Question 1

Question 1 **(30 marks)**

(a) $\frac{(4-2i)}{(2+4i)} = 0 + ki$, where $k \in \mathbb{Z}$, and $i^2 = -1$. Find the value of k.

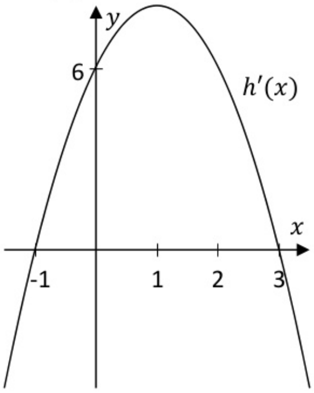
(b) Find $\sqrt{-5+12i}$.
Give both of your answers in the form $a + bi$, where $a, b \in \mathbb{R}$.

(c) Use De Moivre's theorem to find the **three** roots of $z^3 = -8$.
Give each of your answers in the form $a + bi$, where $a, b \in \mathbb{R}$, and $i^2 = -1$.

Question 2

Question 6 **(30 marks)**

The diagram below shows the graph of $h'(x)$ the derivative of a cubic function $h(x)$.



(a) Show that $h'(x) = -2x^2 + 4x + 6$.

(b) Use $h'(x)$ to find the maximum positive value of the **slope** of a tangent to $h(x)$.

(c) The graph of $h(x)$ passes through the point $(0, -2)$.
Find the equation of $h(x)$.

Question 2 **(30 marks)**

(a) $f(x) = x^2 + bx + c$, where $b, c \in \mathbb{R}$.
$f(x)$ has a local minimum point at $(3, -1)$.

Find the value of b and the value of c.

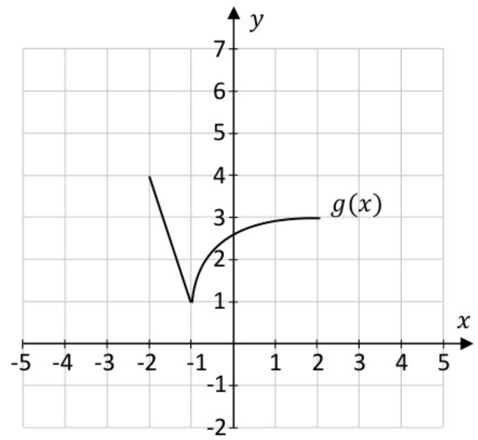
$b =$ ____________ and $c =$ ____________

(b) Find the value of the following limit, where $n \in \mathbb{N}$:

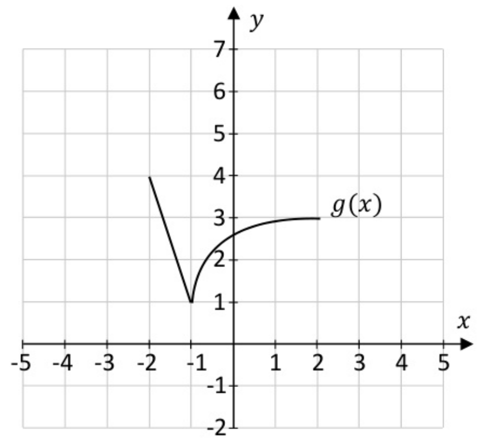
$$\lim_{n \to \infty} \left[\frac{n}{n+1} + \frac{n+1000}{n} + \left(\frac{1}{3}\right)^n \right]$$

(c) The function $g(x)$ is defined for $-2 \leq x \leq 2$, $x \in \mathbb{R}$.
Its graph is shown in each of the two diagrams below.

(i) Draw the graph of $g(x) - 2$ on the co-ordinate diagram below, for $x \in \mathbb{R}$, on as large a domain as possible.



(ii) Draw the graph of $g(x+3)$ on the co-ordinate diagram below, for $x \in \mathbb{R}$, on as large a domain as possible.



Question 3 **(30 marks)**

(a) Prove that $\sqrt{2}$ is **not** a rational number.

(b) t is a positive real number, with:

$$\log_3 t + \log_9 t + \log_{27} t + \log_{81} t = 10$$

Find the value of t. Give your answer in the form 3^r, where $r \in \mathbb{Q}$.

