



Province of the
EASTERN CAPE
EDUCATION

**NATIONAL
SENIOR CERTIFICATE /
NASIONALE
SENIOR SERTIFIKAAT**

GRADE 11 / *GRAAD 11*

NOVEMBER 2016

**MATHEMATICS P2 / *WISKUNDE V2*
MEMORANDUM**

MARKS / *PUNTE*: 150

This memorandum consists of 16 pages./
Hierdie memorandum bestaan uit 16 bladsye.

NOTE:

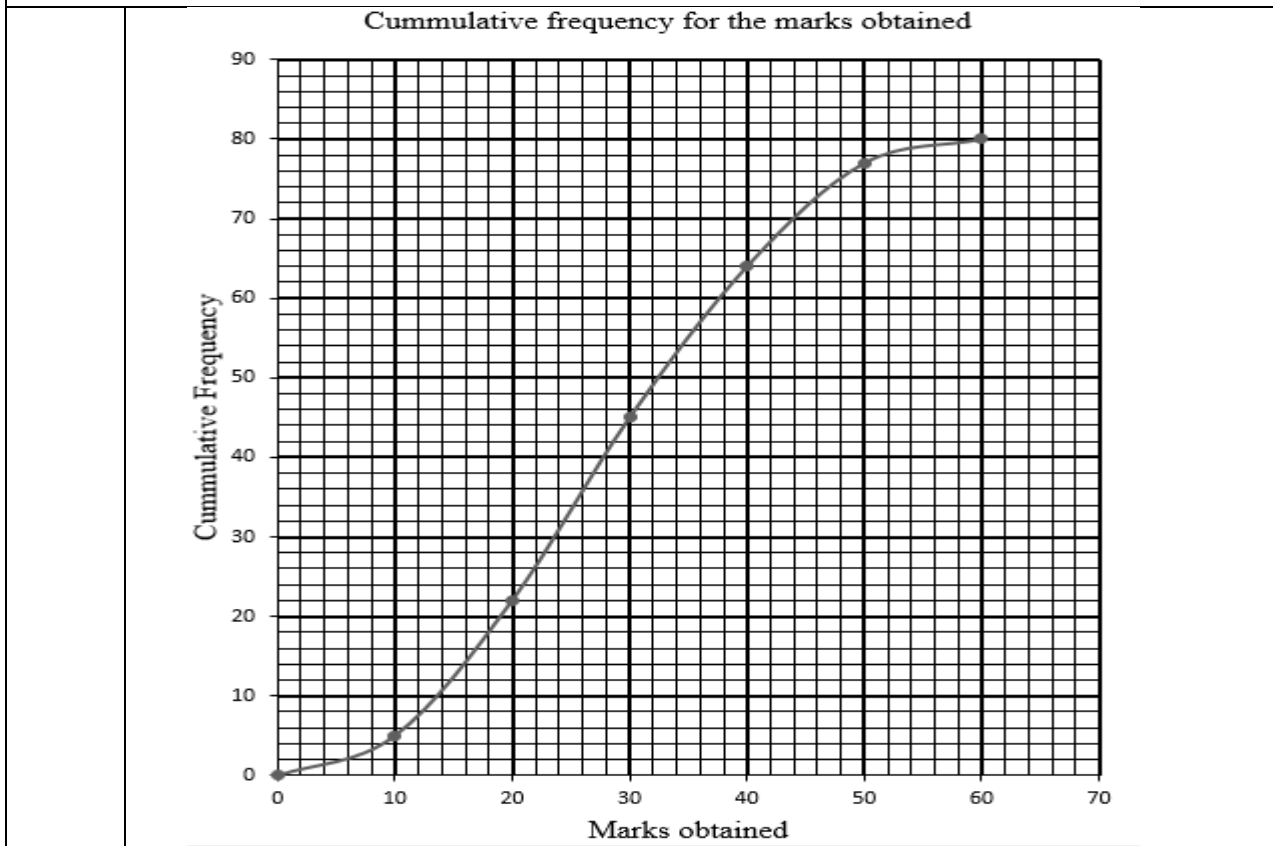
- If a candidate answers a question **TWICE**, only mark the **FIRST** attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in **ALL** aspects of the marking memorandum.
- Assuming answers/values in order to solve a problem is **NOT** acceptable.

