basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

GREENBURY SECONDARY SCHOOL

GRADE 11

MATHEMATICS PAPER TWO

NOVEMBER 2015

MARKS: 150 **EXAMINER :** R. DEMRUGARAM

TIME: 3 Hour **MODERATOR:** L. PILLAY

This question paper consists of 9 pages

INSTRUCTIONS AND INFORMATION:

Read the following instructions carefully before answering the questions:

1. This question paper consists of 12 questions and 9 pages.
2. Answer all the questions.
3. Clearly show all calculations, diagrams, graphs, etc which you have used in determining the answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non- programmable and non-graphical), unless stated otherwise.
6. If necessary, round off your answers to TWO decimal places, unless stated otherwise.
7. Number your answers correctly according to the numbering system used in this question paper.
8. Diagrams are not necessarily drawn to scale.
9. It is in your own interest to write neatly and legibly.

p.t.o. Question One....

QUESTION ONE:

The following data represents a set of mathematics marks of 19 learners in a Mathematics test. The maximum mark is 50.

32 9 49 11 7 31 22 14 25 38 36 40 42 10 19 42 8 6 16

1.1. Write down the 5 number summary. (5)
1.2. Draw a box and whisker plot of the data on the diagram sheet provided. (3)
1.3. Comment on the distribution of the data. (1)
1.4. Calculate the mean. (2)
1.5. Determine the standard deviation. (2)
1.6. Determine the percentage of scores that lie within one standard deviation of the mean. (2) [15]

QUESTION TWO:

The table below shows the performance of learners in a grade 11 Mathematics test.

Class Interval	Frequency	Cumulative Frequency
$0 \leq x < 10$	1	
$10 \leq x < 20$	3	
$20 \leq x < 30$	6	
$30 \leq x < 40$	13	
$40 \leq x < 50$	12	
$50 \leq x < 60$	15	
$60 \leq x < 70$	8	
$70 \leq x < 80$	2	
$80 \leq x < 90$	3	
$90 \leq x < 100$	1	

2.1. Complete the table provided. (3)

2.2. Draw an Ogive curve to represent this information. (4)

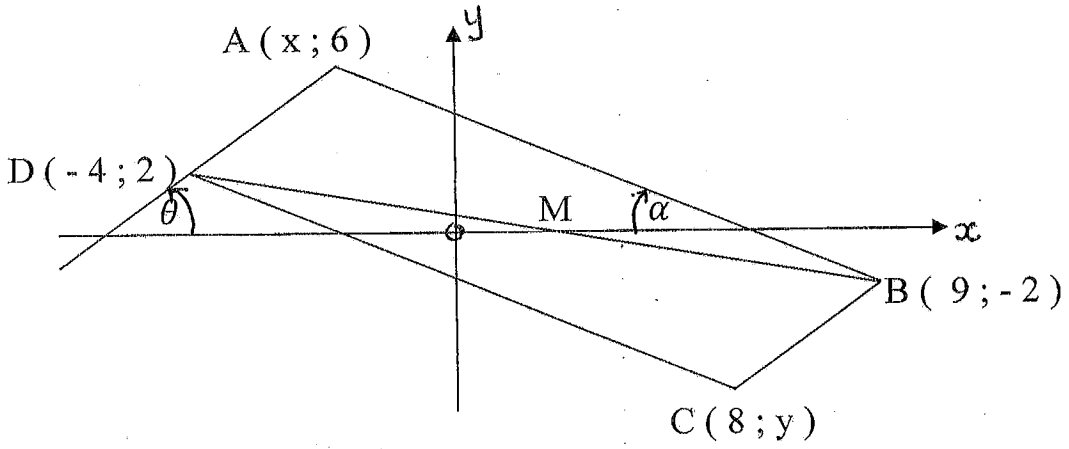
2.3. Estimate the median from the graph. (1) [8]

p.t.o. Question Three....