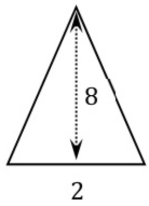
Hint: use the formula $\log_a b = \dfrac{\log_c b}{\log_c a}$.

(c) **(i)** Explain what $\log_6 m$ means, where m is a positive real number.

(ii) m is a real number, and $m > 6$.

What information does this give about the value of $\log_6 m$?

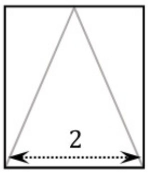
Question 10 **(50 marks)**



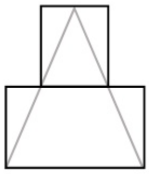
A triangle has a base of length 2 units and a perpendicular height of 8 units, as shown in the diagram on the right.

The diagrams below show T_1, T_2, and T_3, the first three shapes in a sequence of shapes based on this triangle.

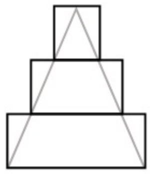
For each value of $n \in \mathbb{N}$, the shape T_n is made up of n rectangles of equal height laid on top of each other. T_n is the collection of the smallest such rectangles that completely covers the triangle.



T_1
1 rectangle

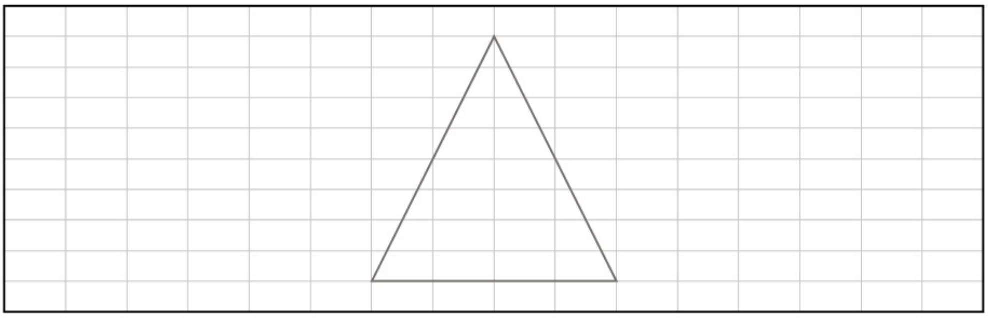


T_2
2 rectangles
of equal height



T_3
3 rectangles
of equal height

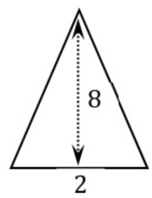
(a) Draw T_4 in the grid below, based on the triangle given on the grid.



(b) Show that the **total area** of the three rectangles in T_3 is $\frac{32}{3}$ square units.

(c) Find the **total area** of the n rectangles in T_n, for $n \in \mathbb{N}$.
Give your answer in square units in terms of n, in its simplest form.

(d) The total area of the rectangles in the nth term of a **different** sequence of groups of rectangles is as follows, for $n \in \mathbb{N}$:



$$\text{Total area} = A_n = \frac{8\,(n-1)}{n}$$

Work out the first value of n for which A_n is **greater than** 95% of the area of the triangle on the right.

This question continues on the next page.

(e) **Diagram A** below shows a square-based pyramid, with base sides of length c units. The base of the pyramid is horizontal, and its perpendicular height is h units (where c, $h \in \mathbb{R}$).

Diagram B below shows the same pyramid. It also shows a horizontal square that lies within the pyramid, a distance of x units down from the top of the pyramid, where $x \in \mathbb{R}$, $0 < x < h$.

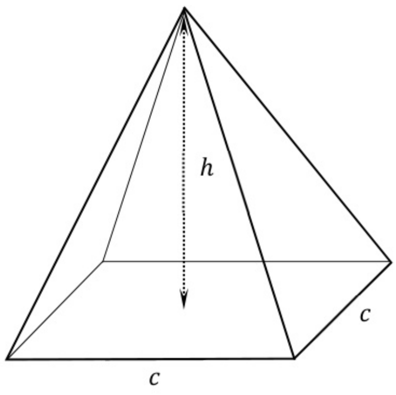


Diagram A

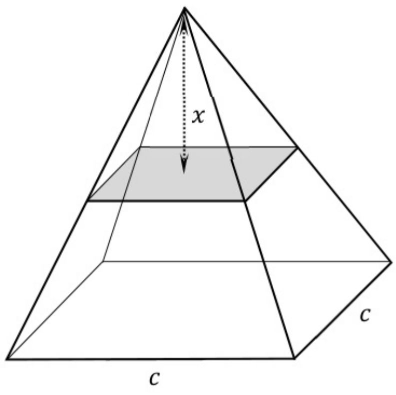


Diagram B

The area of the shaded square in **Diagram B** is $S(x) = \dfrac{x^2\,c^2}{h^2}$.