LET WEL:

- Indien 'n kandidaat 'n vraag twee keer beantwoord, merk slegs die eerste poging.
- Indien 'n kandidaat 'n antwoord doodgetrek het, maar nie oorgedoen het nie, merk die doodgetrekte antwoord.
- Volgehoue akkuraatheid geld in ALLE aspekte van die memorandum.
- Aanname van antwoorde/waardes om 'n probleem op te los, is Onaanvaarbaar.

QUESTION 1 / VRAAG 1															
12	15	15	19	22	23	26	26	32	33	33	33	33	35	35	
1.1.1	26										✓ answer / antwoord (1)				
1.1.2	IQR = 33-19 = 14		Answer only – full marks Slegs antwoord – volpunte						✓ Q ₁ ✓ Q ₂ ✓ answer / antwoord (3)						
1.1.3	$\bar{x} = \frac{392}{15} = 26,13$										Answer only – full marks Slegs antwoord – volpunte			✓ $\frac{392}{15}$ ✓ answer / antwoord (2)	
1.1.4	$\delta = 7.77$										✓ answer / antwoord ✓ (2)				
1.2	$(\bar{x} - \delta; \bar{x} + \delta) = (18,36 ; 33,90)$ 5 people that are outside one standard deviation of the mean 5 mense is buite een standaardafwyking vanaf die gemiddelde \therefore It will be / Dit sal: $\frac{5}{15} \times 100 = 33,33\%$										✓ interval / interval ✓ no. of people outside one SD / aantal mense buite een SA ✓ percentage / persentasie (3)				
[11]															

