



**NATIONAL
SENIOR CERTIFICATE**

GRADE 11

NOVEMBER 2015

**MATHEMATICS P2/WISKUNDE V2
MEMORANDUM**

MARKS/PUNTE: 150

This memorandum consists of 8 pages.
Hierdie memorandum bestaan uit 8 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:

- Indien 'n kandidaat 'n vraag twee keer beantwoord, merk slegs die eerste poging.
- Indien 'n kandidaat 'n antwoord doodgetrek het, maar nie oorgedoen het nie, merk die doodgetrekte antwoord.
- Volgehoue akkuraatheid geld in ALLE aspekte van die memorandum.
- Aanneme van antwoorde/waardes om 'n probleem op te los, is Onaanvaarbaar.

QUESTION/VRAAG 1: [12]																				
1.1	<table border="1"> <thead> <tr> <th>Height (cm) <i>Hoogte (cm)</i></th> <th>Frequency <i>Frekwensie</i></th> <th>Cumulative frequency <i>Kumulatiewe frekwensie</i></th> </tr> </thead> <tbody> <tr> <td>$150 < x \leq 155$</td> <td>4</td> <td>4</td> </tr> <tr> <td>$155 < x \leq 160$</td> <td>22</td> <td>26</td> </tr> <tr> <td>$160 < x \leq 165$</td> <td>56</td> <td>82</td> </tr> <tr> <td>$165 < x \leq 170$</td> <td>32</td> <td>114</td> </tr> <tr> <td>$170 < x \leq 175$</td> <td>6</td> <td>120</td> </tr> </tbody> </table>	Height (cm) <i>Hoogte (cm)</i>	Frequency <i>Frekwensie</i>	Cumulative frequency <i>Kumulatiewe frekwensie</i>	$150 < x \leq 155$	4	4	$155 < x \leq 160$	22	26	$160 < x \leq 165$	56	82	$165 < x \leq 170$	32	114	$170 < x \leq 175$	6	120	<p>✓26</p> <p>✓120</p> <p>(2)</p>
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1.2	<p>Ogive/Ogief</p>	<p>✓ grounding at (150;0)/anker by (150;0)</p> <p>✓ plotting (155;4)/plot van (155;4)</p> <p>✓ plotting with upper limits/plot by boonste limiete</p> <p>✓ joining the points to form a smooth curve/verbind punte om glade kurwe te vorm</p> <p>(4)</p>																		
1.3	<p>(150, 160,5, 163, 166, 175)</p> <p>OR/OF</p> <p>Min = 150</p> <p>$Q_1 = 160,5$</p> <p>$Q_2 = 163$</p> <p>$Q_3 = 166$</p> <p>Max = 175</p>	<p>✓ 150</p> <p>✓ 160,5</p> <p>✓ 163</p> <p>✓ 166</p> <p>✓ 175</p> <p>(5)</p>																		
1.4	Skewed to the left / <i>Skeefgetrek na links.</i>	<p>✓ answer/antwoord</p> <p>(1)</p>																		
			[12]																	

