Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE / GRAAD 11

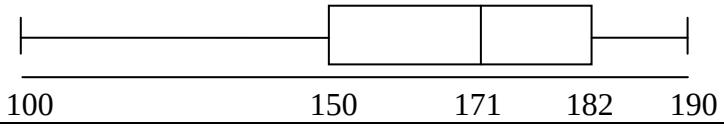
NOVEMBER 2013

MATHEMATICS P2 / WISKUNDE V2 MEMORANDUM

MARKS / PUNTE: **150**

This memorandum consists of 9 pages./
Hierdie memorandum bestaan uit 9 bladsye.

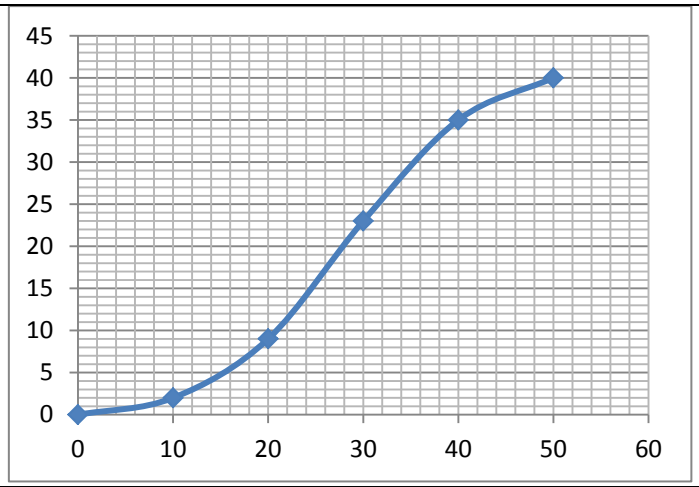
QUESTION / *VRAAG* 1

1.1	100;143;150;155;164;171;171;180;182;188;190 min = 100, Q_1=150, Q_2=171, Q_3= 182, max / *maks*=190  	✓min and max / *min en maks* ✓Q_1 ✓Q_2 ✓Q_3 (4)
1.2	Skewed to the left / *skuins na links*	✓answer / *antwoord* (1)
1.3	100	✓answer / *antwoord* (1) **[6]**

QUESTION / *VRAAG* 2

2.1	Mean / *Gemiddelde* $= \frac{25+47+40+34+28+x+37+28+55+30}{10}$ $= \frac{324+x}{10}$	✓answer / *antwoord* (1)
2.2	$\frac{324+x}{10} = 36$ $x = 36$	✓equating / *gelykstel* ✓answer / *antwoord* (2)
2.3	8,88	✓✓answer / *antwoord* (2)
2.4	outside / *buite* [36 – 8,88 ; 36 + 8,88] = [27,12 ; 44,88] ∴ 3 people / *persone*	✓method / *metode* ✓answer / *antwoord* (2) **[7]**

QUESTION / *VRAAG* 3

3.1		✓first three correct / *eerste drie korrek* ✓remaining two correct / *oorblywende twee korrek* (2)
3.2	 	✓ (0;0) ✓points accurate / *punte akkuraat* ✓shape / *vorm* (3)

Interval	Frequency *Frekwensie*	Cumulative frequency *Kumulatiewe frekwensie*
$\mathbf{0 \leq x < 10}$	**2**	2
$\mathbf{10 \leq x < 20}$	**7**	9
$\mathbf{20 \leq x < 30}$	**14**	23
$\mathbf{30 \leq x < 40}$	**12**	35
$\mathbf{40 \leq x < 50}$	**5**	40

3.3	35 learners / *leerders* Accept 36 or 34 learners / *Aanvaar* 36 *of* 34 *leerders*	✓✓answer / *antwoord* (2) **[7]**