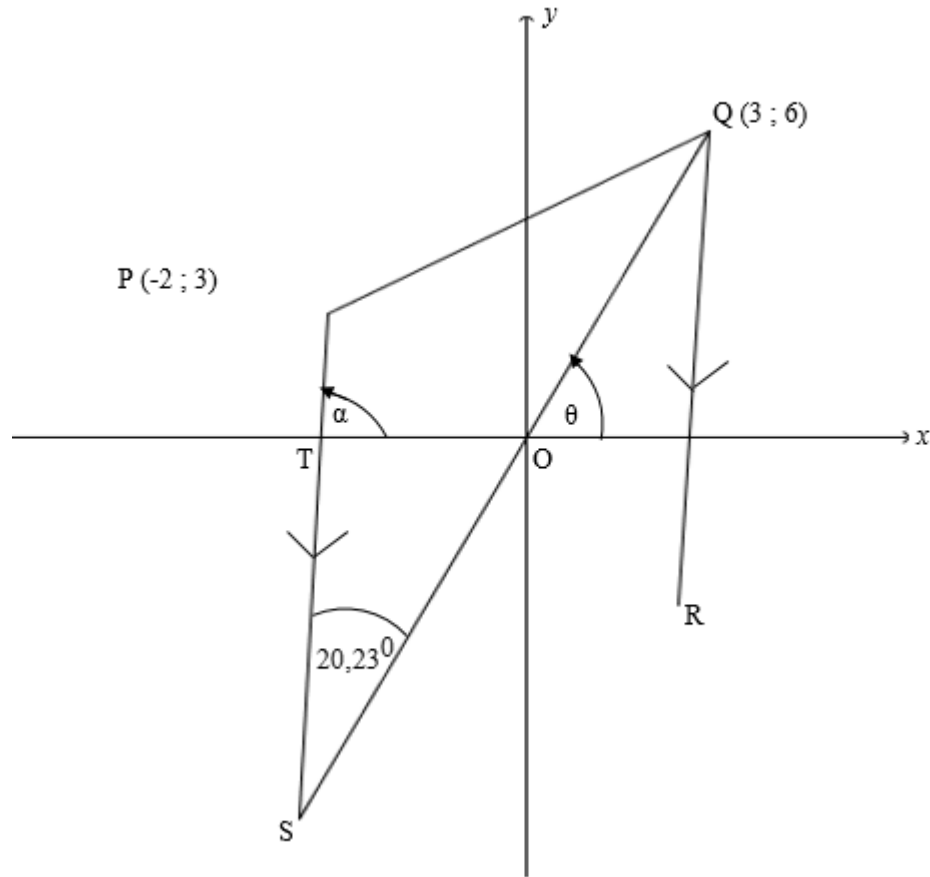
QUESTION 2 / VRAAG 2



2.1	80	✓ answer / antwoord (1)														
2.2	$80 - 22 = 58$	✓ $80 - 22$ ✓ answer / antwoord (2)														
2.3	$Q_2 = 28$	✓✓ answer / antwoord (2)														
2.4	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: center;">Marks obtained <i>Punte behaal</i></th> <th style="text-align: center;">Frequency <i>Frekwensie</i></th> </tr> </thead> <tbody> <tr><td style="text-align: center;">$0 < x \leq 10$</td><td style="text-align: center;">5</td></tr> <tr><td style="text-align: center;">$10 < x \leq 20$</td><td style="text-align: center;">17</td></tr> <tr><td style="text-align: center;">$20 < x \leq 30$</td><td style="text-align: center;">23</td></tr> <tr><td style="text-align: center;">$30 < x \leq 40$</td><td style="text-align: center;">19</td></tr> <tr><td style="text-align: center;">$40 < x \leq 50$</td><td style="text-align: center;">13</td></tr> <tr><td style="text-align: center;">$50 < x \leq 60$</td><td style="text-align: center;">3</td></tr> </tbody> </table>	Marks obtained <i>Punte behaal</i>	Frequency <i>Frekwensie</i>	$0 < x \leq 10$	5	$10 < x \leq 20$	17	$20 < x \leq 30$	23	$30 < x \leq 40$	19	$40 < x \leq 50$	13	$50 < x \leq 60$	3	✓ 17 ✓ 17 ✓ 23 ✓ 3 ✓ 19 ✓ 13 ✓ 3 (5) (2)
Marks obtained <i>Punte behaal</i>	Frequency <i>Frekwensie</i>															
$0 < x \leq 10$	5															
$10 < x \leq 20$	17															
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$30 < x \leq 40$	19															
$40 < x \leq 50$	13															
$50 < x \leq 60$	3															
2.5	$20 < x \leq 30$	✓ (1)														

[11]

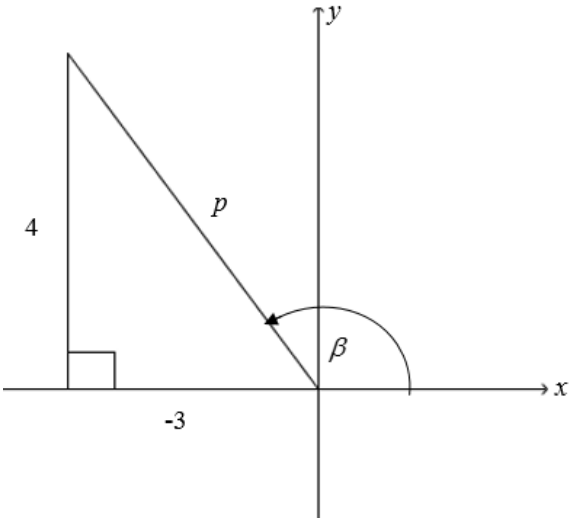
QUESTION 3 / VRAAG 3



3.1	$m_{QS} = \frac{6-0}{3-0} = 2$	<ul style="list-style-type: none"> ✓ subst. / vervang. ✓ gradient / gradiënt (2)
3.2	$\tan \theta = 2$ $\therefore \theta = 63,43^\circ$	<ul style="list-style-type: none"> ✓ $\tan \theta = m_{QS}$ ✓ $\theta = 63,43^\circ$ (2)
3.3		
3.3.1	$\widehat{TOS} = \theta = 63,43^\circ$ [vert. opp. \angle_s / regoorst. \angle^{ie}] $\alpha = 63,43^\circ + 20,23^\circ = 83,66^\circ$ [ext. \angle of a Δ / ext. \angle of a Δ / buite \angle van Δ] $m_{PS} = \tan(83,66^\circ)$ $= 9$	<ul style="list-style-type: none"> ✓ $\widehat{TOS} = 63,43^\circ$ ✓ $\alpha = 83,66^\circ$ ✓ $m_{PS} = \tan(83,66^\circ)$ ✓ $m_{PS} = 9$ (4)