QUESTION/VRAAG 2 [7]		
2.1	$\bar{x} = \frac{2250+2250+3000+3300+3300+3300+3600+3900+4350+4350+5250}{10}$ $\bar{x} = \frac{35550}{10}$ <p style="text-align: center;">ANSWER ONLY : FULL MARKS SLEGS ANTWOORD : VOLPUNTE</p>	✓ $\frac{35550}{10}$ ✓ answer/antwoord (2)
2.2	$\sigma = R 900,12$	✓ answer/antwoord (1)
2.3	$(3555 - 900,12 ; 3555 + 900,12) = (2654,88 ; 4455,12)$ $\therefore 7$ workers lie within one standard deviation <i>werkers lê binne een standaardafwyking</i> $\therefore \frac{7}{10} = 70\%$ of workers lie within one standard deviation. <i>van die werkers lê binne een standaardafwyking.</i>	✓ ✓ interval ✓ 7 ✓ answer/antwoord (4)
[7]		
QUESTION/VRAAG 3 [13]		
3.1	$m_{AB} = m_{BC} = m_{AC}$ $m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $\frac{-1 - 3}{4 + 2} = -\frac{2}{3}$ $\frac{-1 - (a + 2)}{4 - (2a - 11)} = -\frac{2}{3}$ $\frac{-3 - a}{15 - 2a} = -\frac{2}{3}$ $30 - 4a = 9 + 3a$ $7a = 21$ $a = 3$	✓ $m_{AB} = m_{BC} = m_{AC}$ ✓ substitute into equation/vervang in vgl ✓ substituting into m_{AC} /vervang in vgl m_{AC} ✓ answer/antw (4)
3.2	$y - y_1 = m(x - x_1)$ $y - 5 = -\frac{2}{3}(x + 5)$ $y = -\frac{2}{3}x + \frac{5}{3}$	✓ equation/vgl ✓ subst of m and $(-5;5)$ into form/vervang van m en $(4;-1)$ in formule ✓ equation/vgl (3)
3.3	$\text{Midpt} = \left[\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2} \right]$ $= \left[\frac{-2 - 5}{2}; \frac{3 + 5}{2} \right]$ $= \left[-\frac{7}{2}; 4 \right]$	✓ ✓ subst into correct formula/vervang in korrekte formule ✓ coordinates/coordinate (3)
3.4	$m_{CD} = 0$ $\frac{p - 7 + 1}{4p - 4} = \frac{0}{1}$ $p = 6$ OR/OF $p - 7 = -1$ $p = 6$	✓ correct $m = 0$ ✓ substitution into eqn/vervang in vgl ✓ answer/antw ✓ ✓ equating/ ✓ answer (3)
[13]		

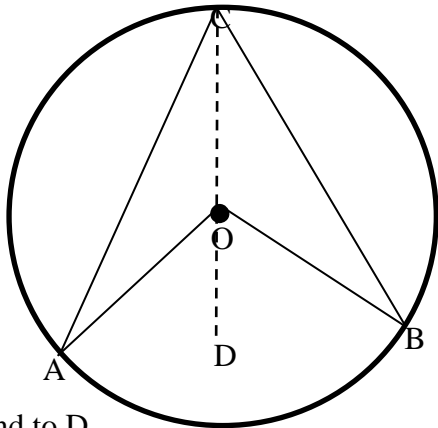
QUESTION/VRAAG 4: [13]		
4.1	$m_{NQ} = \frac{-12}{-6}$ $= 2$	✓ answer/antw (1)
4.2	$M_{NQ} \times M_{MP} = -1 \quad [NQ \perp MP]$ $m_{MP} = -\frac{1}{2}$	✓ answer/antw (1)
4.3	$m_{MP} = -\frac{1}{2}$ $\tan \theta = -\frac{1}{2}$ $Ref = 26,57^\circ$ $\theta = 180^\circ - 26,57^\circ$ $\theta = 153,43^\circ$	✓ $m_{MP} = -\frac{1}{2}$ ✓ $\tan \theta = -\frac{1}{2}$ ✓ value of θ /waarde van θ (3)
4.4	$y - y_1 = m(x - x_1)$ $y - 6 = -\frac{1}{2}(x - 0)$ $y = -\frac{1}{2}x + 6$	✓ equation/vgl ✓✓ subst of $m = -\frac{1}{2}$ and (0;6) into eqn/vervang $m = -\frac{1}{2}$ en (0;6) in vgl ✓ answer/antw (4)
4.5	$m_{NQ} \times m_{MP} = -1$ $m_{MP} = -\frac{1}{2}$ $\frac{y_2 - y_1}{x_2 - x_1} = -\frac{1}{2}$ $\frac{y - 6}{x - 0} = -\frac{1}{2}$ $x = 2; y = 5$ $P(2;5)$	✓ $m_{NQ} \times m_{MP} = -1$ ✓ $m_{MP} = -\frac{1}{2}$ ✓ subst into eqn/vervang in vgl ✓ P (2;5) (4)
4.6	$MR = NR$ $R = \left[\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2} \right]$ $R = \left[\frac{0-6}{2}; \frac{6-12}{2} \right]$ $R = [-3; -3]$	✓ Substitution/vervang. ✓ x value/waarde ✓ y value/waarde (3)
		[16]
QUESTION/VRAAG 5: [23]		
5.1.1	$\tan \theta = 3$ $\theta = 71,57^\circ$	✓ $\tan \theta$ ✓ answer/antwoord (2)
5.1.2	$OP^2 = (1)^2 + (3)^2$ $OP = \sqrt{10}$	✓ OP^2 ✓ answer/antwoord (2)
5.1.3a	$\sin \theta = \frac{3}{\sqrt{10}}$	✓ answer/antwoord (1)
5.1.3b	$\cos(180^\circ + \theta) = -\cos \theta$ $= -\frac{1}{\sqrt{10}}$	✓ $-\cos \theta$ ✓ answer/antwoord (2)
ANSWER ONLY FULL MARKS/ SLEGS ANTWOORD:VOLPUNTE		