QUESTION / *VRAAG* 4

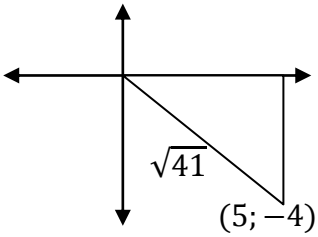
4.1	4.1.1	A(-4 ; 1), B(-1 ; -2), F(k ; p) $-1 = \frac{-4+k}{2}$ $k = 2$ $-2 = \frac{1+p}{2}$ $p = -5$	✓ method / *metode* ✓ $k = 2$ ✓ $p = -5$ (3)
	4.1.2	$m_{AF} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{AF} = \frac{-5-1}{2+4}$ $m_{AF} = -1$ **OR / *OF*** $m_{AF} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{AF} = \frac{-2-1}{-1+4}$ $m_{AF} = -1$	✓ formula / *formule* ✓ substitution / *instelling* ✓ answer / *antwoord* ✓ formula / *formule* ✓ substitution / *instelling* ✓ answer / *antwoord* (3)
	4.1.3	B(-1 ; -2) and / *en* $m_{perp} \times m_{AF} = -1$ $m_{perp} = 1$ $y - y_1 = m(x - x_1)$ $y + 2 = 1(x + 1)$ $y = x - 1$	✓$m_{perp} = 1$ ✓formula / *formule* ✓substitution / *instelling* ✓equation in any form / *vergelyking in enige vorm* (4)
4.2	C(2 ; 5), A(-4 ; 1), F(2 ; -5) $CA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $CA = \sqrt{(-4-2)^2 + (1-5)^2}$ $CA = \sqrt{52}$ $CF = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $CF = \sqrt{(2-2)^2 + (-5-5)^2}$ $CF = \sqrt{100} = 10$ $FA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $FA = \sqrt{(-4-2)^2 + (1+5)^2}$ $FA = \sqrt{72}$ ΔCAF is scalene / *ongelyksydig*	✓formula / *formule* ✓substitution / *instelling* ✓ CA = $\sqrt{52}$ ✓ CF = 10 ✓AF = $\sqrt{72}$ ✓scalene / *ongelyksydig* (6)
4.3	tanθ = m tanθ = -1 θ = 135° $A\hat{F}C = 135° - 90°$ $A\hat{F}C = 45°$	✓ tanθ = -1 ✓ θ = 135° ✓ $A\hat{F}C = 135° - 90°$ ✓ $A\hat{F}C = 45°$ (4)

4.4	If BC were perpendicular to AF, the triangle would have to be isosceles. / *As* BC *loodreg op* AF *was, sou die driehoek gelykbenig moes wees.* **OR / *OF*** C does not satisfy the equation of the perpendicular bisector. / C *maak nie die vergelyking van die middelloodlyn waar nie.*	✓✓explanation / *verduideliking* (2)
4.5	$y = 0$ as D must be the midpoint of FC / *aangesien D die middelpunt van FC is.*	✓✓$y = 0$ (2) **[24]**

QUESTION / *VRAAG* 5

5.1	$x + 2y - 6 = 0$ $y = -\frac{1}{2}x + 3$ $m = -\frac{1}{2}$ $y - y_1 = m(x - x_1)$ **OR / *OF*** $y = mx + c$ $y - 5 = -\frac{1}{2}(x + 2)$ $5 = -\frac{1}{2}(-2) + c$ $y = -\frac{1}{2}x + 4$	✓ $y = -\frac{1}{2}x + 3$ ✓$m = -\frac{1}{2}$ ✓substitution / *instelling* ✓answer / *antwoord* (4)
5.2	K(-3 ; 5), L(2 ; -3), N(5 ; -9) $m_{KL} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-5}{2+3} = -\frac{8}{5}$ $m_{KN} = \frac{-9-5}{5+3} = -\frac{7}{4}$ **OR / *OF*** $m_{LN} = \frac{-9+3}{5-2} = -2$ K, L, M are NOT collinear / *is NIE saamlynig NIE*.	✓substitution / *instelling* ✓$m_{KL} = -\frac{8}{5}$ ✓$m_{KL} = -\frac{7}{4}$ OR / *OF* $m_{LN} = 2$ ✓not collinear / *nie saamlynig nie* (4) **[8]**

QUESTION 6 / VRAAG 6

6.1	6.1.1	$5\tan\alpha + 4 = 0$ $\tan\alpha = -\frac{4}{5}$ $r^2 = x^2 + y^2$ $r^2 = (5)^2 + (-4)^2$ $r = \sqrt{41}$ $y = -5$ $2\cos(180^o - \alpha) = -2\cos\alpha$ $= -2 \times \frac{5}{\sqrt{41}}$ $= -\frac{10}{\sqrt{41}}$ 	✓ $x = 5$, $y = -4$ in diagram ✓ $r = \sqrt{41}$ ✓ $-2\cos\alpha$ ✓ answer / *antwoord* (4)
	6.1.2	$\sin^2(\alpha - 90^o) - \sin^2\alpha = \cos^2\alpha - \sin^2\alpha$ $= \left(\frac{5}{\sqrt{41}}\right)^2 - \left(\frac{-4}{\sqrt{41}}\right)^2$ $= \frac{9}{41}$	✓$\cos^2\alpha$ ✓substitution / *instelling* ✓answer / *antwoord* (3)