3.3.2	$y - 3 = 9(x + 2)$ $y = 9x + 21$	<ul style="list-style-type: none"> ✓ $m_{PS} = 9$ ✓ subst. m and $(-2;3)$ / vervang. m en $(-2 ; 3)$ ✓ equation of PS / vergelyking van PS 	(3)
3.4	$9x + 21 = 2x$ $7x = -21$ $x = -3$ $y = 2(-3)$ $= -6$ $S(-3 ; -6)$	<ul style="list-style-type: none"> ✓ $9x + 21 = 2x$ ✓ simplification / vereenvoudiging ✓ x-value / x-waarde ✓ y-value / y-waarde 	(4)
3.5	$QS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(3 + 3)^2 + (6 + 6)^2}$ $= 6\sqrt{5}$	<ul style="list-style-type: none"> ✓ correct formula / korrekte formule ✓ correct subst. / korrekte vervanging ✓ $QS = 6\sqrt{5}$ 	(3)
3.6	$PS = \sqrt{(-2 + 3)^2 + (3 + 6)^2}$ $= \sqrt{82}$ $\text{Area } \Delta PQS = \frac{1}{2} \cdot QS \cdot PS \sin 20,23$ $= \frac{1}{2} \cdot 6\sqrt{5} \cdot \sqrt{82} \sin 20,23$ $= 21,01$	<ul style="list-style-type: none"> ✓ correct subst. / korrekte vervanging ✓ $PS = \sqrt{82}$ ✓ correct formula for Area of ΔPQS korrekte formule vir Opp. van ΔPQS ✓ correct subst. / korrekte vervang. ✓ 21,01 	(5)
3.7	$\frac{-2+x}{2} = 0; \frac{3+y}{2} = 0$ [PO = OR, diag. of a parm / hoeklyne van 'n ^m] $x = 2 \quad y = -3$ $R(2 ; -3)$ OF/OR Alternate proof $\frac{y-6}{x-3} = \frac{-6-3}{-3+2}$ $y - 6 = -9$ and $x - 3 = -1$ $y = -3$ and $x = -3$ $R(2;-3)$	<ul style="list-style-type: none"> ✓ equation for mid point / verg. van middelpunt. ✓ x-value/x-waarde ✓ y-value/y-waarde ✓ equation for gradient / verg. vir gradiënt. ✓ x-value/x-waarde ✓ y-value/y-waarde 	(3)

3.8	$m_{AB} = m_{BC} \quad (\text{collinear points have equal gradients})$ $\frac{4+1}{5-0} = \frac{2+1}{t-0} \quad (\text{ko-liniêre punte het gelyke gradiënte})$ $1 = \frac{3}{t}$ $\therefore t = 3$	$\checkmark m_{AB} = m_{BC}$ \checkmark substitution / vervanging $\checkmark m_{AB} = 1$ $\checkmark t = 3$ <div style="text-align: right;">(4)</div>
		[30]

QUESTION 4 / VRAAG 4			
4.1	4.1.1	$\sin \beta = \frac{4}{p} \text{ and/en}$ $\cos \beta = -\frac{3}{p}$ <p>β is in the 2nd quadrant where the ratio of $\sin \beta$ is positive and ratio of $\cos \beta$ is negative. / β is in die 2^{de} kwadrant waar die verhouding van $\sin \beta$ positief is en die verhouding van $\cos \beta$ negatief is.</p>	<ul style="list-style-type: none"> ✓ $\sin \beta = \frac{4}{p}$ ✓ $\cos \beta = -\frac{3}{p}$ ✓ Any relevant explanation./ <i>Enige relevante verduideliking</i> <p style="text-align: right;">(3)</p>
	4.1.2	$\tan \beta = \frac{\sin \beta}{\cos \beta}$ $= \frac{4}{p} \times -\frac{p}{3}$ $= -\frac{4}{3}$ <p>OR/OF</p> 	<ul style="list-style-type: none"> ✓ identity / <i>identiteit</i> ✓ $\frac{4}{p} \times -\frac{p}{3}$ <p style="text-align: right;">(2)</p> <p>OR/OF</p> <ul style="list-style-type: none"> ✓ diagram / <i>diagram</i> ✓ <p style="text-align: right;">(2)</p>
	4.1.3	$p = \sqrt{(-3)^2 + 4^2}$ $= 5$	<ul style="list-style-type: none"> ✓ subst. in Pyth. / <i>vervang. in Pyth.</i> ✓ value of p / <i>waarde van p</i> <p style="text-align: right;">(2)</p>