QUESTION THREE:

In the diagram ABCD is a parallelogram with coordinates

A (x ; 6) ; B (9 ; - 2) ; C (8 ; y) and D (- 4 ; 2)



3.1. Calculate the coordinates of M, the midpoint of BD. (2)
3.2. Hence, or otherwise show that x = - 3 units and y = - 6 units (3)
3.3. Calculate the gradient of BC. (2)
3.4. Calculate the gradient of BD. (2)
3.5. Hence show that ΔDBC is not right angled. (2)
3.6. Calculate the equation of AB in the form y = mx + c. (4)
3.7. Determine the value of $D\hat{A}B$ (4) [19]

QUESTION FOUR:

4.1. The distance between the points A (- 6 ; - 5) and B (2 ; x) is 10 units. Determine the value(s) of x. (4)

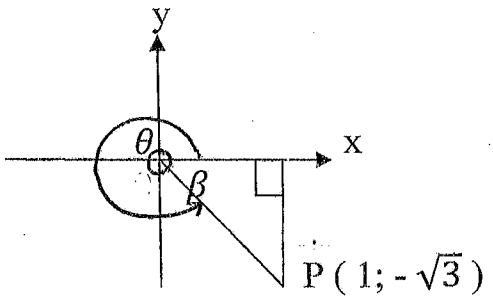
4.2. The equation of a straight line is y = px + 6. Determine the value of p in each of the following cases if the given line:

4.2.1. passes through the point B (- 1 ; - 4) (2)
4.2.2. is perpendicular to the line 2y + x = 4 (2)
4.2.3. is parallel to the x - axis. (1) [9]

p.t.o. Question Five....

QUESTION FIVE:

In the figure point P has coordinates (1 ; $-\sqrt{3}$)
Reflex $X\hat{O}P = \theta$ and $X\hat{O}P = \beta$



Determine the following without the use of a calculator:

5.1. OP (3)

5.2. $\sin\theta$ (2)

5.3. the value of β (3) [8]

QUESTION SIX:

6.1. Determine the value of the following expression without using a calculator:

$$\frac{\cos 10^\circ . \cos 120^\circ}{\sin 80^\circ . \sin 330^\circ}$$ (5)

6.2. Express as a single ratio of θ:

$$\frac{\sin(90^\circ - \theta).\tan(180^\circ - \theta)}{\cos(-\theta) . \sin(180^\circ + \theta)}$$ (5)

6.3. Prove the following identity:

6.3.1. $\sin x + \frac{\cos^2 x}{\sin x} = \frac{1}{\sin x}$ (4)

6.3.2. For which value(s) of x is the identity undefined if $x \in [0^\circ ; 360^\circ]$ (3) [17]

p.t.o. Question Seven....

QUESTION SEVEN:

Given $f(x) = \sin(x - 30°)$
$g(x) = \cos 2x$

7.1. Determine the general solution of $f(x) = g(x)$ (7)

7.2. Draw a neat sketch of f and g on the same set of axes for $x \in [-180°; 180°]$.
Clearly indicate all intercepts with the axes, turning points and end points. (6)

7.3. Write down the period of g. (1)

7.4. Determine the values of x for which $g(x)$ is strictly decreasing as x increases. (2)

7.5. If f is shifted 60° to the left to obtain the graph of $h(x)$, write down the resulting equation of $h(x)$. (2) [18]

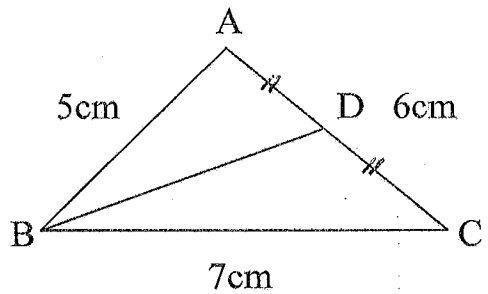
QUESTION EIGHT:

8.1. Complete the following rules for Δ PQR:

8.1.1. Area Δ PQR = $\frac{1}{2}$ pr _____ (1)

8.1.2. $\cos Q = \frac{p^2 + r^2 - q^2}{\_\_\_\_}$ (1)

8.2. In the figure D is the midpoint of AC.
AB = 5 cm, BC = 7 cm and AC = 6 cm.



8.2.1 Show by calculation that $\hat{A} = 78.46°$ (3)

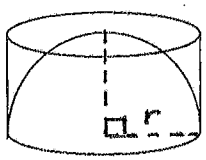
8.2.2. Calculate the area of ΔABD (3)

8.2.3 Calculate the size of $\hat{C}$ (3) [11]

p.t.o. Question Nine....

