvert^2 =\right) 25+9+4 \Rightarrow \sqrt{38}$ scores B1 (we see how 38 is found)

$25+9+4=\sqrt{38}$ scores B0 (they are not equal)

$\left(\lvert\overrightarrow{OA}\rvert^2 =\right) 38 \Rightarrow \left(\lvert\overrightarrow{OA}\rvert =\right)\sqrt{38}$ scores B0 (no method seen to show how 38 is found)

(b)

M1: Attempts to find $\lvert\overrightarrow{OB}\rvert$ (or $\lvert\overrightarrow{OB}\rvert^2$) in terms of a and substitutes in a positive integer for a to find a value for $\lvert\overrightarrow{OB}\rvert$ (or $\lvert\overrightarrow{OB}\rvert^2$). e.g. $\lvert\overrightarrow{OB}\rvert = \sqrt{20+a^2} \Rightarrow$ when $a=2 \Rightarrow \sqrt{24}$. Also accept e.g. $\lvert\overrightarrow{BO}\rvert$ (or $\lvert\overrightarrow{BO}\rvert^2$).

Alternatively sets up an equation or an inequality e.g. $\sqrt{20+a^2} > \sqrt{38}$ and proceeds to $a^2 > ...$ (or $a^2 = ...$). Condone sign slips in their rearrangement only.

Allow the use of =, < or > for this mark. $"20"+a^2 ... 38 \Rightarrow a^2 ... "18"$

(may be implied by sight of $\sqrt{18} = 4.24...$)

A1: 5 cso (answer on its own with no incorrect working seen scores M1A1). Withhold this mark if $\lvert\overrightarrow{OB}\rvert$ (or $\lvert\overrightarrow{OB}\rvert^2$) is incorrect (or $\lvert\overrightarrow{BO}\rvert$ or $\lvert\overrightarrow{BO}\rvert^2$). Do not be concerned with the notation as long as the intention is clear or implied as to what they are trying to calculate (the calculations must be correct)

Withhold this mark if at any point they set $\lvert\overrightarrow{OB}\rvert < \lvert\overrightarrow{OA}\rvert$ but accept an argument leading to an answer of 5 from either $\lvert\overrightarrow{OB}\rvert > \lvert\overrightarrow{OA}\rvert$ or $\lvert\overrightarrow{OB}\rvert = \lvert\overrightarrow{OA}\rvert$

Question 15

Question	Scheme	Marks	AOs
2(a)	$(\mathrm{f}(a)=)\ 4a^3+5a^2-10a+4a=0 \Rightarrow a(....)=0$	M1	3.1a
	$a(4a^2+5a-6)=0$ *	A1*	1.1b
		(2)	
(b)(i)	$a=\frac{3}{4}$	B1	2.2a
(ii)	$4x^3+5x^2-10x+4\times"\frac{3}{4}"=3 \Rightarrow 4x^3+5x^2-10x\ (=0)$	M1	1.1b
	$x=0$	B1	1.1b
	$x=\frac{-5\pm\sqrt{185}}{8}$	A1	1.1b
		(4)	
			(6 marks)

Notes

(a)

M1: Attempts $\mathrm{f}(a)=0$ leading to an equation in a only and attempts to take a factor of a out. Condone slips. The $=0$ may appear on a later line which is fine, but must be seen at some point in their solution in part (a).

A1*: Achieves the given answer with no errors including brackets.
Minimum acceptable is $4a^3+5a^2-10a+4a=0 \Rightarrow a(4a^2+5a-6)=0$
If the $=0$ is absent at the start of their solution, it must appear before achieving the given answer.
Do not allow attempts to find the value of a and substitute that into $\mathrm{f}(x)$

More difficult alternative methods may be seen:

Alt 1: You may see attempts via division / inspection

$$
\begin{array}{r|llll}
 & 4x^2 & +(5+4a)x & & +(-10+5a+4a^2) \\
\hline
x-a & 4x^3 & +5x^2 & -10x & +4a \\
 & 4x^3 & -4ax^2 & & \\
 & & (5+4a)x^2 & -10x & \\
 & & (5+4a)x^2 & -a(5+4a)x & \\
 & & & (-10+5a+4a^2)x & +4a \\
 & & & (-10+5a+4a^2)x & -a(-10+5a+4a^2) \\
 & & & & -6a+5a^2+4a^3
\end{array}
$$

Then sets remainder $-6a+5a^2+4a^3=0$

M1: For dividing the cubic by $(x-a)$ leading to a quadratic quotient in x and a cubic remainder in a which is then set $=0$ and attempts to take a factor of a out.