4.2	$\begin{aligned} & \sin 143^\circ \cdot \cos 127^\circ - \sin 53^\circ \cdot \cos(37^\circ) \\ &= \sin(180^\circ - 37^\circ) \cdot \cos(90^\circ + 37^\circ) - \sin(90^\circ - 37^\circ) \cdot \cos 37^\circ \\ &= \sin 37^\circ \cdot (-\sin 37^\circ) \cdot (-\cos 37^\circ) \cdot \cos 37^\circ \\ &= -\sin^2 37^\circ - \cos^2 37^\circ \\ &= -(\sin^2 37^\circ + \cos^2 37^\circ) \\ &= -1 \end{aligned}$	<ul style="list-style-type: none"> ✓ $\sin 37^\circ$ ✓ $-\sin 37^\circ$ ✓ $-\cos 37^\circ$ ✓ $-(\sin^2 37^\circ + \cos^2 37^\circ)$ ✓ -1 <p style="text-align: right;">(5)</p>
4.3	$\begin{aligned} \text{L.H.S/LK.} &= \left(\tan y + \frac{1}{\tan y} \right) (1 - \cos^2 y) \\ &= \left(\frac{\sin y}{\cos y} + \frac{\cos y}{\sin y} \right) (\sin^2 y) \\ &= \left(\frac{\sin^2 y + \cos^2 y}{\cos y \cdot \sin y} \right) \sin^2 y \\ &= \frac{1}{\cos y \cdot \sin y} \cdot \sin^2 y \\ &= \frac{\sin y}{\cos y} \\ &= \tan y \end{aligned}$	<ul style="list-style-type: none"> ✓ $\frac{\sin y}{\cos y}$ ✓ $\frac{\cos y}{\sin y}$ ✓ $\sin^2 y$ ✓ $\frac{\sin^2 y + \cos^2 y}{\cos y \cdot \sin y}$ ✓ 1 ✓ $\frac{\sin y}{\cos y}$ <p style="text-align: right;">(6)</p>
4.4	$\begin{aligned} \cos \theta - \frac{1}{\cos \theta} &= \frac{5}{6} \\ 6 \cos^2 \theta - 5 \cos \theta - 6 &= 0 \\ (3 \cos \theta + 2)(2 \cos \theta - 3) &= 0 \\ \cos \theta = -\frac{2}{3} \text{ or/of } \cos \theta = \frac{3}{2} &\text{ no solution/geen oplossing} \\ \theta = \pm 131,81^\circ + 360^\circ \cdot k; k \in Z \end{aligned}$	<ul style="list-style-type: none"> ✓ standard form <i>standaardvorm</i> ✓ factors / <i>faktore</i> ✓ both equations <i>beide oplossings</i> ✓ no solution / <i>geen oplossing</i> ✓ $\theta = \pm 131,81^\circ$ ✓ $360^\circ \cdot k$ and/en $k \in Z$ <p style="text-align: right;">(6)</p>
[24]		

QUESTION 5 / VRAAG 5		
5.1		<p><i>f:</i></p> <ul style="list-style-type: none"> ✓ <i>x</i>-intercepts ✓ <i>x</i>-afsnit ✓ min. and max. <i>min. en maks.</i> ✓ shape / vorm <p><i>g:</i></p> <ul style="list-style-type: none"> ✓ intercepts / afsnitte ✓ asymptotes <i>asimptote</i> ✓ shape / vorm
5.2	$-90^{\circ} < x \leq 45^{\circ}$	<ul style="list-style-type: none"> ✓ correct critical value / <i>korrekte kritiese waarde</i> ✓ notation / <i>notasie</i> <p style="text-align: right;">(2)</p>
5.3	$h(x) = -\{-\cos(45^{\circ} - x + 45^{\circ})\}$ $= -\{-\cos(90^{\circ} - x)\}$ $= \sin x$	<ul style="list-style-type: none"> ✓ subst. / <i>vervang.</i> ✓ $\sin x$ <p style="text-align: right;">(2)</p>
		[10]

QUESTION 6 / VRAAG 6		
6.1	$c^2 = a^2 + b^2 - 2ab \cos \hat{C}$	✓ answer / antwoord (1)
6.2		
6.2.1	$UR^2 = TU^2 + TR^2 - 2TR \cdot TU \cdot \cos \hat{T}$ $= 16^2 + 12^2 - 2(16)(12) \cos 41^\circ$ $= 110,191$ $\therefore UR = 10,50$	✓ cosine rule / <i>cosinusreël</i> ✓ correct subst./ <i>korrekte vervang.</i> ✓ $UR = 10,50$ (3)
6.2.2	$\frac{\sin \hat{U}_2}{12} = \frac{\sin \hat{T}}{10,50}$ $\sin \hat{U}_2 = \frac{12 \cdot \sin 41^\circ}{10,5}$ $\hat{U}_2 = 48,57^\circ$ <p style="text-align: center;">OR/OF</p> $TR^2 = UR^2 + TU^2 - 2 \cdot UR \cdot TU \cdot \cos \hat{U}_2$ $(12)^2 = (10,50)^2 + (16)^2 - 2(10,50)(16) \cos \hat{U}_2$ $\cos \hat{U}_2 = \frac{(10,5)^2 + (16)^2 - (12)^2}{2(10,50)(16)}$ $\hat{U}_2 = 48,57^\circ$	✓ use of sine rule / <i>gebruik van sinusreël</i> ✓ correct subst./ <i>korrekte vervang.</i> ✓ \hat{U}_2 (3) ✓ use of cosine rule <i>gebruik van cosinusreël</i> ✓ correct substitution <i>korrekte vervanging</i> ✓ \hat{U}_2 (3)