5.2	$2 \cos^2 x + 5 \sin x = 4$ $2 \cos^2 x + 5 \sin x - 4 = 0$ $2(1 - \sin^2 x) + 5 \sin x - 4 = 0$ $2 \sin^2 x - 5 \sin x + 2 = 0$ $(2 \sin x - 1)(\sin x - 2) = 0$ $2 \sin x = 1 \quad \text{or} \quad \cos x = 2$ $\sin x = \frac{1}{2} \quad \text{no soln}$ $x = 30^\circ + k.360^\circ$ $x = 150^\circ + k.360^\circ, k \in Z$	<ul style="list-style-type: none"> ✓ standard form/st vorm ✓ $1 - \sin^2 x$ ✓ $\sin x = \frac{1}{2}$ or $\sin x = 2$ ✓ no soln ($\sin x = 2$) ✓ $30^\circ ; 150^\circ$ ✓ $k.360, k \in Z$ <p style="text-align: right;">(6)</p>
5.3	$\frac{\sin x - \sin x}{\cos^2 x}$ $\frac{-\sin^2 x}{\cos^2 x}$ $= -\tan x$	<ul style="list-style-type: none"> ✓ $\sin x$ ✓ $-\sin x$ ✓ $\cos^2 x$ ✓ $\frac{-\sin^2 x}{\cos^2 x}$ ✓ $\tan x$ <p style="text-align: right;">(5)</p>
5.4	$\frac{\sin^2 x + (1 + \cos x)^2}{\sin x(1 + \cos x)}$ $\frac{\sin^2 x + 1 + 2 \cos x + \cos^2 x}{\sin x(1 + \cos x)}$ $\frac{2 + 2 \cos x}{\sin x(1 + \cos x)}$ $\frac{2(1 + \cos x)}{\sin x(1 + \cos x)}$ $\frac{2}{\sin x}$	<ul style="list-style-type: none"> ✓ numerator / teller ✓ denominator / noemer ✓ expansion / uitbreiding ✓ simplification / vereenvoudiging ✓ factorisation / faktorisering <p style="text-align: right;">(5)</p>
[23]		
QUESTION/VRAAG 6: [11]		
6.1	$b^2 = a^2 + c^2 - 2ac \cos B$	<ul style="list-style-type: none"> ✓ $a^2 + c^2$ ✓ $2ac \cos B$ <p style="text-align: right;">(2)</p>
6.2.1	$\frac{PS}{RS} = \tan \hat{P}RS$ $PS = RS \cdot \tan 65^\circ$ $PS = 158 \cdot \tan 65^\circ$ $PS = 338,83$	<ul style="list-style-type: none"> ✓ Ratio for <i>tan/verhou. vir tan</i> ✓ substitution 65° / <i>verv van 65°</i> ✓ answer/<i>antwoord</i> <p style="text-align: right;">(3)</p>
6.2.2	<p>In ΔPQS</p> $SQ^2 = PS^2 + PQ^2 - 2PS \cdot SQ \cdot \cos 30^\circ$ $= 338,83^2 + 1500^2 - 2(338,83)(1500) \cdot \cos 30^\circ$ $= 1484499,606$ $SQ = 1218,40 \text{ m}^2$	<ul style="list-style-type: none"> ✓ use of <i>cos formula/gebruik van cos formule</i> ✓ Substitution/<i>vervang</i> ✓ SQ <p style="text-align: right;">(3)</p>