A1*: Completely correct with $a(4a^2+5a-6)=0$

Alt 2: You may also see a grid or an attempt at factorisation via inspection

	$4x^2$	$+(5+4a)x$	$+(-10+5a+4a^2)$
x	$4x^3$	$+(5+4a)x^2$	$(-10+5a+4a^2)x$
$-a$	$-4ax^2$	$-a(5+4a)x$	$-a(-10+5a+4a^2)$

OR $4x^3+5x^2-10x+4a \equiv (x-a)(4x^2+px-4)$ which should be followed by equating the x terms and x^2 terms to form two equations which can be solved simultaneously.

$-10=-ap-4$ and $5=-4a+p \Rightarrow p=5+4a$

$\Rightarrow -10=-a(5+4a)-4 \Rightarrow 4a^2+5a-6=0 \Rightarrow a(4a^2+5a-6)=0$

The above are examples. There may be other correct attempts so look at what is done.

M1: For an attempt to set up two simultaneous equations by equating coefficients for x and equating coefficients for x^2. Condone slips.

A1*: $4a^2+5a-6=0 \Rightarrow a(4a^2+5a-6)=0$ Completely correct with no errors.

(b) Mark (i) and (ii) together

(i)

B1: Deduces that $a=\frac{3}{4}$ only. May be implied by their resultant cubic. If they do (b)(ii) multiple times using other roots for which $a \neq \frac{3}{4}$, then the solutions arising from using the other roots $a \neq \frac{3}{4}$ must be rejected

(ii)

M1: Attempts to substitute their $a=\frac{3}{4}$ (which must be positive) into $f(x)$, sets their $f(x)=3$ and collects terms on one side (= 0 may be implied). Condone arithmetical and sign slips. Condone if they repeat this step using their other root(s).

B1: $(x=)\ 0$

A1: $(x=)\ \frac{-5\pm\sqrt{185}}{8}$ (**and these values only**) or exact equivalent (ignore 0 for this mark). Withhold this mark if the fraction line was clearly not intended to be under both terms. This mark cannot be scored if they proceed directly to the roots from $4x^3+5x^2-10x$ without taking a factor of or dividing by x first to **see the quadratic factor**. Isw once the correct answers are seen if they proceed to provide rounded answers after.

e.g. $4x^3+5x^2-10x+4\times"\frac{3}{4}"=3 \Rightarrow 0, \frac{-5\pm\sqrt{185}}{8}$ is M0B1A0

e.g. $4x^3+5x^2-10x=0 \Rightarrow 0, \frac{-5\pm\sqrt{185}}{8}$ is M1B1A0

e.g. $4x^3+5x^2-10x=0 \Rightarrow 4x^2+5x-10=0 \Rightarrow \frac{-5\pm\sqrt{185}}{8}$ is M1B0A1

e.g. $4x^3+5x^2-10x=0 \Rightarrow x(4x^2+5x-10)=0 \Rightarrow 0, \frac{-5\pm\sqrt{185}}{8}$ is M1B1A1

Question 16

Question	Scheme	Marks	AOs
1	$x^{\frac{1}{2}}(2x-5)=...x^{\frac{3}{2}}+...x^{\frac{1}{2}}$ or $\dfrac{x^{\frac{1}{2}}(2x-5)}{3}=\dfrac{...x^{\frac{3}{2}}+...x^{\frac{1}{2}}}{3}$	M1	1.1b
	$\dfrac{2x^{\frac{3}{2}}}{3}-\dfrac{5x^{\frac{1}{2}}}{3}$	A1	1.1b
	$\displaystyle\int \dfrac{2x^{\frac{3}{2}}}{3}-\dfrac{5x^{\frac{1}{2}}}{3}\,\mathrm{d}x=...x^{\frac{5}{2}}\pm...x^{\frac{3}{2}}\ (+c)$	dM1	1.1b
	$\dfrac{4}{15}x^{\frac{5}{2}}-\dfrac{10}{9}x^{\frac{3}{2}}+c$	A1	1.1b
		(4)	
			(4 marks)

Notes

M1: Attempts to multiply out the brackets of the numerator and either writes the expression (or just the numerator) as a sum of terms with **indices**. Award for either one correct index of $...x^{\frac{3}{2}}+...x^{\frac{1}{2}}$ which comes from a correct method. Condone appearing as terms on separate lines for this mark. The correct index may be implied later when e.g. $\sqrt{x}\to x^{\frac{3}{2}}$ or $\left(\sqrt{x}\right)^3\to x^{\frac{5}{2}}$ after they have integrated.