(i) The volume of the pyramid is $\int_0^h S(x)\,dx$.

Use integration to find the volume of the pyramid in cubic units, in terms of c and h.

(ii) x starts to increase at a rate of 3 units per second.
This causes $S(x)$ to increase as well.

Find the rate of change of $S(x)$ with respect to time, at the instant when x is half the perpendicular height of the pyramid. Give your answer in square units per second, in terms of c and h.

Question 3 **(30 marks)**

(a) Find the integral:

$$\int \cos 6x \ dx$$

(b) The function f is defined for $x \in \mathbb{R}$ as:

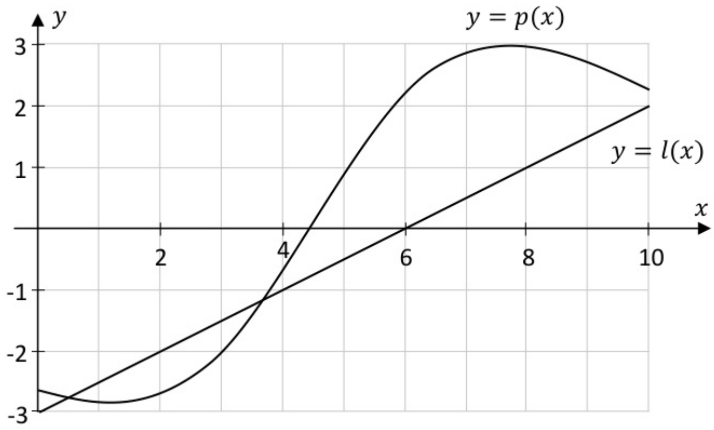
$$f(x) = 2x^3 - 9x^2 + 5x - 11$$

(i) Find the equation of the tangent to the graph of f at the point where $x = 2$. You do **not** need to simplify your answer.

(ii) Find the x co-ordinate of the point of inflection of f.

$x =$ __________

(c) The diagram below shows the curve $y = p(x)$ and the line $y = l(x)$, for $0 \leq x \leq 10$, $x \in \mathbb{R}$.



There are two values of x in the domain $0 \leq x \leq 10$ for which:

$$p'(x) = l'(x)$$

where $p'(x)$ is the derivative of $p(x)$.

Use the infomation in the diagram to estimate these two values of x, as accurately as you can. Show your work on the diagram.

$x =$ __________ **or** ___________

Question 8 **(50 marks)**

(a) The table in **Part (a)(ii)** below shows some of the values of the function:

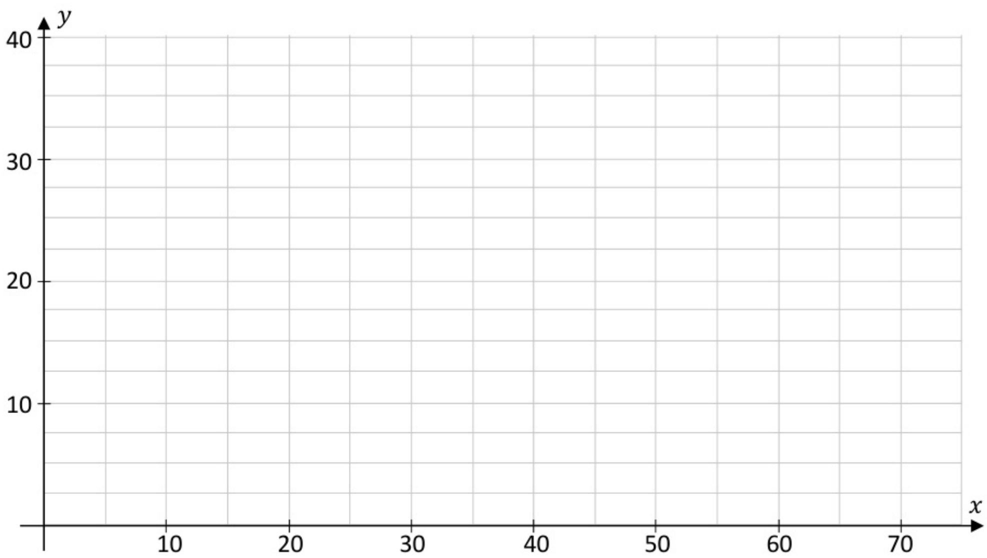
$$h(x) = 0{\cdot}001x^3 - 0{\cdot}12x^2 + px + 5, \quad x \in \mathbb{R},$$