6.2.3	In $\triangle RSQ$: $\frac{RS}{SQ} = \tan \theta$ $\tan \theta = \frac{158}{1218,40}$ $\tan \theta = 0,129678266666$ $\theta = 7,39$	$\checkmark \frac{RS}{SQ} = \tan \theta$ $\checkmark \tan \theta$ \checkmark value of θ /waarde van θ (3)
6.2.4	Area of $\triangle SPQ = \frac{1}{2} SP \cdot PQ \cdot \sin \hat{P}$ $= \frac{1}{2} (338,83)(1\ 500)\sin 30^\circ$ $= 127061,25 \text{ m}^2$	\checkmark correct formula / korrekte formule \checkmark substitute / vervang (338,83), (1500) $\checkmark \sin 30^\circ$ \checkmark answer / antwoord (4)
[15]		

QUESTION/VRAAG 7: [12]

7.1		f \checkmark asymptotes / asimptote \checkmark min value / min waarde \checkmark max value / maks waarde g $\checkmark (-45^\circ; -1)$ $\checkmark (45^\circ; 1)$ $\checkmark (135^\circ; -1)$
(6)		
7.2	$x \in (-45; 0) \cup (45; 90) \cup (135; 180)$	$\checkmark -45$ $\checkmark 45$ $\checkmark 0$ $\checkmark 90$ $\checkmark 135$ $\checkmark 180$
(6)		
7.3	90°	\checkmark answer / antwoord (1)
[13]		

QUESTION/VRAAG 8: [8]		
8.1	bisects the chord.	✓ answer/antwoord (1)
8.2.1	$OD^2 = OF^2 + DF^2$ (Pythagoras) $= 3^2 + 4^2$ (substitution/vervang) $= 25$ $OD = 5 \text{ cm}$	✓ $OD^2 = OF^2 + DF^2$ ✓ method/metode ✓ answer/antwoord (3)
8.2.2	$AE^2 = AO^2 - OE^2$ (Pythagoras) $AE^2 = 5^2 - 4^2$ (substitution/vervang) $AE^2 = 9$ $AE = 3 \text{ cm}$ But $AB = 2AE$ ($OE \perp AB$) $AB = 2(3)$ $= 6 \text{ cm}$	✓ $AE^2 = AO^2 - OE^2$ method/metode ✓ $AE = 3 \text{ cm}$ ✓ S/R ✓ answer/antwoord (4)
[8]		
QUESTION/VRAAG 9: [13]		
9.1	 <p>CONSTR: Join CO, extend to D</p> <p>PROOF: In ΔAOC</p> <p>i) $\hat{O}_1 = \hat{A}_1 + \hat{C}_1$ (ext \angle of Δ/buitehoek van Δ) ii) $\hat{O}_2 = \hat{B}_2 + \hat{C}_2$ (ext \angle of Δ/buitehoek van Δ) iii) $\hat{O}_1 = 2\hat{C}_1$ ($AO = OC$) iv) $\hat{O}_2 = 2\hat{C}_2$ ($BO = OC$) $\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{C}_1 + 2\hat{C}_2$ $\therefore \hat{AOC} = 2(\hat{C}_1 + \hat{C}_2)$ $= 2\hat{ACB}$</p>	✓ construction/konstr ✓ S/R ✓ S/R ✓ S/R ✓ S/R ✓ conclusion / gevolgtrekking (6)
9.2.1	$\hat{D}_1 = 25^\circ$ (radii equal/radiusse gelyk)	✓ S ✓ R (2)
9.2.2	$\hat{O}_1 = 50^\circ$ (ext \angle of Δ /buitehoek van Δ)	✓ S ✓ R (2)
9.2.3	$\hat{A}_1 = 25^\circ$ (angles in same segment/ hoeke in dieselfde segment)	✓ S ✓ R (2)
9.2.4	$\hat{E} = 155^\circ$ (opp angles of cyclic quad/ teenoorstaande hoeke van 'n koordevierhoek)	✓ S ✓ R (2)
[14]		