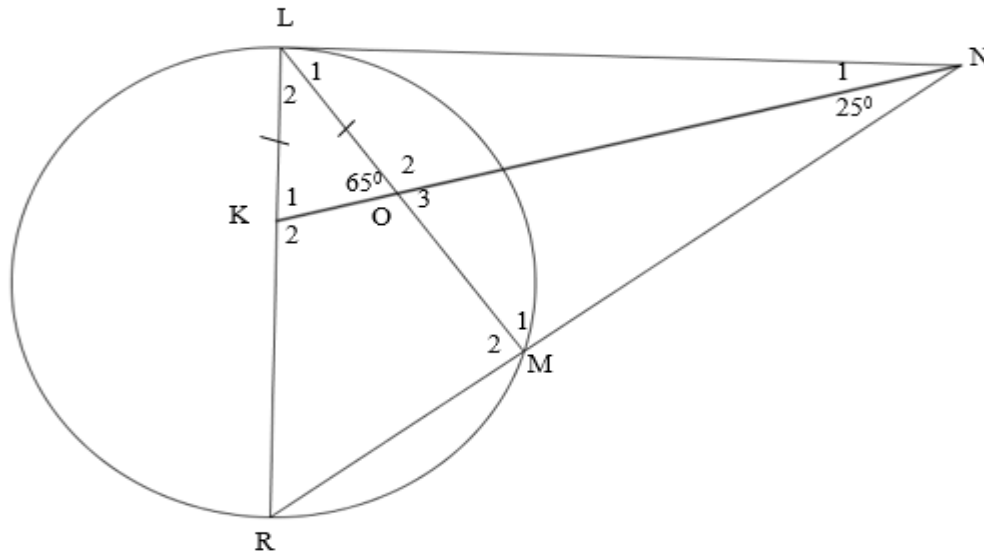
6.2.3	$\hat{U}_3 = 41^\circ$ [tan chord theorem] [raaklyn koord stelling] $\therefore \hat{U}_{2+3} = 89,57^\circ$ $\hat{S} = 49,43^\circ$ [\angle_s in Δ URS] $\frac{RS}{\sin 41^\circ} = \frac{10,5}{\sin 49,43^\circ}$ $RS = 9,07$ $\therefore \hat{TRS} = 21,07$	✓ S ✓ S ✓ S ✓ correct subst. in sine rule / <i>korrekte vervang. in sinusreël</i> ✓ \hat{TRS} (5)
		[12]

QUESTION 7 / VRAAG 7		
7.1	<p>Surface area of a hemisphere: <i>Buite oppervlak van hemisfeer:</i></p> $= 2\pi r^2$ $= 2 \times \pi \times 10^2 = 628,32 \text{ m}^2$ <p>or/of</p> $= 2\pi rh$ $= 2 \times \pi \times 10 \times 10$ $= 628,32 \text{ m}^2$ <p>Surface area of a cylinder: <i>Buite oppervlak van silinder:</i></p> $= 2 \times \pi \times 10 \times 75 + \pi \times 10^2 = 5026,55 \text{ m}^2$ <p>\therefore TSA of the tank / <i>TBO van die tenk</i> $= 5654,87 \text{ m}^2$</p>	<p>✓ correct subst./korrekte vervang.</p> <p>✓ Area of hemisphere / <i>Oppervlakte van hemisfeer</i></p> $\text{Using } \pi = 3,14$ $\text{SA/BO} = 628 \text{ m}^2$ <p>✓ correct subst./korrekte vervang.</p> <p>✓ Area of a cylinder / <i>Oppervlakte van silinder</i></p> $\text{Using } \pi = 3,14$ $\text{SA/BO} = 5024 \text{ m}^2$ <p>✓ TSA / TBO</p> $\text{Using } \pi = 3,14$ $\text{TSA/TBO} = 5652 \text{ m}^2$ <p style="text-align: right;">(5)</p>
7.2	<p>V. hemisphere/V. van hemisfeer</p> $= \frac{2}{3} \times \pi \times 10^3 = 2094,40 \text{ m}^3$ <p>V. cylinder/V. van silinder</p> $= \pi \times 10^2 \times 75 = 23561,94 \text{ m}^3$ <p>V. of the tank/V. van die tenk $= 25656,34 \text{ m}^3$</p>	<p>✓ correct subst./korrekte vervang.</p> <p>✓ V. of hemi. / V. van hemisfeer</p> $\text{Using } \pi = 3,14$ $\text{Volume} = 2\,093 \text{ m}^3$ <p>✓ Correct subst./korrekte vervang.</p> <p>✓ V. of cylinder / V. van silinder</p> $\text{Using } \pi = 3,14$ $\text{V}_{\text{cyl/sil}} = 23\,550 \text{ m}^3$ <p>✓ V. of the tank / V. van tenk</p> $\text{Using } \pi = 3,14$ $\text{Vol}_{\text{tank/tenk}} = 25\,643 \text{ m}^3$ <p style="text-align: right;">(5)</p>
[10]		