6.2	$4\cos^2 x - \tan 45° = 0$ $4\cos^2 x = 1$ $\cos^2 x = \frac{1}{4}$ $\cos x = \pm\frac{1}{2}$ $x = 60°$ or / *of* $x = 120°$ or / *of* $x = 240°$ or / *of* $x = 300°$	✓ $\tan 45° = 1$ ✓$\cos x = \pm\frac{1}{2}$ ✓any two correct angles / *enige twee korrekte hoeke* ✓remaining two correct angles / *oorblywende twee korrekte hoeke* (4) **[11]**

QUESTION / *VRAAG* 7

7.1	$\frac{\sin 117^o}{\cos 27°} + \cos(-x).\tan(180° - x).\sin(360° + x)$ $= \frac{\cos 27°}{\cos 27°} + \cos x.(-\tan x).\sin x$ $= 1 - \sin^2 x$ $= \cos^2 x$		✓ $\cos 27°$ ✓ $\cos x$ ✓ $-\tan x$ ✓ $\sin x$ ✓ $1 - \sin^2 x$ ✓ $\cos^2 x$ (6)
7.2	7.2.1	$LHS = \frac{\cos x}{1 - \sin x} - \frac{\cos x}{1 + \sin x}$ $LHS = \frac{cosx + sin\,x\,cos\,x - cos\,x + sin\,x\,cos\,x}{(1 - \sin x)(1 + \sin x)}$ $LHS = \frac{2sin\,x\,cos\,x}{1 - sin^2 x}$ $LHS = \frac{2sin\,x\,cos\,x}{cos^2 x}$ $LHS = \frac{2sin\,x}{cos\,x}$ $LHS = 2tan\,x$ $= RHS$	✓$(1 - \sin x)(1 + \sin x)$ ✓$2sin\,x\,cos\,x$ ✓$1 - sin^2 x$ ✓$cos^2 x$ ✓$\frac{2sin\,x}{cos\,x}$ (5)
	7.2.2	$x = -90^o$ $x = 90^o$	✓ -90^o ✓ 90^o (2)
7.3	$\sqrt{\tan\theta} = x + \frac{1}{x}$ if $x^2 + \frac{1}{x^2} = 1$ $\tan\theta = \left(x + \frac{1}{x}\right)^2$ $\tan\theta = x^2 + 2 + \frac{1}{x^2}$ $\tan\theta = 2 + 1 \quad (x^2 + \frac{1}{x^2} = 1)$ $\tan\theta = 3$ $\theta = 71{,}57° + k.\,180° \quad (k \in Z)$		✓$\tan\theta = \left(x + \frac{1}{x}\right)^2$ ✓$x^2 + 2 + \frac{1}{x^2}$ ✓$\tan\theta = 3$ ✓$\theta = 71{,}57° +$ ✓ $k\,180°$ ✓ $k \in Z$ (6) **[19]**

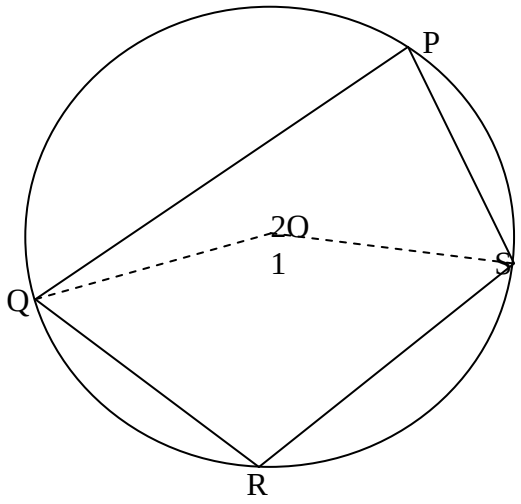
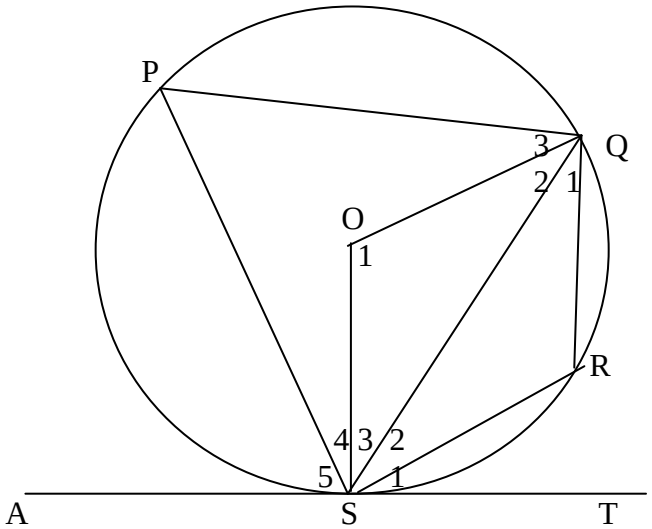
QUESTION / *VRAAG* 8