QUESTION/VRAAG 10: [6]			
10.1	$\widehat{B}_1 = \widehat{C}_2 = x$ $\widehat{B}_2 = \widehat{C}_1 = x$	(angles in the same segment/ <i>hoeke in dieselfde segment</i>) (tan chord/ <i>tan koord</i>)	✓ S ✓ R ✓ S ✓ R (4)
10.2	$\widehat{C}_1 = \widehat{C}_2$ $\therefore DC$ bisects/ <i>halveer</i> \widehat{ACF}	(both equal to x / <i>albei gelyk aan x</i>)	✓ S/R ✓ conclusion / <i>gevolgtrekking</i> (2)
			[6]
QUESTION/VRAAG 11 [11]			
11.1	Are supplementary OR add to 180° .		✓ answer/ <i>antwoord</i> (1)
11.2.1	$\widehat{C}_2 = \widehat{A}$ $\widehat{A} = \widehat{D}_3 = x$ $\therefore MC = MD$	(Ext \angle of cyclic quad/ <i>buitehoek van koordevhk</i>) (corresponding angles, $AB \parallel DC$ / <i>Ooreenkomstige hoeke $AB \parallel DC$</i>) (base angles of Δ equal/ <i>basis hoeke van Δ gelyk</i>)	✓ S ✓ R ✓ S ✓ R ✓ R (5)
11.2.2	$\widehat{M} = 180^\circ - 2x$	(angles of Δ / <i>hoeke van Δ</i>)	✓ S ✓ R (2)
11.2.3	$\widehat{O}_1 = 2x$ $\widehat{M} + \widehat{O}_1 = 180^\circ$ $\therefore BODM$ is a cyclic quad. $\therefore BODM$ is koordevierhoek	(\angle at centre = $2 \angle$ at circumference/ \angle by middle = $2 \angle$ by omtreks)	✓ S ✓ R ✓ S (3)
			[11]
QUESTION/VRAAG 12 [11]			
12.1	$Sh^2 = 8^2 + 3^2$ $Sh^2 = 64 + 9$ $Sh = \sqrt{73}$ $= 8,54 \text{ cm}$	(Pythagoras) (substitution/ <i>vervang</i>)	✓ S ✓ method/ <i>metode</i> ✓ answer/ <i>antwoord</i> (3)
12.2	Area of Δ face = $\frac{1}{2} b \cdot h$ $= \frac{1}{2} (6)(\sqrt{73})$ $= 25,63 \text{ cm}^2$		✓ formula/ <i>formule</i> ✓ substitution/ <i>vervang</i> ✓ answer/ <i>antwoord</i> (3)
12.3	TSA = area of slanted faces + area of right prism $TBO =$ oppervlakte van skuinsvlakke + oppervlakte van reghoekige prisma $= 3(25,63) + 6^2 + 4(6 \times 12)$ $= 76,89 + 36 + 288$ $= 400,89 \text{ cm}^2$		✓✓ TSA ✓✓ substitute/ <i>vervang</i> ✓ answer/ <i>antwoord</i> (5)
			[11]
			TOTAL/TOTAAL: 150