QUESTION NINE:

A solid wooden hemisphere is carved from a cylindrical piece of wood. The radius of the hemisphere and the cylinder is 12 cm.



Calculate the volume of:

9.1. the wooden hemisphere. (3)

9.2. the volume of wood that has been carved away. (4) [7]

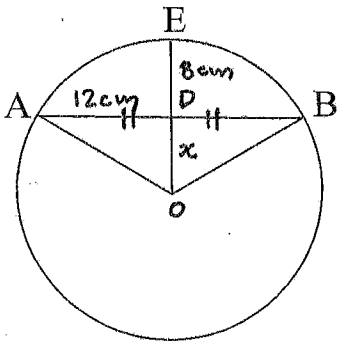
In Questions 10, 11and 12 please note:

1. **Diagrams are not drawn to scale.**
2. **Give a reason for each new statement.**
3. **Show all necessary working in full.**

QUESTION TEN:

10.1. Complete the statement so that it is valid:
The line drawn from the centre of a circle perpendicular to the chord _____ (1)

10.2. AB is the chord of circle centre O.
OE bisects chord AB.
AD = 12 cm, ED = 8 cm and OD = x cm



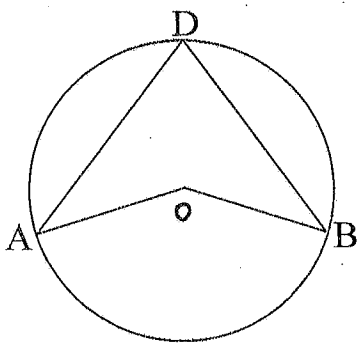
10.2.1. Determine the radius OB in terms of x. (1)

10.2.2. Hence calculate the diameter of the circle. (4) [6]

p.t.o... Question 11....

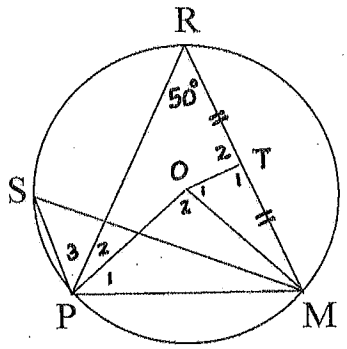
QUESTION ELEVEN:

11.1. In the diagram, O is the centre of the circle.
A, B and D are points on the circle.



Using Euclidean geometry methods prove the theorem which states that $A\hat{O}B = 2\,A\hat{D}B$. Clearly indicate any construction lines. (5)

11.2. In the diagram, O is the centre of the circle RMPS.
T is the midpoint of RM. $\hat{R} = 50°$



11.2.1. Calculate, with reasons the value of the following:

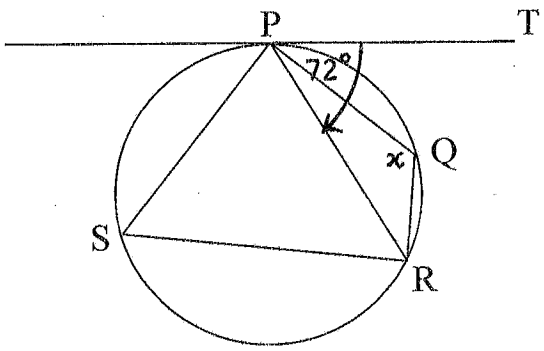
a) $\hat{O}_2$ (2)
b) $\hat{S}$ (2)
c) $\hat{P}_1$ (2)
d) $\hat{T}_1$ (2)

11.2.2. Show by calculations, that TOPM is **NOT** a cyclic quadrilateral.(2) [15]

p.t.o. Question Twelve....