8.1	$a = -2$ $b = 1$ $p = 1$ $r = 45°$	✓ $a = -2$ ✓ $b = 1$ ✓ $p = 1$ ✓ $r = 45°$ (4)
		✓
8.2	$f(x) - g(x) = 0$ $f(x) = g(x)$ $x = -75°$ or / *of* $105°$	✓ $-75°$ ✓ $105°$ (2)
8.3	$360°$	✓ $360°$ (1)
8.4	$g(x) = \sin(x + 45^o)$ $h(x) = \sin\frac{1}{2}(x + 45° - 45°)$ $\therefore h(x) = \sin\frac{x}{2}$	✓$\frac{1}{2}$ ✓$-45°$ answer only: full marks / *slegs antwoord: vol punte* (2) **[9]**

QUESTION / *VRAAG* 9

9.1		$\sin(90° - \beta) = \frac{MN}{LN}$ $MN = LN.\sin(90° - \beta)$ But $\frac{LN}{\sin\alpha} = \frac{x}{\sin(180° - (\alpha + \beta))}$ $LN = \frac{x.\sin\alpha}{\sin(\alpha + \beta)}$ $\therefore MN = \frac{x.\sin\alpha\,.\cos\beta}{\sin(\alpha + \beta)}$	✓$\sin(90^0 - \beta) = \frac{MN}{LN}$ ✓$MN = LN.\sin(90° - \beta)$ ✓Sine rule / *sinus reël* ✓$\sin(180° - (\alpha + \beta)) = \sin(\alpha + \beta)$ ✓$LN = \frac{x.\sin\alpha}{\sin(\alpha+\beta)}$ ✓answer / *antwoord* (6)
9.2	9.2.1	$\alpha = 76°$, $\beta = 72°$ and / *en* $x = 48$ metres / *meter* $MN = \frac{x.\sin\alpha\,.\cos\beta}{\sin(\alpha + \beta)}$ $MN = \frac{48.\sin 76°\,.\cos 72°}{\sin(76° + 72°)}$ MN = 27,16 metres / *meter*	✓substitution / *instelling* ✓answer / *antwoord* (2)
	9.2.2	Area of $\Delta KLN = \frac{1}{2}KL \times LN \sin K\hat{N}L$ $= \frac{1}{2}(48)(88)\sin 72^o$ $= 2\,008{,}63\ m^2$	✓ formula / *formule* ✓ substitution / *instelling* ✓ answer / *antwoord* (3) **[11]**

QUESTION / *VRAAG* 10

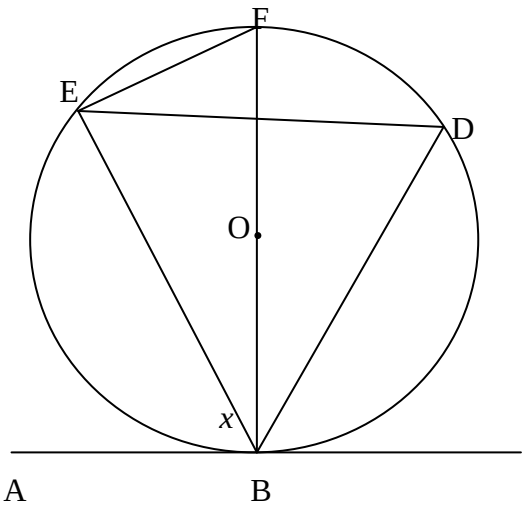
10.1	10.1.1	Equal to angle in the alternate segment *Gelyk aan die hoek in die teenoorstaande segment*	✓answer / *antwoord* (1)
	10.1.2	Interior opposite angle / *teenoorstaande binnehoek*	✓answer / *antwoord* (1)
10.2		 Constr.: Join OQ and OS / *Teken OQ en OS* Proof / *Bewys*: $\widehat{O_1} = 2\,\hat{P}$ (angle at centre / *hoek by middelpunt*) $\widehat{O_2} = 2\,\hat{R}$ (angle at centre / *hoek by middelpunt*) But / *Maar* $\widehat{O_1} + \widehat{O_2} = 360°$ $2\,\hat{P} + 2\,\hat{R} = 360°$ Hence / *dus* $\hat{P} + \hat{R} = 180°$	✓construction / *konstruksie* ✓$\widehat{O_1} = 2\,\hat{P}$ ✓reason / *rede* ✓$\widehat{O_2} = 2\,\hat{R}$ ✓$2\,\hat{P} + 2\,\hat{R} = 360°$ **(5)**
10.3	10.3.1	 $S\hat{Q}R = \hat{S}_1 = 23°$ (tan-chord / *raaklyn-koord*) $Q\hat{S}R = S\hat{Q}R = 23°$ (QR = RS)	✓$\hat{Q}_1 = 23°$ ✓reason / *rede* ✓$\hat{S}_2 = 23°$ ✓reason / *rede* (4)
	10.3.2	$\hat{R} + 23° + 23° = 180°$ (sum of angles of a triangle) $\hat{R} = 134^0$	✓reason / *rede* ✓$\hat{R} = 134°$ (2)
	10.3.3	$\hat{P} + \hat{R} = 180°$ (cyclic quad. / *koordevierhoek*) $\therefore \hat{P} = 46°$	✓reason / *rede* ✓$\hat{P} = 46°$ (2)