The $\frac{1}{3}$ does not need to be considered for this mark.

A1: $\dfrac{2x^{\frac{3}{2}}}{3}-\dfrac{5x^{\frac{1}{2}}}{3}$ or equivalent e.g. $\dfrac{1}{3}\left(2x^{\frac{3}{2}}-5x^{\frac{1}{2}}\right)$. Condone $\dfrac{2x^{\frac{3}{2}}-5x^{\frac{1}{2}}}{3}$ May be implied by further work.

The correct index may be implied later when e.g. $\sqrt{x}\to x^{\frac{3}{2}}$ or $\left(\sqrt{x}\right)^3\to x^{\frac{5}{2}}$ after they have integrated.

Ignore incorrect integration notation around the terms. Ignore any presence or absence of dx.

Be aware that a factor of $\frac{1}{3}$ may be taken outside of the integral so you may need to look at further work to award the first A mark if work on the two terms is done separately or in a list.

May be unsimplified and the two terms may appear in a list which is fine.

Coefficients must be exact.

dM1: Increases the power by one on an x^n term where n is a fraction. The index does not need to be processed.

e.g. $...x^{\frac{3}{2}+1}$ or $...x^{\frac{1}{2}+1}$ It is dependent on the previous method mark so at least one of the terms must have had a correct index.

Note that integrating the numerator and denominator e.g. $\dfrac{2x^{\frac{3}{2}}}{3}-\dfrac{5x^{\frac{1}{2}}}{3}\to\dfrac{...x^{\frac{5}{2}}}{3x}-\dfrac{...x^{\frac{3}{2}}}{3x}$ is dM0.

A1: $\frac{4}{15}x^{\frac{5}{2}}-\frac{10}{9}x^{\frac{3}{2}}+c$ and including the constant or simplified exact equivalent such as

$\frac{4}{15}\sqrt{x^5}-\frac{10}{9}\sqrt{x^3}+c$ or $\frac{4}{15}x^{2.5}-\frac{10}{9}x^{1.5}+c$ or $\frac{1}{45}\left(12x^{\frac{5}{2}}-50x^{\frac{3}{2}}\right)+c$ or $\frac{x^{\frac{3}{2}}}{45}(12x-50)+c$. Fractions must be in their lowest terms and indices processed.

Do not accept e.g. $0.267x^{\frac{5}{2}}-1.11x^{\frac{3}{2}}+c$ but allow $0.2\dot{6}x^{\frac{5}{2}}-1.\dot{1}x^{\frac{3}{2}}+c$

Isw once a correct answer is seen but withhold this mark if there is spurious notation around their final answer e.g. $\int \frac{4}{15}x^{\frac{5}{2}}-\frac{10}{9}x^{\frac{3}{2}}+c \ \mathrm{d}x$ is M1A1dM1A0

Alternative method using integration by parts example

M1: e.g. $\int x^{\frac{1}{2}}(2x-5)\,\mathrm{d}x=...x^{\frac{3}{2}}(2x-5)-\int ...x^{\frac{3}{2}}\,\mathrm{d}x$ (applies integration by parts correctly to typically achieve this form – the $(2x-5)$ may also be split up as well – send to review if unsure how to mark)

This may also be done the other way round e.g. $\int x^{\frac{1}{2}}(2x-5)\,\mathrm{d}x=...x^{\frac{1}{2}}(x^2-5x)-\int ...x^{\frac{3}{2}}\pm ...x^{\frac{1}{2}}\,\mathrm{d}x$

The $\frac{1}{3}$ does not need to be considered for this mark.

A1: A correct intermediate stage applying integration by parts with correct coefficients.

e.g. $\int \frac{x^{\frac{1}{2}}(2x-5)}{3}\,\mathrm{d}x=\frac{2}{3}x^{\frac{3}{2}}\left(\frac{2x-5}{3}\right)-\int \frac{4}{9}x^{\frac{3}{2}}\,\mathrm{d}x$ (or unsimplified equivalent).

Coefficients must be exact. (See main scheme notes above) The other way round this could appear as

e.g. $\int \frac{x^{\frac{1}{2}}(2x-5)}{3}\,\mathrm{d}x=\frac{1}{3}x^{\frac{1}{2}}(x^2-5x)-\frac{1}{6}\int x^{\frac{3}{2}}-5x^{\frac{1}{2}}\,\mathrm{d}x$. Condone a missing d$x$. May be implied.

dM1: Increases the power by one on an x^n term where n is a fraction e.g. $\int ...x^{\frac{3}{2}}\,\mathrm{d}x \to ...x^{\frac{5}{2}}$ The index does not need to be processed. It is dependent on the previous method mark.