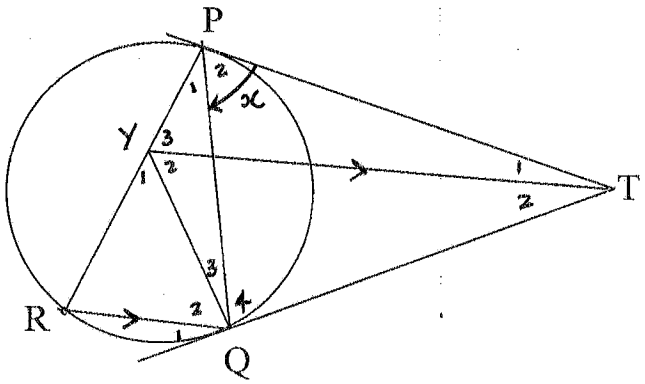
QUESTION TWELVE:

12.1. PT is a tangent to PQRS.
$T\hat{P}R = 72°$. $\hat{Q} = x$



Calculate the value of x. (4)

12.2. TP and TQ are tangents to the circle at P and Q respectively.
TY is drawn parallel to RQ with Y on RP. $T\hat{P}Q = x$



12.2.1. Name with reasons 3 other angles equal to x. (6)
12.2.2. Prove that TPYQ is a cyclic quadrilateral. (2)
12.3.3. Prove that TY bisects $P\hat{Y}Q$. (2)
12.2.4. Prove that ΔRQY is an isosceles triangle. (3) [17]

TOTAL : 150

04/11/2015

GRADE 11 MATHEMATICS PAPER 2

MARKING MEMORANDUM

QUESTION ONE:

1.1. 6 7 8 9 (10) 11 14 16 19 (22) 25 31 32 36 (38) 40 42 42 49

MIN = 6 ✓A

Q_1 = 10 ✓A

Q_2 = 22 ✓A

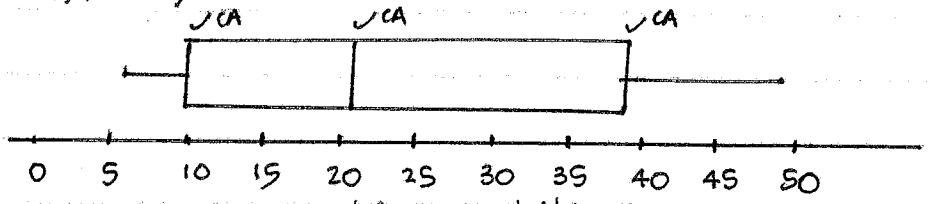
Q_3 = 38 ✓A

MAX = 49 ✓A

If learners used formulae to obtained answers (full credit)

(5)

1.2.



(3)

1.3. SKEWED TO THE RIGHT / POSITIVELY SKEWED ✓CA / 50% marks lie between 10 & 38 ; etc (1)

1.4. MEAN = $\frac{457}{19}$ = 24,05 ≈A / 24 ✓A (2)

1.5. σ = 13,83 ≈A (attempt with use of a table – 1 mark) (2)

1.6. $(\bar{x} \pm \sigma)$

$= (\bar{x} - \sigma ; \bar{x} + \sigma)$

$= (24,05 - 13,83 ; 24,05 + 13,83)$

$= (10,22 ; 37,88)$ ✓CA

$= \frac{9}{19} \times 100$

= 47,37 % ✓CA / 47% / 47,4% (2) [15]

✱. 1.3. 50% marks lie between 6 and 22 or
50% " " " 22 and 38
accept any one of the answers.

QUESTION TWO:

2.1.