QUESTON 8 / VRAAG 8

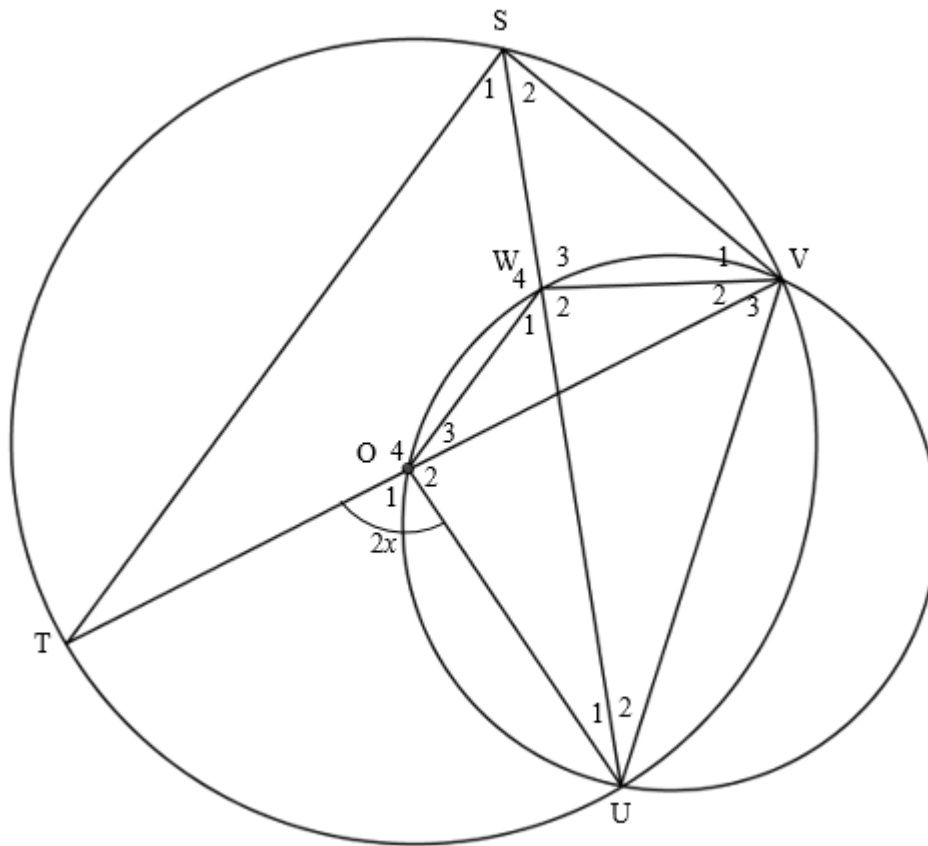
8.1	$\hat{D}_1 = 90^\circ$ [line from centre to midpoint of chord] [lyn vanaf middelpnt van sirkel na middelpnt van koord]	✓ S ✓ R (2)
8.2	$OA^2 = OD^2 + AD^2$ $= (3 \text{ cm})^2 + (4 \text{ cm})^2$ $\therefore OA = 5 \text{ cm}$	✓ subst. in Pyth. vervang. in Pyth ✓ answer/ antwoord (2)
8.3	$\hat{A} = 90^\circ$ [tan \perp rad] / [raaklyn \perp radius]	✓ S ✓ R (2)
8.4	$(x+5)^2 = (x+4)^2 + 5^2$ $x^2 + 10x + 25 = x^2 + 8x + 16 + 25$ $2x = 16$ $x = 8$	✓ subst. into Pyth / vervang. in Pyth ✓ simplification / vereenvoudiging ✓ standard form / standaardvorm ✓ x-value / x-waarde (4)
		[10]