A1: $\frac{4}{15}x^{\frac{5}{2}}-\frac{10}{9}x^{\frac{3}{2}}+c$ or exact simplified equivalent. (See main scheme notes above)

Alternative method using the substitution method

M1: e.g. let $u=x^{\frac{1}{2}} \Rightarrow \int ...u^4+...u^2\,\mathrm{d}u$ (uses a substitution to express the integral in terms of another variable. Allow slips with the coefficients, but the indices should be correct for their substitution)

The $\frac{1}{3}$ does not need to be considered for this mark.

A1: e.g. $\int \frac{4u^4}{3}-\frac{10u^2}{3}\,\mathrm{d}u$ or unsimplified equivalent. Coefficients must be exact. See main scheme notes above). May be implied by further work. Condone a missing dx.

dM1: $\int ...u^4+...u^2\,\mathrm{d}u \to ...u^5+...u^3$ (increases the power by one on at least one of their indices – does not need to be processed. It is dependent on the previous method mark.

A1: $\frac{4}{15}x^{\frac{5}{2}}-\frac{10}{9}x^{\frac{3}{2}}+c$ or exact simplified equivalent. (See main scheme notes above)

There may be alternative substitutions, but the same marking principles apply.

Question 17

Question	Scheme	Marks	AOs
9(a)	$\dfrac{9^{7-2k}}{3^{4k-5}}=\dfrac{3^{2(k-1)}}{9^{7-2k}} \Rightarrow \dfrac{3^{2(7-2k)}}{3^{4k-5}}=\dfrac{3^{2(k-1)}}{3^{2(7-2k)}}$	M1	3.1a
	$\left(3^{2(7-2k)}\right)^2=3^{4k-5}\times 3^{2(k-1)}$ $\Rightarrow 28-8k=6k-7 \Rightarrow k=\ldots$ $3^{2(7-2k)-(4k-5)}=3^{2(k-1)-2(7-2k)}$ $\Rightarrow 19-8k=6k-16 \Rightarrow k=\ldots$	dM1	1.1b
	$k=\dfrac{5}{2}$*	A1*	2.1
		(3)	
(b)	$a=3^{4(2.5)-5}$ and $r=\dfrac{9^{7-2(2.5)}}{3^{4(2.5)-5}} \Rightarrow$ one of $a=243$ or $r=\dfrac{1}{3}$	M1	2.2a
	$S_\infty=\dfrac{a}{1-r}=\dfrac{"243"}{1-"\frac{1}{3}"}$	M1	1.1b
	$S_\infty=\dfrac{729}{2}\,(364.5)$ cao	A1	1.1b
		(3)	
			(6 marks)

Notes

(a) **Special cases:**

SC 1: For those that verify rather than prove a SC 100 is awarded for substituting $k=\dfrac{5}{2}$ into all three terms to correctly obtain 243, 81 and 27 **with** a statement that this is **geometric** with $r=\dfrac{1}{3}$ (or equivalent reason). All statements must be correct.

SC 2: Be aware that e.g. $\dfrac{9^{7-2k}}{3^{4k-5}}=\dfrac{3^{2(k-1)}}{9^{7-2k}} \Rightarrow 81^{2(7-2k)}=9^{6k-7}$ is an incorrect process (without some indication that they have intentionally squared both sides) that fortuitously leads to the correct answer and may score maximum SC 010.

M1: Uses the 3 terms to set up an equation in k **and**

- **either** reaches a common base by replacing 9 with 3^2 or by replacing 3 with $9^{0.5}$ and uses the power law of indices correctly
- **or** uses the laws of indices correctly to reach $9^{14-4k}=3^{6k-7}$ condoning slips in e.g. expanding brackets.

Writing down e.g. $2(7-2k)-(4k-5)=2(k-1)-2(7-2k)$ is sufficient to imply the M1.

dM1: Correct processing leading to a value for k.

A1*: Correct value following correct working. Allow $k=2.5$ in place of $k=\dfrac{5}{2}$

Condone missing/invisible brackets provided they are recovered correctly.