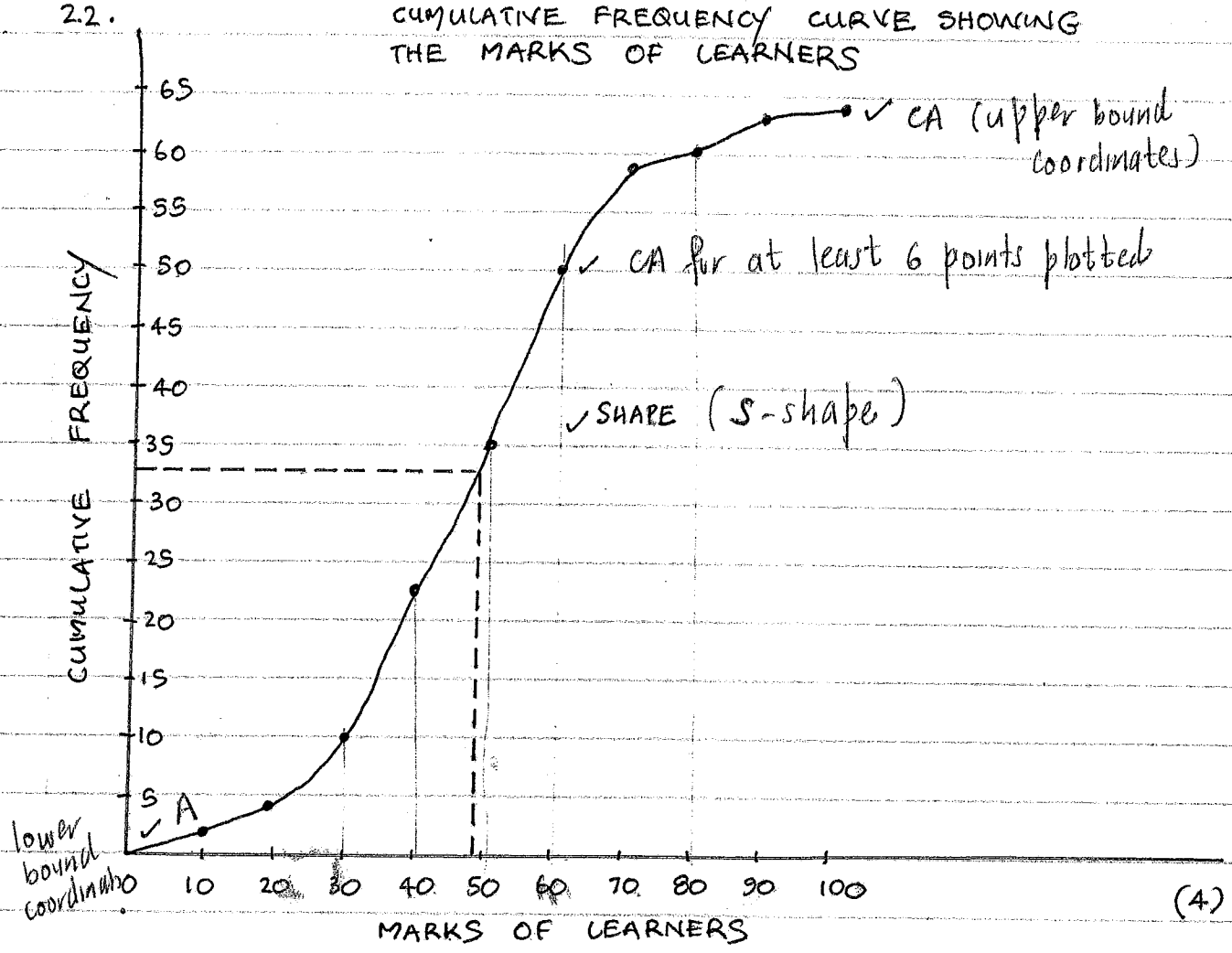
CLASS INTERVAL	FREQ	CUMULATIVE FREQUENCY
$0 \leq x < 10$	1	1
$10 \leq x < 20$	3	4
$20 \leq x < 30$	6	10
$30 \leq x < 40$	13	23
$40 \leq x < 50$	12	35
$50 \leq x < 60$	15	50
$60 \leq x < 70$	8	58
$70 \leq x < 80$	2	60
$80 \leq x < 90$	3	63
$90 \leq x < 100$	1	64

- all correct 3 marks
- 1 Error 2 marks
- 2 Errors 1 mark

✓✓A More than 2 Errors ø marks

(3)

2.2.



(4)

2.3. MEDIAN = ±48 accept answers: {46 / 47 / 48 / 49} or fractions thereof. (1) [8]

QUESTION THREE:

3.1. $B(9;-2)$; $D(-4;2)$

$M\left(\frac{9-4}{2};\frac{-2+2}{0}\right)$ ✓A

$M\left(\frac{5}{2};0\right)$ ✓A (2)

3.2. $A(x;6)$; $C(8;y)$

$\frac{x+8}{2} = \frac{5}{2}$ ✓A ; $\frac{6+y}{2} = 0$ ✓A

$x = -3$; $y = -6$ ✓A (3)

3.3. $m_{BC} = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{-2+6}{9-8}$

$= 4$ ✓A (2)

3.4. $m_{BD} = \frac{2+2}{-4+9}$

$= \frac{4}{5}$ ✓A (2)

3.5. $m_{BC} \times m_{BD} = \frac{4}{1} \times \frac{4}{5}$

$= \frac{8}{5}$ ✓A

$\neq 1$ ✓A

$\therefore \triangle DBC$ is not rt angled. (2)