	10.3.4	$\hat{O}_1 = 2\hat{P}$ (angle at centre / *middelpunthoek*) $\therefore \hat{O}_1 = 92°$	✓ reason / *rede* ✓ $\hat{O}_1 = 92°$ (2) **[17]**

QUESTION / VRAAG 11

11.1	OD = 25 cm ∴ OC = 25 cm – 18cm = 7 cm $AC^2 + OC^2 = OA^2$ $AC^2 + (7)^2 = (25)^2$ $AC^2 = 576$ $\therefore AC = 24$ cm $AB = 2 \times AC$ (OD⊥AB) $\therefore AB = 48$ cm		✓OC = 7 cm ✓Pythagoras ✓AC = 24 cm ✓ reason / *rede* (OD⊥AB) ✓answer / *antwoord*(5)
11.2	11.2.1	$B\hat{P}R = 25°$ (PR||QB, alt angles / *verw. hoeke*) $R\hat{Q}B = 25°$ (Subtended by / *onderspan deur* RB) $P\hat{R}Q = 25°$ (Subtended by / *onderspan deur* PQ OR / *OF* alt angles / *verw hoeke*)	✓$B\hat{P}R = 25°$ ✓reason / *rede* ✓$R\hat{Q}B = 25°$✓reason / *rede* ✓$P\hat{R}Q = 25°$✓reason / *rede* (6)
	11.2.2 (a)	$R\hat{O}B = 2 \times R\hat{Q}B$ (angle at centre / *middelpunthoek*) $\therefore R\hat{O}B = 50°$	✓reason / *rede* ✓$R\hat{O}B = 50°$ (2)
	11.2.2 (b)	$O\hat{R}T + R\hat{O}T + R\hat{T}O = 180°$(angles of / *hoeke van*Δ) $O\hat{R}T + 50° + 90° = 180°$ $\therefore O\hat{R}T = 40°$	✓ $O\hat{R}T = 40°$ ✓ reason / *rede* (2)
	11.2.2 (c)	$R\hat{O}S = 100°$ (ΔROT ≡ ΔSOT)	✓100^o ✓reason / *rede* (2)
	11.2.2 (d)	$R\hat{P}Q = 115°$($B\hat{P}Q = 90°$, angle in semi-circle / *hoek in half-sirkel*)	✓ answer / *antwoord* ✓ reason / *rede* (2) **[19]**

QUESTION / *VRAAG* 12

12.1	12.1.1 (a)	 $F\hat{B}E = 90° - x$ (tangent is perp. to a diameter)	✓ $90° - x$ (1)
	12.1.1 (b)	$\hat{F} = x$	✓ x (1)

	12.1.2	$E\hat{O}B = 2x$ (angle at centre / *middelpunthoek*) $\therefore A\hat{B}E \neq E\hat{O}B$	✓$E\hat{O}B = 2x$ ✓reason / *rede* ✓$\therefore A\hat{B}E \neq E\hat{O}B$ (3)
12.2	$C\hat{A}R = A\hat{B}D$ (alt angles / *verw hoeke*, AC || DB) $C\hat{A}R = C\hat{P}R$ (subtended by / *onderspan deur* CR) $\therefore R\hat{B}D = C\hat{P}R$ (both / *albei* = $C\hat{A}R$) Hence PDBR is a cyclic quadrilateral (Ext. angle = int. opp. angle / *buitehoek = teenoor. binnehoek*)		✓$C\hat{A}R = A\hat{B}D$ ✓reason / *rede* ✓$C\hat{A}R = C\hat{P}R$ ✓reason / *rede* ✓$R\hat{B}D = C\hat{P}R$ ✓conclusion / *slotsom* ✓ reason / *rede* (7) **[12]**

TOTAL: / TOTAAL: 150