in the domain $0 \leq x \leq 75$.

(i) Use $h(10) = 30$ to show that $p = 3{\cdot}6$

(ii) Complete the table below and hence draw the graph of $h(x)$ in the domain $0 \leq x \leq 75$ on the grid below.

x	0	10	20	30	40	50	60	70	75
$h(x)$		30			21		5		21·875



(b) The function $h(x)$ can be used to model the height above level ground (in metres) of a section of the path followed by a rollercoaster track, where x is the horizontal distance from a fixed point.

(i) Find $h'(x)$, the derivative of $h(x)$.

(ii) Show that this section of the track reaches its maximum height above level ground when $x = 20$.

This question continues on the next page.

(iii) Find, using calculus, the height above ground, in metres, at the instant the track passes through an inflection point. The function $h(x) = 0{\cdot}001x^3 - 0{\cdot}12x^2 + 3{\cdot}6x + 5$.

(c) Use the function $h(x) = 0{\cdot}001x^3 - 0{\cdot}12x^2 + 3{\cdot}6x + 5$, $x \in \mathbb{R}$, to find the average height of this section of the track above level ground, from $x = 0$ to $x = 75$.
Give your answer in metres correct to 2 decimal places.

Question 8 **(50 marks)**

Olga, Chen, Fiona, and Rohan all have bank accounts.

(a) Olga puts €3000 in a savings account. Interest is added annually at a rate of 2·4% per year. Work out the amount in Olga's account after 5 years, correct to the nearest cent.

(b) **(i)** Explain what is meant by the "present value" of a payment of €1000 in 1 year's time, at a particular interest rate.

(ii) Chen puts a different amount in a savings account with the same interest rate (2·4% per year). After 6 years, Chen has €4000 in the account.

Work out how much money Chen put in the account initially, correct to the nearest cent.

This question continues on the next page

(c) Fiona is taking out a loan at the same annual interest rate (2·4% per year).
Fiona makes payments quarterly (that is, 4 times per year).
Work out the quarterly interest rate that would be equivalent to an APR of 2·4%.
Give your answer as a **percentage**, correct to 2 decimal places.

(d) Rohan wants to put the same amount of money in a savings account at the start of each month for 36 months so that, at the end of 3 years, he will have a total of €$12\ 000$ in the account. Interest is calculated at a rate of $0{\cdot}11\%$ per month.

(i) Taking €A to be the amount Rohan puts in his account at the start of each month, write down a geometric series in €A to show the total amount of money in the account at the end of the 3 years. Include the first two and the last two terms.

(ii) Hence, find the value of €A that will give a total of €$12\ 000$ in the account after 3 years. Give your answer correct to the nearest cent.

(e) A park sells three types of ticket: child, student, and adult. The table below gives information on the price of each ticket and the percentage of tickets sold.
For example, 15% of all tickets sold are student tickets.

Type of ticket	**Child**	**Student**	**Adult**
Price of ticket	€11	€5 less than an adult ticket	€ x
Percentage	52%	15%	33%

The expected value of the price of a ticket is €$13{\cdot}85$.
Work out the value of x, the price of an adult ticket.

(f) When an item is being sold:

- the **mark up** is the profit as a percentage of the cost price, and

- the **margin** is the profit as a percentage of the selling price.

A shop sells an item with a **margin** of 18%. Work out the **mark up** for this item.
Give your answer as a percentage, correct to the nearest percent.