3.7. $m_{AD} = \frac{6-2}{-3+4}$

$= 4$

$\theta = 75,96°$ ✓CA

$m_{AB} = \frac{-2-6}{9+3}$

$= \frac{-2}{3}$

$\alpha = 33,69°$ ✓CA

$\hat{DAB} = 70,35°$ ✓CA (as of $\triangle$) (4)

3.6. $m_{AB} = -\frac{2}{3}$

$y - y_1 = m(x-x_1)$ ✓

$y - 6 = -\frac{2}{3}(x+3)$ ✓

$y - 6 = -\frac{2}{3}x - 2$

$y = -\frac{2}{3}x + 4$ ✓ ✓ (4)

[19]

QUESTION FOUR:

4.1. $AB = \sqrt{(2+6)^2 + (x+5)^2}$ ✓A

$10 = \sqrt{64 + x^2 + 10x + 25}$

$100 = 64 + x^2 + 10x + 25$ ✓A

$x^2 + 10x + 64 + 25 - 100 = 0$

$x^2 + 10x - 11 = 0$ ✓CA

$x = 1$ or $x = -11$ ✓CA (4)

4.2.1. $y = px + 6$

$(-1;-4)$: $-4 = p(-1) + 6$ ✓A

$p = 10$ ✓CA (2)

422. $2y + x = 4$

$2y = -x + 4$

$y = -\frac{1}{2}x + 2$

$m_1 = -\frac{1}{2}$ ✓A

$p = 2$ ✓CA (2)

4.2.3. $p = 0$ ✓A (1)

[9]

QUESTION FIVE:

5.1.1. $OP^2 = (1)^2 + (-\sqrt{3})^2$ (Th of Pyth) ✓ ✓

$OP = 2$ ✓ (3)

5.1.2. $\sin\theta = \frac{-\sqrt{3}}{2}$ ✓ ✓ (2)

5.1.3. Reflex $X\hat{O}B = 300°$ ✓

$\hat{B} = 60°$ ✓ (3)

[8]

QUESTION SIX :

6.1. $\dfrac{\cos 10^\circ \cos 120^\circ}{\sin 80^\circ \sin 330^\circ}$

$= \dfrac{\sin 80^\circ \cos(180^\circ - 60^\circ)}{\sin 80^\circ \sin(360 - 30^\circ)}$ ✓A ✓A

$= \dfrac{-\cos 60}{-\sin 30^\circ}$ ✓A ✓A

$= -\frac{1}{2}$ ✓CA

$= -\frac{1}{2}$

$= 1$ ✓CA

OR $\dfrac{\cos 10^\circ . (-\sin 30^\circ)}{\cos 10^\circ (-\sin 30^\circ)}$ ✓1 ✓1 ✓1

$= 1$ ✓1

(5)

6.2. $\dfrac{\sin(90 - \theta)\tan(180 - \theta)}{\cos(-\theta)\sin(180 + \theta)}$

$= \dfrac{\cos\theta \cdot -\tan\theta}{\cos\theta \cdot -\sin\theta}$ ✓A ✓A

$= \dfrac{\sin\theta}{\cos\theta}$ ✓A $\times \dfrac{1}{\sin\theta}$ ✓A

$= \dfrac{1}{\cos\theta}$ ✓CA

(5)

6.3.1 LHS $= \sin x + \dfrac{\cos^2 x}{\sin x}$ ✓A

$= \dfrac{\sin^2 x + \cos^2 x}{\sin x}$ ✓A ✓A

$= \dfrac{1}{\sin x}$ ✓A

(4)

6.3.2. $\sin x = 0$ ✓A ✓A ✓A

$x = 0^\circ, 180^\circ, 360^\circ$

(3) [17].

QUESTION SEVEN:

7.1. $\cos 2x = \sin(x-30^\circ)$.