QUESTION 9 / VRAAG 9



9.1	$\hat{O}_3 = 65^\circ$ [vert. opp \angle s] / [regeorst. \angle e] $\hat{M}_1 = 90^\circ$ [sum of \angle s of a Δ] / [som van \angle e van Δ] \therefore LN is a diameter to circle LMN [chord subtends 90° or converse \angle in a semi-circle] \therefore LN is 'n middellyn van sirkel LMN [koord onderspan 90° of omgekeerde van \angle in semi-sirkel]	\checkmark S \checkmark R \checkmark S \checkmark R \checkmark R	(5)
9.2	$\hat{N}_1 = 25^\circ$ [ON bisects \hat{N}] / [ON halveer \hat{N}] $\hat{L}_1 = 40^\circ$ [ext \angle of a Δ LON = sum of int opp \angle s] / [buite \angle van Δ LON = som vanteenorst. binne \angle e] $\hat{K}_1 = 65^\circ$ [\angle s opp = sides] / [\angle e teenoor gelyke sye] $\hat{L}_2 = 50^\circ$ [sum of \angle s of a Δ] / [som van \angle e van 'n Δ] $\therefore \hat{L} = 90^\circ$	\checkmark S/R \checkmark S/R \checkmark S \checkmark R \checkmark S/R	(5)
9.3	$\hat{R} = 40^\circ$ [sum of \angle s of a Δ] / [som van \angle e van 'n Δ] $\therefore \hat{R} = \hat{L}_1$ \therefore LN is a tangent [\angle between a line and a chord] / [\angle tussen lyn en koord]	\checkmark S \checkmark R \checkmark R	(3)
9.4	Yes. LR subtends 90° OR $\hat{M}_2 = 90^\circ$ [LR is a diameter] JA. LR onderspan 90° OF $\hat{M}_2 = 90^\circ$ [LR is 'n middellyn]	\checkmark Yes / Ja \checkmark Reason / Rede	(2)
			[15]

QUESTION 10 / VRAAG 10



10.1	$\hat{S}_1 = x$	$[\angle \text{ at centre} = 2 \times \angle \text{ at circumf.}]$ $[\text{Middelpts } \angle = 2 \times \text{Omtreks } \angle]$	✓S	✓R
	$\hat{V}_3 = x$	$[\angle \text{ s in same segment }]/[\angle \text{ e in dieselfde segment}]$	✓S	✓R
	$\hat{U} = x$	$[\text{ext. } \angle \text{ of a } \Delta \text{ OVU} = \text{sum of opp int } \angle \text{ s}]$ $[\text{buite } \angle \text{ van } \Delta \text{ OVU} = \text{som van teenoorst. binne } \angle \text{ e}]$	✓S	✓R
	$\hat{W}_1 = \hat{V}_3 = x$	$[\angle \text{ s in same segment }]/[\angle \text{ e in dieselfde segment}]$	✓S	✓R (8)
10.2	$\hat{O}_2 = 180^\circ - 2x$	$[\angle \text{ s on a straight line }]/[\angle \text{ e op 'n reguitlyn}]$	✓S	✓R
	$\hat{W}_2 = \hat{O}_2 = 180^\circ - 2x$	$[\angle \text{ s in same segment }]/[\angle \text{ e in dieselfde segment}]$	✓S	✓R (4)

10.3	$\hat{S}_{1+2} = 90^\circ$ [∠ in a semi – circle] / [∠ in semi – sirkel] $\therefore \hat{S}_2 = 90^\circ - x$ $\hat{V}_1 = \hat{W}_2 - \hat{S}_2$ [ext. ∠ of a Δ] / [buite ∠ van 'n Δ] $= 180^\circ - 2x - (90^\circ - x)$ $= 90^\circ - x$ $\therefore SW = WV$ [sides opp. = ∠s] / [syte teenoor gelyke hoeke]	✓S ✓R ✓S ✓R ✓R (5)
		[17]
TOTAL/TOTAAL:		150