Alt 1 Using Base 9:

$$\frac{9^{7-2k}}{3^{4k-5}}=\frac{3^{2(k-1)}}{9^{7-2k}} \Rightarrow \frac{9^{7-2k}}{9^{2k-2.5}}=\frac{9^{k-1}}{9^{7-2k}} \text{ o.e. scores M1}$$

$$\Rightarrow 9^{9.5-4k}=9^{3k-8} \Rightarrow 9.5-4k=3k-8 \Rightarrow 7k=17.5 \Rightarrow k=2.5$$

Score dM1 when a value of k is achieved using a correct process and A1* if fully correct.

Alt 2 Finding r in terms of k and using e.g. $u_3 = ar^2$:

$$r = \frac{9^{7-2k}}{3^{4k-5}} = \frac{3^{2(7-2k)}}{3^{4k-5}} \left\{= 3^{19-8k}\right\} \textbf{ or } r = \frac{3^{2(k-1)}}{9^{7-2k}} = \frac{3^{2k-2}}{3^{2(7-2k)}} \left\{= 3^{6k-16}\right\}$$

$$\Rightarrow 3^{4k-5} \times \left(3^{19-8k}\right)^2 = 3^{2k-2} \textbf{ or } \Rightarrow 3^{4k-5} \times \left(3^{6k-16}\right)^2 = 3^{2k-2} \text{ scores M1}$$

$$\Rightarrow 3^{4k-5} \times 3^{2(19-8k)} = 3^{2k-2} \Rightarrow 33 - 12k = 2k - 2 \Rightarrow 14k = 35 \Rightarrow k = 2.5$$

Score dM1 when a value of k is achieved using a correct process and A1* if fully correct.

Alt 3 Using Logs Way 1:

$$\frac{9^{7-2k}}{3^{4k-5}} = \frac{3^{2(k-1)}}{9^{7-2k}} \Rightarrow \left(9^{7-2k}\right)^2 = 3^{6k-7} \Rightarrow 9^{14-4k} = 3^{6k-7} \text{ scores M1}$$

$$\Rightarrow (14-4k)\log_3 9 = 6k - 7$$

$$\Rightarrow 2(14-4k) = 6k - 7$$

$$\Rightarrow k = 2.5$$

Score dM1 when a value of k is achieved using a correct process and A1* if fully correct.

Alt 4 Using Logs Way 2:

$$\frac{9^{7-2k}}{3^{4k-5}} = \frac{3^{2(k-1)}}{9^{7-2k}}$$

$$\Rightarrow (7-2k)\log_3 9 - (4k-5)\log_3 3 = (2k-2)\log_3 3 - (7-2k)\log_3 9 \quad \text{scores M1}$$

$$\Rightarrow 2(7-2k) - (4k-5) = 2k - 2 - 2(7-2k)$$

$$\Rightarrow 19 - 8k = 6k - 16$$

$$\Rightarrow k = 2.5$$

Score dM1 when a value of k is achieved using a correct process and A1* if fully correct.

Alt 5 Recognising that taking $\log_3$ forms an Arithmetic Sequence:

$$\{\log_3\}u_1 = 4k-5,\ \{\log_3\}u_2 = 2(7-2k),\ \{\log_3\}u_3 = 2(k-1)$$

$$\Rightarrow 2(7-2k) - (4k-5) = 2(k-1) - 2(7-2k) \quad \text{scores M1}$$

$$\Rightarrow 19 - 8k = 6k - 16$$

$$\Rightarrow k = 2.5$$

Score dM1 when a value of k is achieved using a correct process and A1* if fully correct.
There is no need to see any mention of log in this approach.

(b)

M1: Deduces expressions for the first term **and** the common ratio using $k = \frac{5}{2}$ in the <u>correct formulae</u> **and** finds at least one of $a = 243$ or $r = \frac{1}{3}$. Allow if seen in (a). May be implied by correct values for a and r. For reference, $a = 3^{4(2.5)-5}$ and $r = \frac{9^{7-2(2.5)}}{3^{4(2.5)-5}}$ $\left\{\text{or } r = \frac{3^{2(2.5-1)}}{9^{7-2(2.5)}}\right\}$

M1: Recalls the sum to infinity formula and substitutes their values for a and r provided $|r| < 1$
Dependent on a correct attempt to find both a and r using $k = 2.5$ but allow if neither value is correct or if they are unprocessed e.g. $\dfrac{3^{4(2.5)-5}}{1 - \dfrac{9^{7-2(2.5)}}{3^{4(2.5)-5}}}$ scores this mark.

A1: cao. Correct sum to infinity. Answer only (with no working) scores full marks. Apply isw.