$\cos 2x = \cos[90-(x-30)]$ ✓

$\cos 2x = \cos(120-x)$ ✓

$2x = 120-x+k360$ ✓ or $2x = 360-(120-x)+k360$, $k\in Z$ ✓

$3x = 120+k360$ ✓ $2x = 360-120+x+k360$

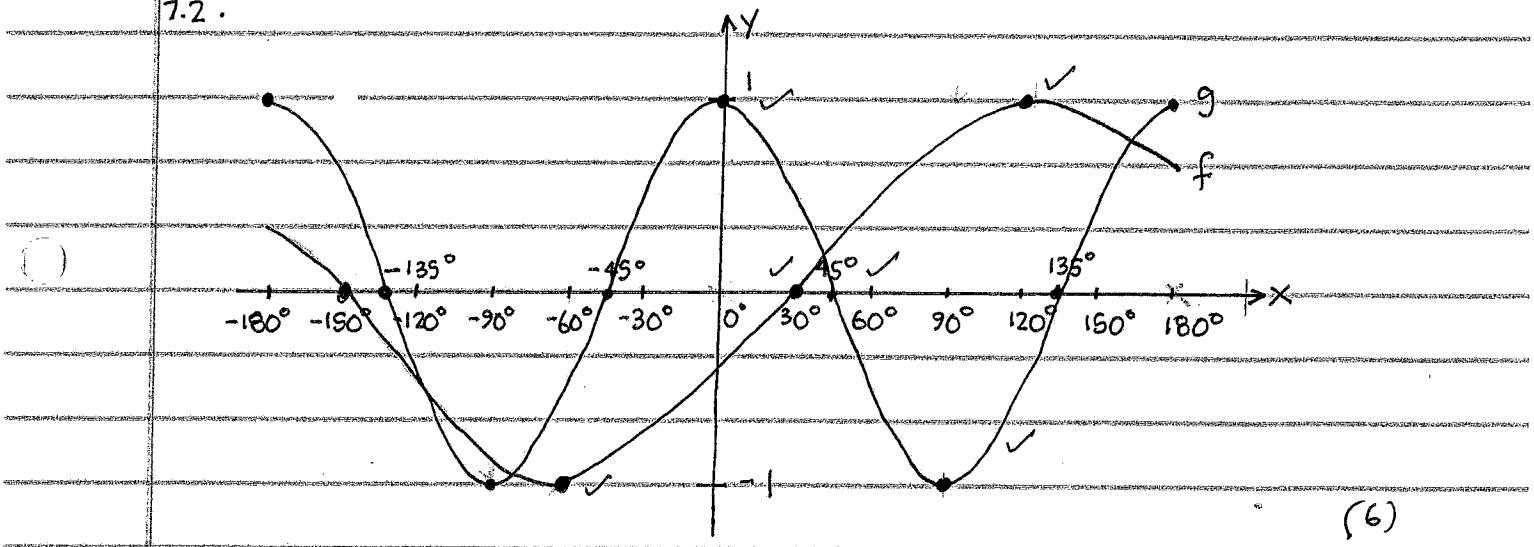
$x = 40^\circ+k120$ ✓ $x = 240+k360$ ✓ / $x = -120^\circ-360^\circ.k$ (7)

Look for shorter techniques on corrections used by learners eg

$x-30^\circ+2x = 90^\circ+360^\circ.k \Rightarrow \ldots$

OR $x-30^\circ-2x = 90^\circ+360^\circ.k \Rightarrow \ldots$

7.2.



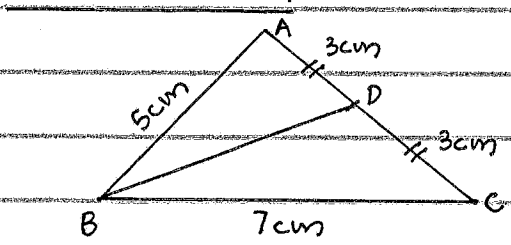
(6)

7.3. 180° ✓A ✓CA ✓CA (1)

7.4. $x\in(-180^\circ;-90^\circ)$; $x\in(0^\circ,90^\circ)$ (2)

7.5. $h(x) = \sin(x+30^\circ)$ ✓A ✓A (2) [18]

QUESTION EIGHT:



8.1.1. AREA $\Delta PQR = \frac{1}{2}pr\sin Q$ ✓A (1)

8.1.2. $\cos Q = \frac{p^2+r^2-q^2}{2pr}$ ✓A (1)

8.2.1. $\cos A = \frac{b^2+c^2-a^2}{2bc}$

$\cos A = \frac{6^2+5^2-7^2}{2(6)(5)}$

$A = 78,46$ (3)

8.2.2. AREA $\Delta ABD = \frac{1}{2}(AB)(AD)\sin 78,46^\circ$

$= \frac{1}{2}(5)(3)\sin 78,46^\circ$ ✓A

$= 7,35\ cm^2$ ✓CA (3)

8.2.3. $\frac{\sin C}{5} = \frac{\sin 78,46}{7}$ ✓A

$\sin C = \frac{5\sin 78,46}{7}$ ✓CA

$\hat{C} = 44,41^\circ$ ✓CA (3)

[11]

QUESTION NINE:

9.1. $V_1 = \frac{1}{2}\left[\frac{4}{3}(\pi)(12)^3\right]$ ✓A ✓A

$= 3619,11 \text{ cm}^3$ ✓CA (3)

9.2. VOLUME OF CYLINDER

$= \pi(12)^2(12)$ ✓A

$= 5428,67 \text{ cm}^3$ ✓CA

VOLUME OF CARVED WOOD

$= 5428,67 - 3619,11$ ✓CA

$= 1809,56 \text{ cm}^3$ ✓CA (4)

[7]

QUESTION TEN:

10.1. bisects the chord ✓A (1)

10.2.1 $OB = x + 8$ ✓A (1)

10.2.2. $(x+8)^2 = 12^2 + x^2$ ✓A (Th of Pyth)

$x^2 + 16x + 64 = 144 + x^2$

$16x = 80$

$x = 5$ ✓

$r = 13 \text{ cm}$

DIAMETER $= 26 \text{ cm}$ ✓ (4) [6]

QUESTION ELEVEN :

11.1. GIVEN : CIRCLE WITH $A\hat{O}B$ AT CENTRE AND $A\hat{D}B$ AT CIRCUMFERENCE.

RTP : $A\hat{O}B = 2A\hat{D}B$

PROOF : DRAW DC THROUGH DO ✓

$DO = AD = OB$ (RADII)

$\hat{O}_1 = \hat{A} + \hat{D}_1$ (EXT ∠ OF Δ)

$= 2D_1$ ✓

$\hat{O}_2 = \hat{D}_2 + B$ (EXT ∠ OF Δ)

$= 2D_2$ ✓

$A\hat{O}B = 2\hat{D}_1 + 2\hat{D}_2$ ✓

$= 2(\hat{D}_1 + \hat{D}_2)$

$= 2A\hat{D}B$ ✓ (5)

11.2.1 a) $\hat{O}_2 = 100°$ ✓A ✓A (∠ at centre) (2)

b) $\hat{S} = 50°$ ✓A ✓A (∠ at centre) or (∠s in same segm) (2)

c) $\hat{P}_1 = 40°$ ✓A (∠s of isosc Δ; radii) ✓A (2)

d) $\hat{T}_1 = 90°$ ✓A (midpt, centre) ✓A (2)

11.2.2. $\hat{P}_1 + \hat{T}_1$

$= 40° + 90°$

$= 130°$ ✓

$\neq 180°$ ✓

∴ TOPM is not cyclic quad (2) [15]

QUESTION TWELVE :

12.1. $\hat{S} = 72^\circ$ ✓A (∠ betw tan, chord)

$x = 108^\circ$ ✓A (opp ∠s of cyclic ✓A quad) (4)

12.2.1. $\hat{R} = x$ ✓A (tan chord theorem) ✓A

$\hat{Y}_3 = x$ ✓A (corresp ∠s = TY ∥ RQ) ✓A

$\hat{Q}_4 = x$ ✓A (tan chord theorem) ✓A (6)

12.2.2. $\hat{Y}_3 = \hat{Q}_4 = x$ (Proved) ✓A ✓A

$\therefore$ TPYQ is cyclic (∠s in same segm) (2)

12.2.3. $\hat{P}_2 = \hat{Y}_2 = x$ ✓A (∠s in same segm QT)

$\therefore$ TY bisects $P\hat{Y}Q$ (2)

12.2.4. $\hat{Y}_2 = \hat{Q}_2 = x$ ✓A (TY ∥ RQ)

Now $\hat{R} = \hat{Q} = x$ ✓A

$\therefore$ $\triangle$RQY is isosceles (3) [17]